OCR
Oxford Cambridge and RSA

H

Tuesday 03 November 2020 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

8182187365

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

J560/04

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

.CG Maths.

Answer **all** the questions.

1 **(a)** Write these numbers in standard form.

(i) 6500

Entering 6500 into the calculator and converting it into ENG notation puts it in standard form, in this case

(a)(i) ..... 6.5×10^{3} ..... **[1]**

(ii) 0.0584

0.0584 must be multiplied by 10 twice to get a number between 1 and 10. So the result must be multiplied by 10^{-2}

(ii) ..... 5.84×10^{-2} ..... **[1]**

(b) Work out $(4.2\times10^{5})\times(1.8\times10^{-2})$, giving your answer in standard form.

Putting it into the calculator and converting the answer into ENG notation puts it in standard form, in this case

(b) ..... 7.56×10^{3} ..... **[1]**

2 James is taking three examination papers in Spanish.
Here are his first two results.

Paper 1: $\frac{43}{80}$ Paper 2: $\frac{38}{65}$

Paper 3 is out of 95.
The marks in each of the three papers are added together.

Find the lowest mark that James needs in Paper 3 to achieve 60% of the total marks.

$(95+80+65)\times\frac{60}{100}-43-38$

Paper 1 can be assumed to be out of 80 marks and Paper 2 can be assumed to be out of 65 marks. Adding together the number of marks each of the papers is out of works out the total number of marks the whole exam is out of. Putting the 60 over 100 converts the percentage into a fraction, which when multiplied by the total number of marks the whole exam is out of finds 60% of the total marks. Subtracting the 43 and 38 marks achieved so far on Paper 1 and Paper 2 leaves the number of marks needed on Paper 3 to get 60% of the total marks

..... 63 ..... **[4]**

3 Three people take $2\frac{1}{2}$ hours to deliver leaflets to 270 houses.

Assuming all people deliver leaflets at the same rate, how long will it take five people to deliver leaflets to 405 houses?
Give your answer in hours and minutes.

$\frac{3 \times 2\frac{1}{2}}{270} \times 405$

5

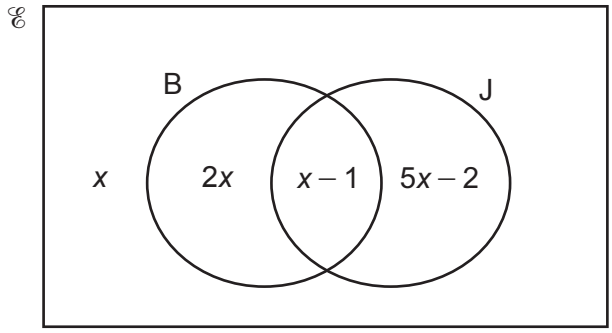
Multiplying the 3 people by the $2^1/_2$ hours works out how many hours worth of work was done. Dividing this by the 270 houses works out how many hours worth of work are needed for each house. Multiplying this by the 405 works out how many hours worth of work are needed for 405 houses. Dividing this by the 5 people works out how long it will take in hours

The answer of 2.25 hours can be converted into hours and minutes using the calculator

.....2..... hours .....15..... minutes **[4]**

4 In a survey, 60 students were asked whether they have a bank account (B) and whether they have a part-time job (J).

The number of students who had neither a bank account nor a part-time job was x.
The Venn diagram shows the results in terms of x.



One of the 60 students is chosen at random.

Find the probability that they have a bank account.
Show your working.

$x+2x+x-1+5x-2$

Adding together all of the expressions in the Venn diagram expresses the total number of students in terms of x

$9x-3=60$

Simplifying the expression by collecting like terms and setting equal to the 60

$9x=63$
$x=7$

Adding 3 to both sides then dividing both sides by 9 to solve x

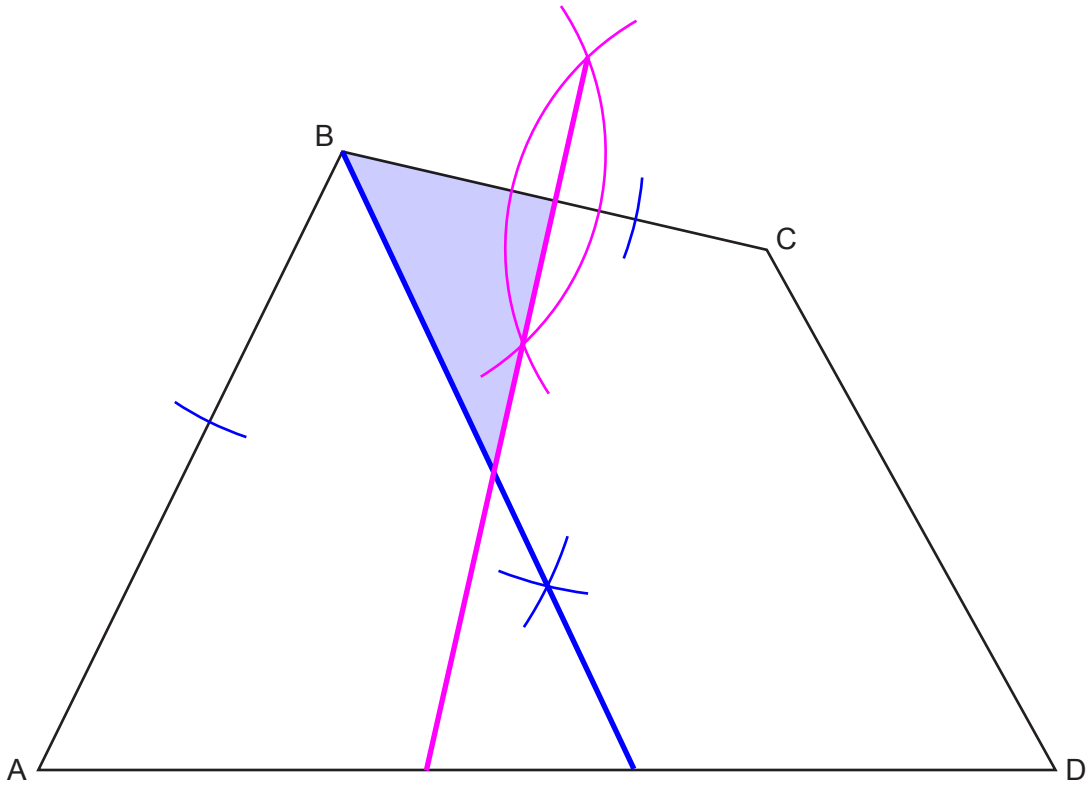
$2x7+7-1$

Substituting 7 for x in 2x + x - 1, the expressions within the B ring added together finds that 20 students have a bank account

$\frac{20}{60}$ ..... **[5]**

20 out of the 60 students have a bank account

5 ABCD is a quadrilateral.



For (a): Scribe two arcs from B using a compass and the same radius. Scribe two arcs of the same radius from the points these arcs cross AB and BC. Draw a straight line from B through the cross where the two second arcs meet.

For (b): Scribe an arc from B which is at more than half of the length of BC. Scribe an arc from C using the same radius. Draw a straight line through the two points where these arcs meet

(a) Construct the bisector of angle ABC.
Show all your construction lines.

Shown in blue

[2]

(b) Construct the perpendicular bisector of BC.
Show all your construction lines.

Shown in pink

[2]

(c) Shade the region which is

- nearer to BC than to AB

and

- nearer to B than to C.

Shaded in blue

[1]

6 A cuboid measures 6 cm by 8 cm by 15 cm.
A cube has the same volume as the cuboid.

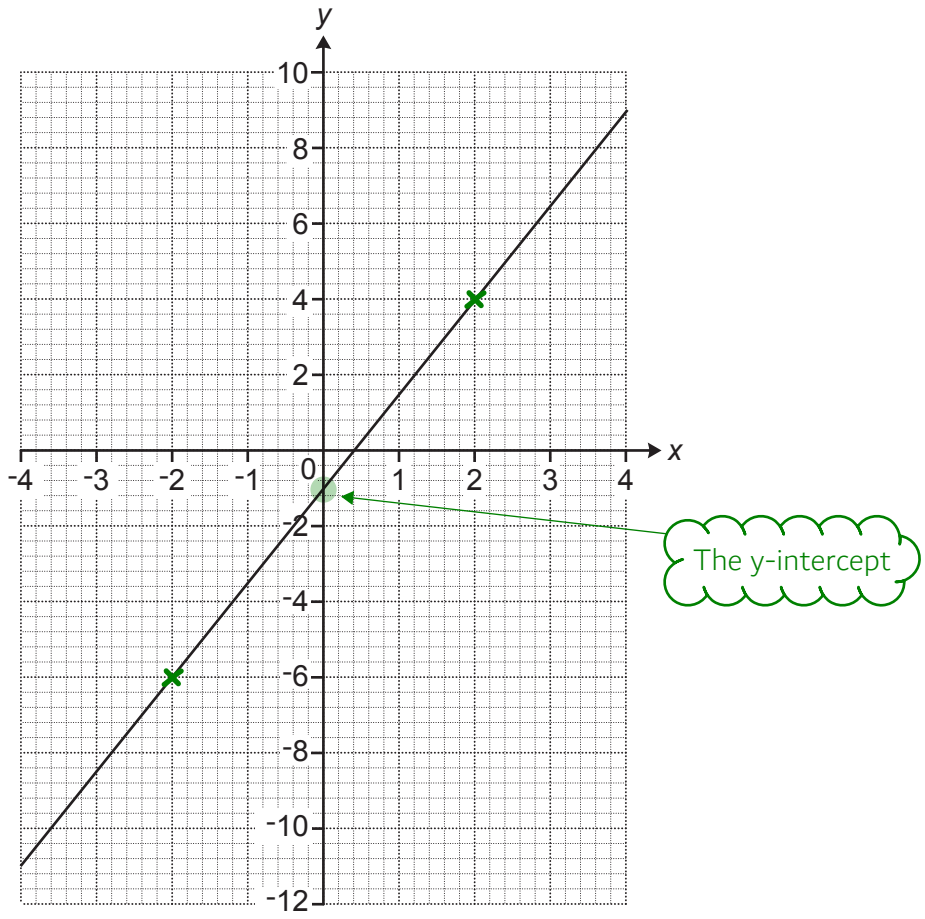
Find the surface area of the cube, giving your answer correct to 3 significant figures.

$(\sqrt[3]{6\times8\times15})^2\times6$

Volume of cuboid = length x width x height. So 6 x 8 x 15 expresses the volume of the cuboid and the cube. Volume of cube = length^3 so cube rooting the volume works out the length of one of the sides on the cube. Each of the faces on the cube is a square. Area of square = length^2 so squaring the side length expresses the area of one of the faces. There are 6 square faces on a cube so multiplying the area of one of the square faces by 6 works out the surface area of the cube

.....482..... cm^2 **[4]**

7 This graph shows part of a straight line.



(a) Show that the gradient of the line is 2.5. **[1]**

$\frac{4--6}{2--2}=2.5$

Gradient = (change in y)/(change in x). Working with the two points marked with green crosses. Change in y is the y-coordinate of the first cross subtracted from the y-coordinate of the second cross. Change in x is the x-coordinate of the first cross subtracted from the x-coordinate of the second cross

(b) Write down the equation of the line.

The general equation of a straight line is y = mx + c, where m is the gradient and c is the y-intercept. The gradient is 2.5 and the y-intercept is -1

(b) $y=2.5x-1$ **[2]**

8 Lily buys and sells microwaves.

She buys each one for £32 and sells it for £60.
She also pays £7 for the delivery of each microwave she sells.

If she sells a microwave that is faulty then Lily must pay for its repair and redelivery.
This costs her another £25 for each faulty microwave.

Last month, 6 out of the 80 microwaves Lily sold were faulty.

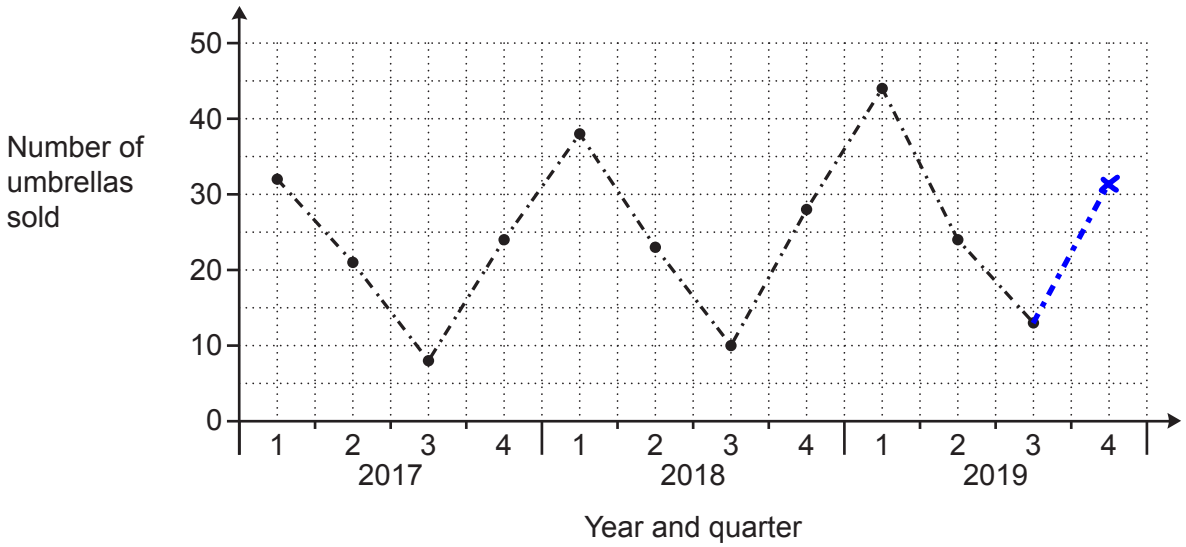
This month she has orders for 133 microwaves.

Calculate her expected percentage profit on this month's order.
Showing your working in the boxes below may help you present your work.

Expected number of faulty microwaves: $\frac{6}{80} \times 133 \rightarrow 10$ Expressing the fraction of the microwaves which were faulty last month gives 6/80. We can estimate that the same fraction of microwaves will be faulty this month. So doing 6/80 of the 133 gives 9.975, which rounds to 10 to the nearest whole number	Expected costs: $(32+7) \times 133 + 10 \times 25 = 5437$ Adding the cost of buying each microwave (£32) and the delivery cost (£7) gives the total initial cost of each microwave sold. Multiplying this by the 133 orders works out the total initial cost of all of the microwaves. Multiplying the 10 faulty microwaves by the £25 cost to repair and redeliver each one gives the total cost of the faulty microwaves. Adding the total initial cost and the cost of the faulty ones gives the total expected costs
Income from sales: $60 \times 133 = 7980$ Multiplying the price they are sold for (£60) by the number of orders (133) gives the total income from the sales	Expected percentage profit: $\frac{7980 - 5437}{5437} \times 100$ Subtracting the costs from the income gives the profit. Expressing this as a fraction of the costs gives the fraction profit. Multiplying this by 100 converts it into a percentage profit

46.8 ..... % **[6]**

9 The graph shows the number of umbrellas sold in Ling's shop for each quarter from quarter 1 of 2017 to quarter 3 of 2019.



(a) The shop sold 32 umbrellas in quarter 4 of 2019.

Complete the graph. **[1]**

(b) Make one comment about the **seasonal** variation shown in this graph.

Peaks in quarter 1

..... **[1]**

(c) Make one comment about the **annual** variation shown in this graph.

Generally increases each year

..... **[1]**

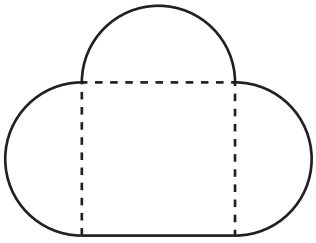
(d) Ling predicts that she will sell 50 umbrellas in quarter 1 of 2020.

What assumption has she made?

The sales will continue to increase at a similar rate

..... **[1]**

10 The diagram shows Jane's lawn.
It is in the shape of a square of side 36 m and three semi-circles.



Not to scale

She is going to spread fertiliser on the lawn at a rate of 30 g per square metre.
The fertiliser is only sold in 10 kg bags costing £15.80 each.

Calculate the cost of buying the bags of fertiliser for her lawn.
You must show all your working.

Area of square = length^2

Area of circle = π x radius^2. The radius is half of the diameter of 36m. Dividing the are of the full circle by 2 works out the area of each semicircle. Multiplying this by 3 as there are 3 semicircles

There are 1000g in 1kg. So dividing the 30g by 1000 converts it into kilograms

$$\frac{\left(36^2+3\left(\frac{\pi\times\left(\frac{36}{2}\right)^2}{2}\right)\right)\times\frac{30}{1000}}{10}$$

Adding the area of the square and the 3 semicircles works out the total area of the garden. Multiplying this by the 30/1000 kg works out how much fertiliser is needed. Dividing this by the 10kg bags works out how many bags are needed

9X15.80

8.46... bags is rounded up to 9 bags. Multiplying this by the cost of each bag works out the cost of buying the fertiliser

£ ..... 142.20 ..... **[6]**

11 **(a)** The length, d, of Jamal's car is 4.72 m, correct to 2 decimal places.

Complete the error interval for the length, d.

$4.72 \pm \frac{0.01}{2}$

Adding and subtracting half of the resolution (what it goes up in) works out the upper and lower bound. The resolution is 0.01 as it is correct to 2 decimal places

(a) ..... 4.715 ..... $\leqslant d <$ ..... 4.725 ..... **[2]**

(b) Jamal travels 430 km, correct to the nearest 10 km.
His average speed is 57.3 km/h, correct to 1 decimal place.

Calculate the shortest possible time for Jamal's journey.
Give your answer correct to the nearest minute.

$S \; \frac{d}{t}$

Writing the formula triangle for distance, speed, time

$\frac{430 - \frac{10}{2}}{57.3 + \frac{0.1}{2}}$

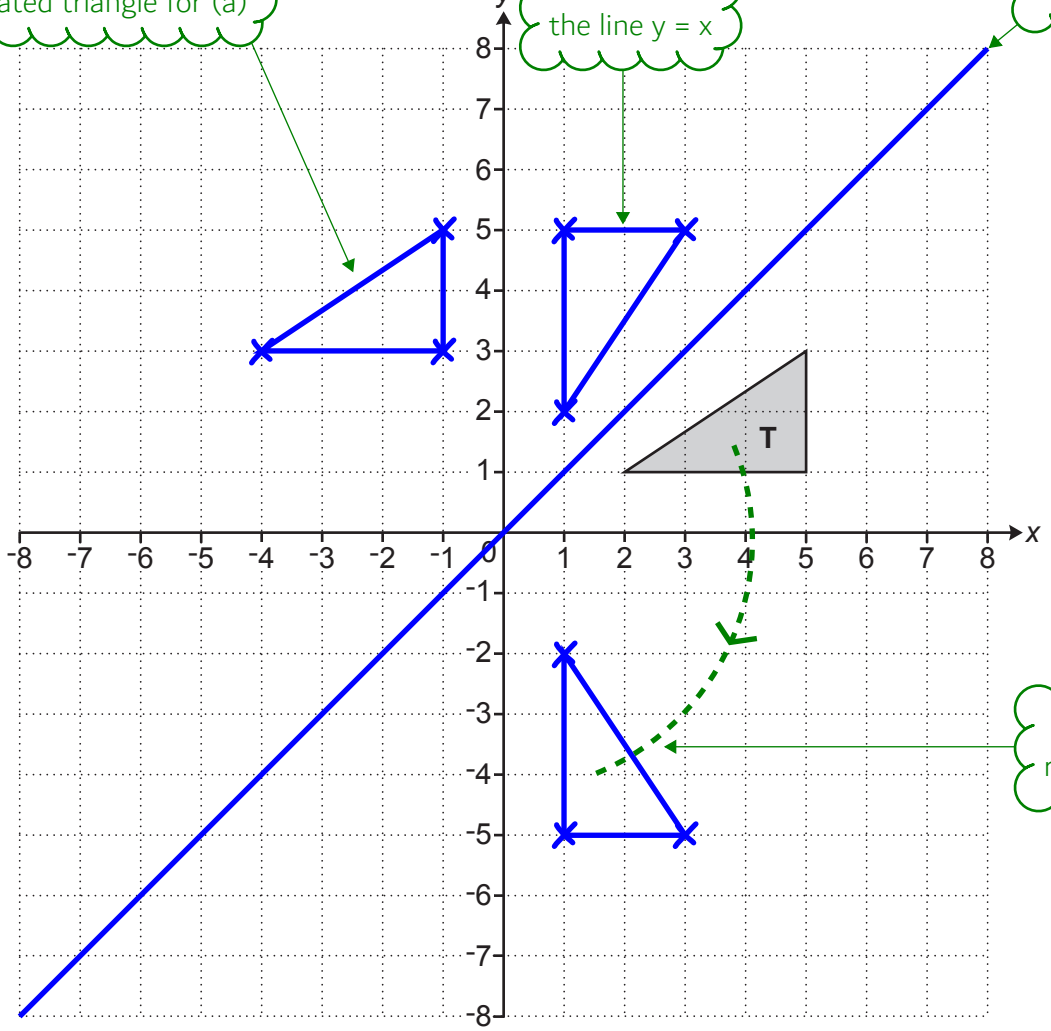
From the formula triangle, covering t tells us that time = distance/speed. In order to get the lower bound of the time, the lower bound of the distance and the upper bound of the speed should be used. To get the lower bound of the distance, half of the resolution is subtracted from the 430. The resolution is 10 as this is what it is to the nearest. To get the upper bound of the speed, half of the resolution is added to the 57.3. The resolution is 0.1 as it is correct to 1 decimal place

The calculator can be used to give the 7.41... hours in time. The result is 7°24'38.29", which means 7 hours 24 minutes 38.29 seconds

The answer rounds to 7 hours 25 minutes to the nearest minute as 38 seconds is over half a minute and causes the minutes to round up

(b) ..... 7 ..... hours ..... 25 ..... minutes **[5]**

12 Triangle **T** is drawn on a coordinate grid.



(a) Translate triangle **T** by vector $\begin{pmatrix} -6 \\ 2 \end{pmatrix}$. 6 to the left and 2 up **[2]**

(b) Describe fully the **single** transformation that is equivalent to:

- a reflection in the line $y = x$, followed by
- a reflection in the x-axis.

To reflect, count the number of perpendicular jumps to the line and do the same number on the other side for each corner then join the corners up

You may use the grid above to help you.

Rotation 90° clockwise centre (0 , 0)

Tracing paper can be used to work out the centre of rotation **[3]**

13 Ali and Beth take it in turns to play a computer game.
On each turn, the player achieves a score out of 50.
Ali and Beth play the computer game many times and record their scores.

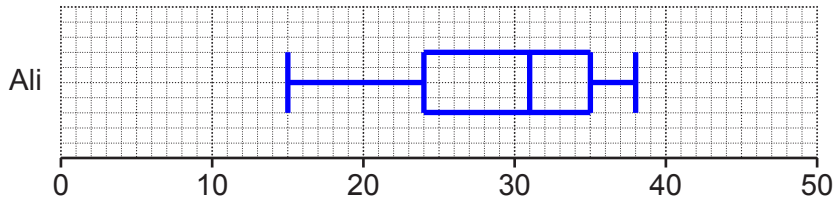
(a) Ali's scores are summarised below.

- median = 31
- highest score = 38
- range = 23
- lower quartile = 24
- interquartile range = 11

Draw a box plot to show the distribution of Ali's scores.

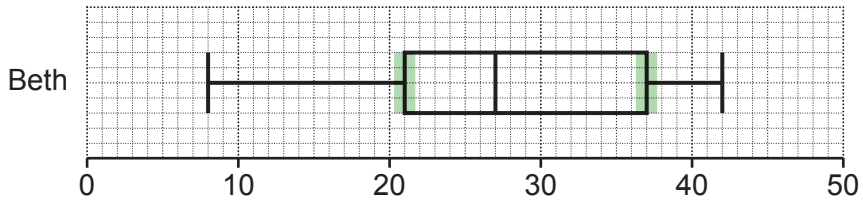
38−23=15 ← Subtracting the range from the highest score works out that the lowest score is 15

24+11=35 ← Adding the interquartile range to the lower quartile works out that the upper quartile is 35



[3]

(b) This box plot shows the distribution of Beth's scores.



Find the interquartile range of Beth's scores.

37−21 ← Interquartile range = upper quartile - lower quartile

(b) 16 **[2]**

(c) Kareem says

Beth was more consistent than Ali because Beth had a lower median score.

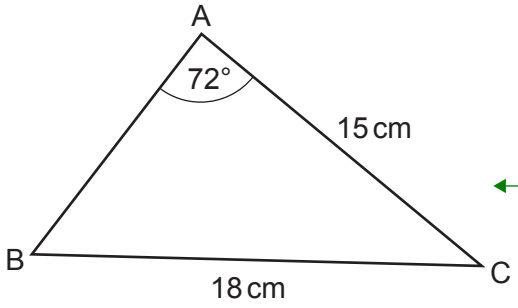
Is his statement correct?
Explain your reasoning.

No, median has nothing to do with consistency

Interquartile range is a measure of consistency

[2]

14 The diagram shows triangle ABC.



Not to scale

The triangle is not right angled so neither Pythagoras' theorem or right-angled trigonometry can be used. There is not enough information to use the sine or cosine rule to work out AB yet. Side a is opposite angle A, side b is opposite angle B and side c is opposite angle C

AC = 15 cm, BC = 18 cm and angle BAC = 72°.

Calculate length AB, giving your answer correct to 3 significant figures.
Show your working.

$\frac{\sin B}{b} = \frac{\sin A}{a}$ — The sine rule can be used to work out angle B as there are opposite sides and angles

$B = \sin^{-1}\left(\frac{b\sin A}{a}\right)$ — Rearranging to make B the subject by multiplying both sides by b and then doing the inverse sin of both sides

$= \sin^{-1}\left(\frac{15\sin 72}{18}\right)$ — Substituting 15 for b, 72 for A and 18 for a

$= 52.4\ldots$ — Storing the exact value of 52.42418285 as B on the calculator

$C = 180 - 72 - 52.4\ldots$ — There are 180° in total in a triangle. So subtracting the other angles from 180 works out angle C. Using the exact value of B

$= 55.5\ldots$ — Storing the exact value of 55.57581715 as C on the calculator

$\frac{c}{\sin C} = \frac{a}{\sin A}$ — The sine rule can now be used to work out side c, length AB

$c = \frac{a\sin C}{\sin A}$ — Rearranged to make c the subject by multiplying both sides by sinC

$= \frac{18\sin 55.5\ldots}{\sin 72}$ — Substituting 18 for a, the exact value of angle C for C and 72 for A

The answer of 15.61184765 is rounded to 3 significant figures

15.6 ..... cm **[6]**

15 Here are two pieces of work.

For each one, describe the error made and give the complete correct solution.

(a)

Question:

Solve by factorisation.

$3x^2 - 2x - 5 = 0$

Solution:

$(3x + 5)(x - 1) = 0$

Therefore $x = -5/3$ or $x = 1$

Error: Not factorised correctly

(3x + 5)(x - 1) expands to $3x^2 + 2x - 5$

Correct solution:

Multiplying the 3 by the -5 gives -15. Finding two numbers which multiply to -15 and add to the -2

Using table mode set f(x) = 15/x. Start: 1. End: 30. Step: 1

This lists out the factor pairs of 15. One of the two numbers must be negative to multiply to -15. 3 and -5 work

$3x^2+3x-5x-5$ ← Splitting the middle x term into 3x and -5x

$3x(x+1)-5(x+1)$ ← Factorising the left two terms and the right two terms separately

$(3x-5)(x+1)=0$ ← Bringing together the 3x and -5 into a bracket to express in factorised form

$x=\frac{5}{3}$ or $x=-1$ ← One of the two brackets must equal to 0 in order to multiply to 0. When 3x - 5 = 0, x = 5/3. When x + 1 = 0, x = -1

[3]

(b)

Question:

Solve, giving your answers correct to 3 significant figures.

$$2x^2 - 8x + 3 = 0$$

Solution:

$$x = -(^-8) \pm \frac{\sqrt{(^-8)^2 - 4 \times 2 \times 3}}{2 \times 2}$$

Therefore $x = 6.42$ or $x = 9.58$

Error: -(-8) should be in the fraction

.....

.....

Correct solution:

$\frac{-(-8)\pm\sqrt{(-8)^2-4\times2\times3}}{2\times2}$

Using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x=0.419$ or $x=3.58$

[3]

16 y is inversely proportional to the square of x.
$y = 2$ when $x = 5$.

Find a formula linking x and y.

$y \propto \frac{1}{x^2}$ ← Inversely means '1 over'. Writing out the proportion

$y = \frac{k}{x^2}$ ← The right side can be multiplied by anything and still be directly proportional. Multiplying by k, which represents a constant value, converts the proportion into an equation

$k = yx^2$ ← Multiplying both sides by x^2 makes k the subject

$= 2 \times 5^2$ ← Substituting 2 for y and 5 for x

$= 50$

Substituting 50 for k in the original equation

$y = \frac{50}{x^2}$

..... **[3]**

17 Expand and simplify.

$(x+1)(x-1)(x+2)$

$x^2 - x + x - 1$ ← Expanding the first two brackets

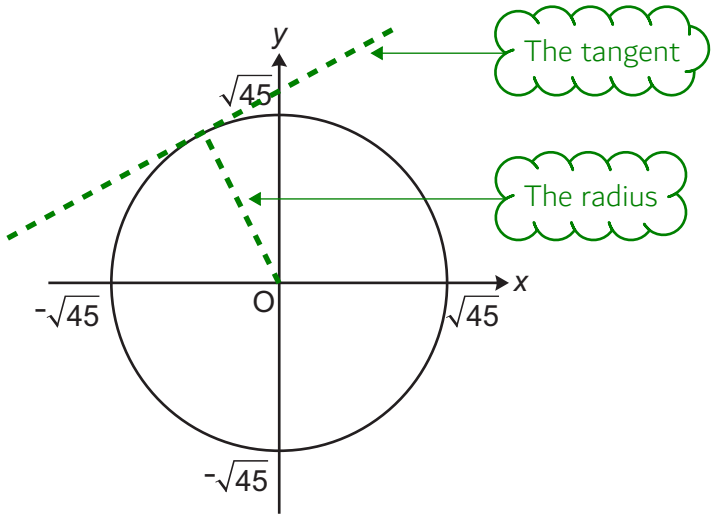
$(x^2-1)(x+2)$ ← Simplifying the expansion by collecting like terms then writing multiplied by the third bracket

Expanding out with the third bracket. There are no like terms so it cannot be simplified

$x^3 + 2x^2 - x - 2$

..... **[3]**

18 Here is a sketch of the circle $x^2 + y^2 = 45$.



(a) Show that the tangent to this circle at the point $(-3, 6)$ has a gradient of $\frac{1}{2}$. **[2]**

$-1 \div \frac{6-0}{-3-0} = \frac{1}{2}$ ← The tangent is perpendicular to the radius. Therefore its gradient is the negative reciprocal of the gradient of the radius. Negative reciprocal means '-1 over'

Gradient of the radius. Gradient = (change in y)/(change in x). The radius goes from the point (-3 , 6) to the origin (0 , 0). Change in y is found by subtracting the y-coordinate of the origin from the y-coordinate of the point. Change in x is found by subtracting the x-coordinate of the origin from the x-coordinate of the point

(b) Find the equation of the tangent at the point $(-3, 6)$.

$y = \frac{1}{2}x + c$ ← The tangent is a straight line so its equation must be y = mx + c, where m is the gradient and c is the y-intercept. Substituting 1/2 for m

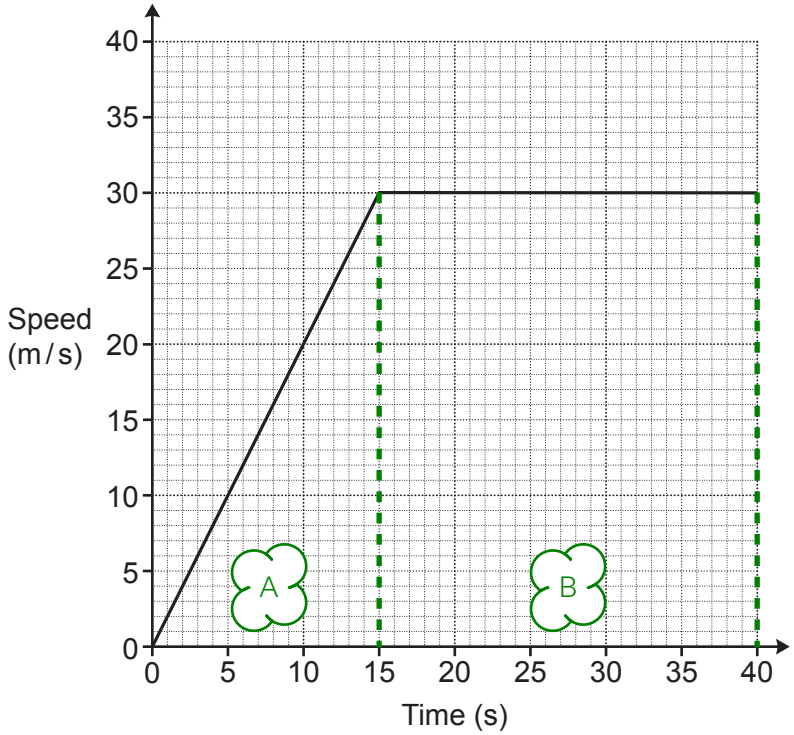
$c = y - \frac{1}{2}x$ ← Rearranged to make c the subject by subtracting 1/2 x from both sides

$= 6 - \frac{1}{2}(-3)$ ← Substituting 6 for y and -3 for x as the point is on the tangent

$= \frac{15}{2}$

(b) $y = \frac{1}{2}x + \frac{15}{2}$ ..... **[2]**

19 **(a)** The graph shows the speed of a vehicle during the first 40 seconds of motion.



Calculate the distance travelled by the vehicle during the 40 seconds.

$\frac{1}{2} \times 15 \times 30 + (40-15) \times 30$

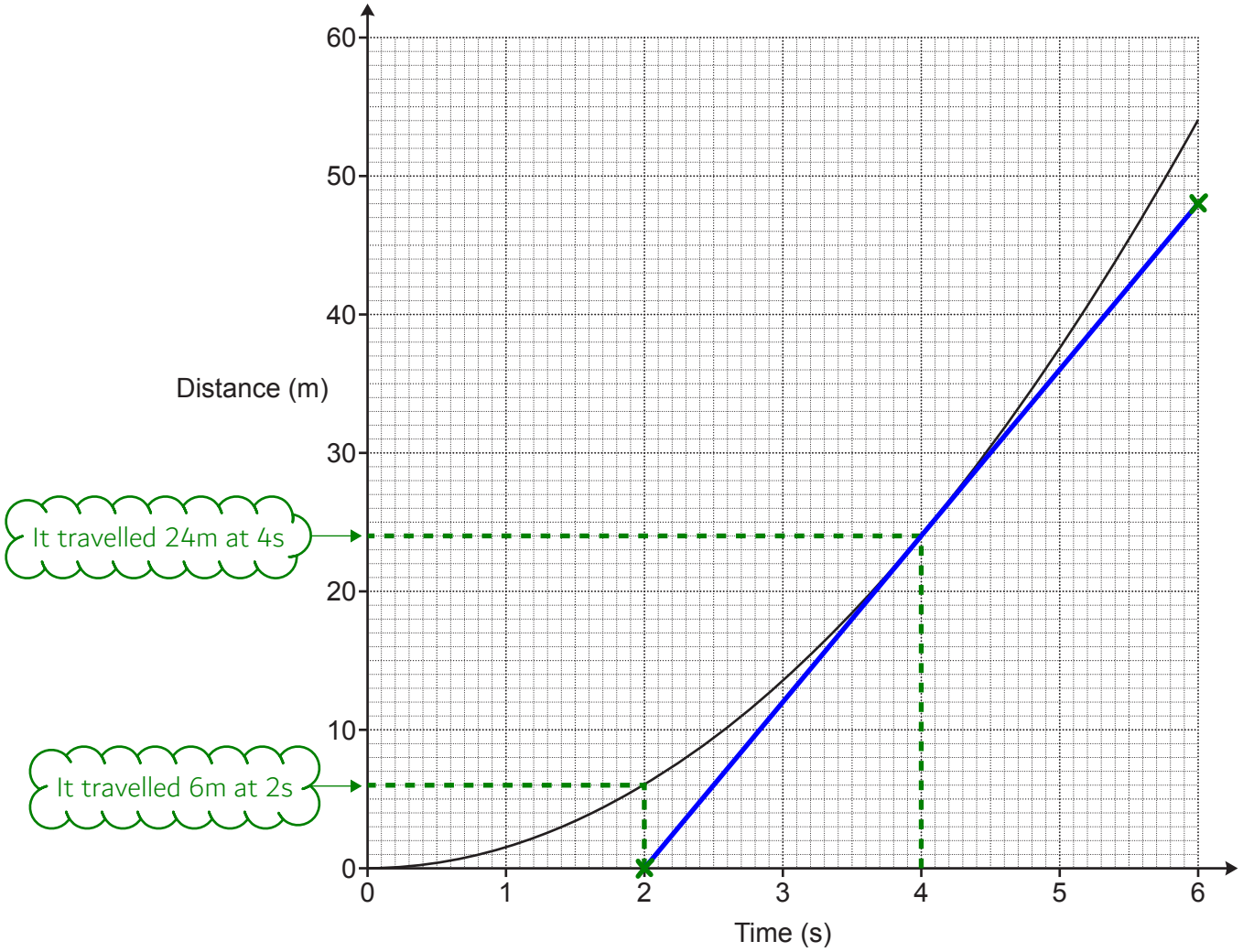
Distance on a speed-time graph is equal to the area under the line. Adding the area of triangle A and rectangle B finds this

Area of triangle A. Area of triangle = 1/2 x base x height. The base is 15 and the height is 30

Area of rectangle B. Area of rectangle = base x height. The base is found with (40 - 15) and the height is 30

(a) ..... 975 ..... m **[3]**

(b) The graph shows the distance travelled by a particle over 6 seconds.



(i) Work out the average speed of the particle between 2 and 4 seconds.

$\frac{24-6}{4-2}$

The unit of m/s means to divide the distance in metres by the time in seconds. 24 - 6 works out the difference in distance between 2 and 4 seconds

(b)(i) ..... 9 ..... m/s **[2]**

(ii) Estimate the speed of the particle at 4 seconds.

$\frac{48-0}{6-2}$

Speed is the gradient on a distance-time graph. Drawing a tangent at 4 seconds and working out its gradient. Gradient = (change in y)/(change in x). Change is between the two green crosses. Change in y is found by subtracting the y-coordinate of the first cross from the y-coordinate of the second cross. Change in x is found by subtracting the x-coordinate of the first cross from the x-coordinate of the second cross

(ii) ..... 12 ..... m/s **[4]**

Turn over for Question 20

20 Solve.

$$x^2 + y^2 = 34$$
$$y = x + 2$$

Show your working.

$(x+2)^2$ ← Squaring both sides of y = x + 2 to work out what y^2 is in terms of x

x^2+4x+4 ← Expanding the square bracket by squaring the first term, doubling the product of the two terms and squaring the last term

$2x^2+4x+4=34$ ← Substituting $x^2 + 4x + 4$ for y^2 in the top equation

$2x^2+4x-30=0$ ← Rearranging into the quadratic form by subtracting 34 from both sides

$x = \frac{-4 \pm \sqrt{4^2-4\times2\times-30}}{2\times2}$ ← Using the quadratic formula to solve x

$x=3$ or $x=-5$

$y=3+2$ or $y=-5+2$ ← Substituting the x values into the second equation to find y

$x =$ ....3.... $y =$ ....5....

$x =$ ....-5.... $y =$ ....-3.... **[6]**

END OF QUESTION PAPER