

Please write clearly in block capitals.

Centre number

Candidate number

Surname \_\_\_\_\_

Forename(s) \_\_\_\_\_

Candidate signature \_\_\_\_\_

# GCSE MATHEMATICS

# H

Higher Tier          Paper 3 Calculator

Tuesday 12 June 2018

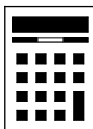
Morning

Time allowed: 1 hour 30 minutes

### Materials

For this paper you must have:

- a calculator
- mathematical instruments.



### Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
<b>TOTAL</b>	

### Advice

- In all calculations, show clearly how you work out your answer.



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to [curtis@cgmaths.co.uk](mailto:curtis@cgmaths.co.uk)

Answer **all** questions in the spaces provided

- 1 Circle the decimal that is closest in value to  $\frac{11}{20}$  [1 mark]

0.56

0.6

0.525

0.5

These calculations work out the distance between  $\frac{11}{20}$  and each decimal. -0.01 has the smallest magnitude

$$\begin{aligned} 11/20 - 0.56 &= -0.01 \\ 11/20 - 0.6 &= -0.05 \\ 11/20 - 0.525 &= 0.025 \\ 11/20 - 0.5 &= 0.05 \end{aligned}$$

- 2 Circle the list of **all** the integers that satisfy  $-2 < x \leq 4$  [1 mark]

-2, -1, 0, 1, 2, 3

-1, 0, 1, 2, 3

-2, -1, 0, 1, 2, 3, 4

-1, 0, 1, 2, 3, 4

The integers which are greater than -2 and less than or equal to 4

- 3 Circle the largest number. [1 mark]

3.27

3.27

3.277

3.207

3.2777... 3.2700 3.2770 3.2077...

Writing each number truncated to 4 decimal places is enough to work out which is largest



4 What is the size of an exterior angle of a regular decagon?

Circle your answer.

[1 mark]

18°

36°

144°

162°

$$360/10 = 36$$

The sum of the exterior angles in any polygon is 360. Therefore, as decagons have 10 sides and 10 exterior angles and the shape is regular meaning all of them are the same, this works out each exterior angle

5  $a$  is a common factor of 72 and 120

$b$  is a common multiple of 6 and 9

Work out the highest possible value of  $\frac{a}{b}$

[4 marks]

$$72 = 2^3 \times 3^2$$

$$120 = 2^3 \times 3 \times 5$$

FACT B

o, 33

Expressing both 72 and 120 as a product of prime factors by entering the number, pressing =, SHIFT then FACT (the button on the left)

$$2^3 \times 3$$

The highest common factor is the lowest power of each prime multiplied together

Answer

$$\frac{24}{18}$$

Expressing the highest common factor of 72 and 120 over the lowest common multiple of 6 and 9. The LCM of 6 and 9 is found by counting up in 9s until a multiple of 6 is reached

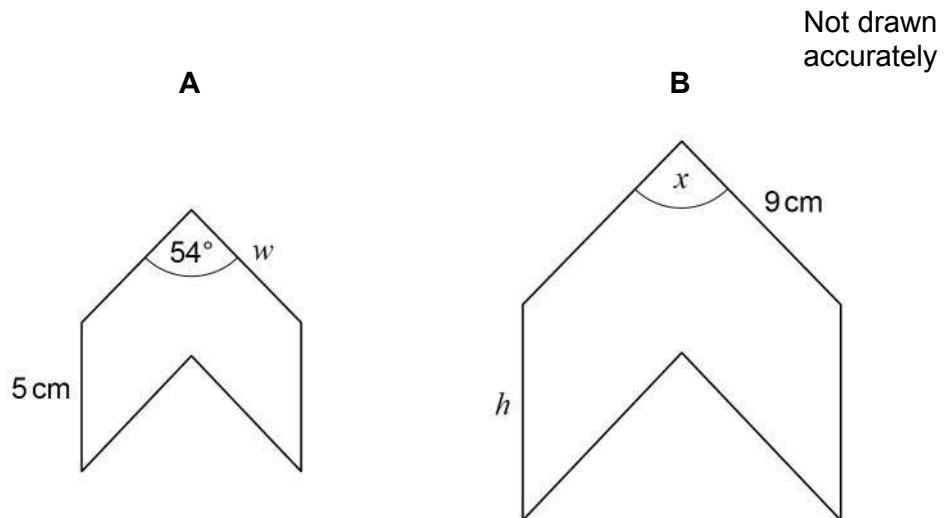
Turn over for the next question



6

A and B are similar shapes.

B is an enlargement of A with scale factor 1.5

Work out the values of  $x$ ,  $h$  and  $w$ .**[3 marks]**

$5 \times 1.5$

This works out  $h$ 

$9 \div 1.5$

This works out  $w$ 

$x$  stays the same as it was in shape A as they are similar. The angles in similar shapes are the same

$x = 54$  degrees

$h = 7.5$  cm

$w = 6$  cm



7 Investment A Save £150 per month for 2 years.  
2.5% interest is added to the total amount saved.

Investment B Invest £3500  
Compound interest is added at 3% per year.

After 2 years, how much **more** is investment B worth than investment A?

[4 marks]

$$\underline{3500 \times \left(\frac{100+3}{100}\right)^2 - 150 \times 12 \times 2 \times \frac{100+2.5}{100}}$$

The worth of investment B.  $100 + 3$  works out the percentage it increases to each year. Dividing this by 100 converts it into a fraction multiplier. Raising this to the power of 2 as it needs to be multiplied by twice as it is after 2 years. Multiplying this by the £3500 increases it by 3% twice

The worth of investment A. There are 12 months in a year and there are 2 years so multiplying the £150 by 12 and 2 works out how much is saved before the interest.  $100 + 2.5$  works out the percentage it increases to. Dividing this by 100 converts it into a fraction multiplier. Multiplying by this by the amount saved before the interest increases it by 2.5%

Subtracting the worth of investment A from the worth of investment B works out the difference and therefore how much more investment B is worth than investment A

Answer £ 23.15

Turn over for the next question



- 8 (a) Show that the lines  $y = 3x + 7$  and  $2y - 6x = 8$  are parallel.

Do **not** use a graphical method.

[3 marks]

$$2y = 6x + 8$$

← Adding 6x to both sides of the second equation

$$y = 3x + 4$$

← Dividing both sides by 2

Both lines have gradient of 3

← Both equations are now in the form  $y = mx + c$ , where  $m$  is the gradient. Parallel lines have the same gradient

- 8 (b) Is the point  $(-5, -6)$  above, below or on the line  $y = 3x + 7$ ?

Tick **one** box.

Above

Below

On the line

You **must** show your working.

Do **not** use a graphical method.

[2 marks]

$$y = 3(-5) + 7 = -8$$

← Substituting the x coordinate of the point into the equation to find what y should be on the line. It should be -8 and -6 is above this



- 9 The cost of a ticket increases by 10% to £19.25

Work out the original cost.

[3 marks]

$$\frac{19.25}{110} \times 100$$

If it is increased by 10%, it is now at 110% of the original value. Dividing by 110 works out 1% of the original value. Multiplying by 100 works out 100%, the full amount, of the original value

Answer £ 17.50

- 10 The  $n$ th term of a sequence is  $12n - 5$

Work out the numbers in the sequence that

have two digits

and

are **not** prime.

[3 marks]

19, 31, 43, 55, 67, 79, 91

Using table mode by pressing MENU then 3.  $f(x) = 12x - 5$ . Ignore  $g(x)$ . Start: 1. End: 30. Step: 1

This lists out the sequence up to the 30th term.  
Writing down the ones which have two digits

FACT B



Enter each number, press = then SHIFT then FACT (the button on the left)

This expresses each number as a product of prime factors.  
If it comes back as itself it must be prime.  $55 = 5 \times 11$  and  
 $91 = 7 \times 13$  so 55 and 91 are not prime

Answer 55, 91





$$11 \quad \mathbf{a} = \begin{pmatrix} 6 \\ -10 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}$$

11 (a) Work out  $\mathbf{a} + \mathbf{b} + \mathbf{c}$

[2 marks]

$$6 + -1 + -4$$

Adding together the x components  
and y components separately

$$-10 + 2 + 7$$

Answer

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

11 (b) Show that  $\mathbf{a} + 2\mathbf{c}$  is parallel to  $\mathbf{b}$

[2 marks]

$$6 + 2(-4)$$

Working out the x components  
and y components separately

$$-10 + 2(7)$$

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix} = 2 \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Expressing the column vector as 2 lots of vector  $\mathbf{b}$



12

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

A force of 40 Newtons is applied to an area of 3.2 square metres.

Work out the pressure.

Give the units of your answer.

[2 marks]

$$\frac{40}{3.2}$$

Dividing the force by the area works out the pressure

Answer

$$12.5 \text{ N/m}^2$$

The force in Newtons was divided by the area in square metres so the unit is Newtons per square metre

13

Tick **all** the statements that are true for any rhombus.

[1 mark]



The diagonals are lines of symmetry



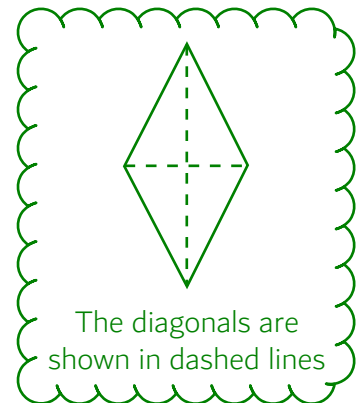
The diagonals bisect each other



The diagonals are perpendicular



The diagonals are equal in length

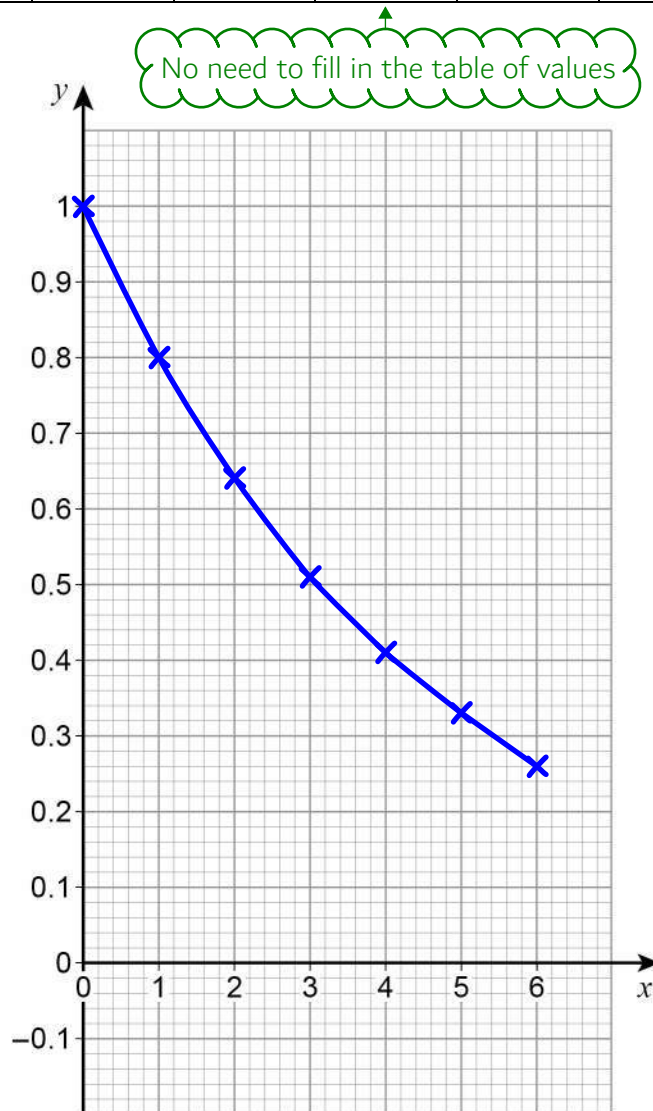


Turn over for the next question



14 Draw the graph of  $y = 0.8^x$  for values of  $x$  from 0 to 6 [3 marks]

$x$	0	1	2	3	4	5	6
$y$							



- 15 Amy has  $x$  beads.  
Billy has three more beads than Amy.  
Carly has four times as many beads as Billy.  
Circle the expression for the number of beads that Carly has.

[1 mark]

$4x + 3$

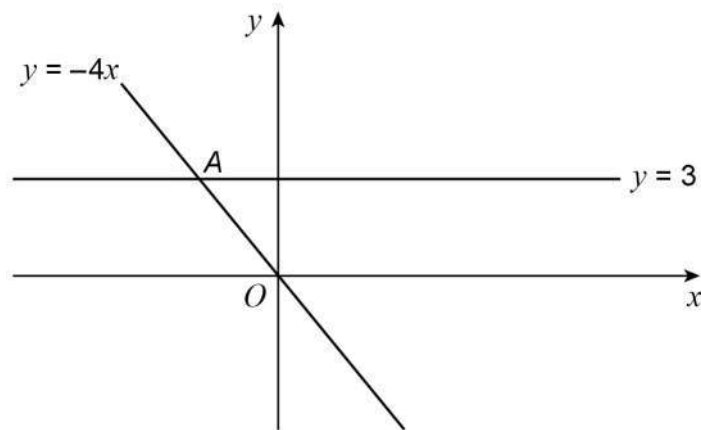
$3x + 4$

$4(x + 3)$

$x + 12$

Billy has  $x + 3$ . Multiplying this by 4 expresses how many Carly has

- 16 Two straight lines intersect at point A.

Not drawn  
accurately

Circle the coordinates of A.

[1 mark]

$(-\frac{3}{4}, 3)$

$(-4, 3)$

$(-12, 3)$

$(-\frac{4}{3}, 3)$

The y coordinate must be 3 as it is on the line  $y = 3$ . Doing simultaneous equations, which finds the coordinates of intersection, gives  $-4x = 3$  as both sides are equal to y. Dividing both sides by -4 gives  $x = -3/4$



- 17 Here are two methods to make a 4-digit code.  
Codes can have repeated digits.

**Method A**

For the first two digits use an odd number between 30 and 100  
For the last two digits use a multiple of 11

**Method B**

Use four digits in the order even odd even odd  
Do **not** use the digit zero

Which method gives the **greater** number of possible codes?  
You **must** show your working.

**[3 marks]**

$$35 \times 9 = 315$$

There are 5 odds in each of the 30s, 40s, 50s, 60s, 70s, 80s and 90s.  $5 \times 7 = 35$  so there are 35 odd numbers between 30 and 100. The 2 digit multiples of 11 are 11, 22, 33, 44, 55, 66, 77, 88 and 99; there are 9 of these. Using the product rule for counting works out the total number of possible codes by multiplying the number of possibilities for the first two and last two digits

$$4 \times 5 \times 4 \times 5 = 400$$

The even digits are 0, 2, 4, 6, 8 but 0 can't be used so there are 4 even digits which can be used. The odd digits are 1, 3, 5, 7, 9 so there are 5 odd digits which can be used. Using the product rule for counting works out the total number of possible codes by multiplying the number of possibilities for each digit

Answer \_\_\_\_\_

**B**

Method A has 315 possible codes and method B has 400 possible codes. 400 is greater than 315



18 Show that, for  $x \neq 0$

$$\frac{x+4}{3x} - \frac{5}{2x}$$

can be written in the form  $\frac{ax+b}{cx}$  where  $a$ ,  $b$  and  $c$  are integers.

[3 marks]

$$\frac{2x+8}{6x} - \frac{15}{6x}$$

Multiplying both the numerator and denominator of the first fraction by 2 and the numerator and denominator of the second fraction by 3 gives a common denominator so that they can be subtracted

Answer  $\frac{2x-7}{6x}$

19 The equation of a straight line is  $3x + 2y = 24$

Circle the point where the line crosses the  $x$ -axis.

[1 mark]

(0, 8)

(12, 0)

(0, 12)

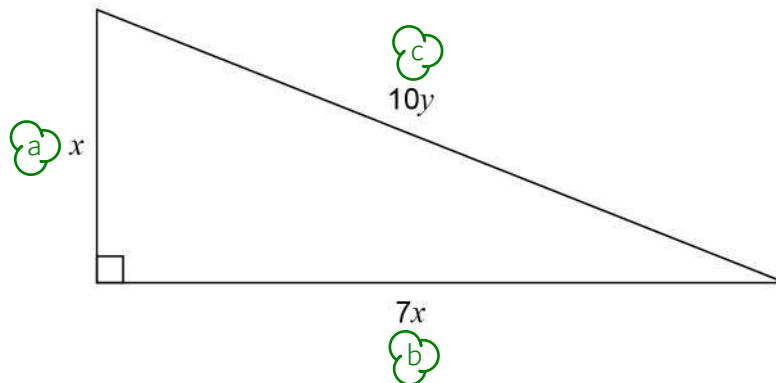
(8, 0)

$y = 0$  when the line crosses the  $x$ -axis.  $3x + 2(0) = 3x = 24$ .  $x = 8$



20

All dimensions are in centimetres.

Not drawn  
accuratelyUse Pythagoras' theorem to work out the exact value of  $\frac{x}{y}$ **[3 marks]**

$$x^2 + (7x)^2 = (10y)^2$$

Using Pythagoras' theorem:  $a^2 + b^2 = c^2$ , where  $c$  is the longest side and  $a$  and  $b$  are the shorter sides

$$50x^2 = 100y^2$$

$(7x)^2 = 49x^2$ . Adding  $x^2$  gives  $50x^2$ .  $(10y)^2 = 100y^2$

$$\frac{x^2}{y^2} = \frac{100}{50}$$

$$= 2$$

Dividing both sides by  $y^2$  and by 50

Square rooting both sides give  $x/y$

Answer

$$\sqrt{2}$$



- 21 The mass of an ornament is  $m$  grams.  
The height of the ornament is  $h$  centimetres.  
 $m$  is directly proportional to the cube of  $h$ .  
 $m = 1600$  when  $h = 8$

- 21 (a) Work out an equation connecting  $m$  and  $h$ .

[3 marks]

$$m = kh^3$$

$m = kh^3$ . The right side of the proportion can be multiplied by anything and still be directly proportional. Using  $k$  to represent what it is multiplied by to convert it into an equation

$$k = \frac{m}{h^3} = \frac{1600}{8^3}$$

Rearranging to make  $k$  the subject then substituting in the values of  $m$  and  $h$  given, which must satisfy the equation, to find  $k$

Answer

$$m = \frac{25}{8}h^3$$

Substituting the value of  $k$  back into the original equation

- 21 (b) Work out the mass of an ornament of height 12 centimetres.

[2 marks]

$$\frac{25}{8} \times 12^3$$

$m$ , the mass, is already the subject in the equation found in part (a) so it tells us how to find  $m$ . Substituting in 12 for  $h$

Answer

5400

grams

Turn over for the next question



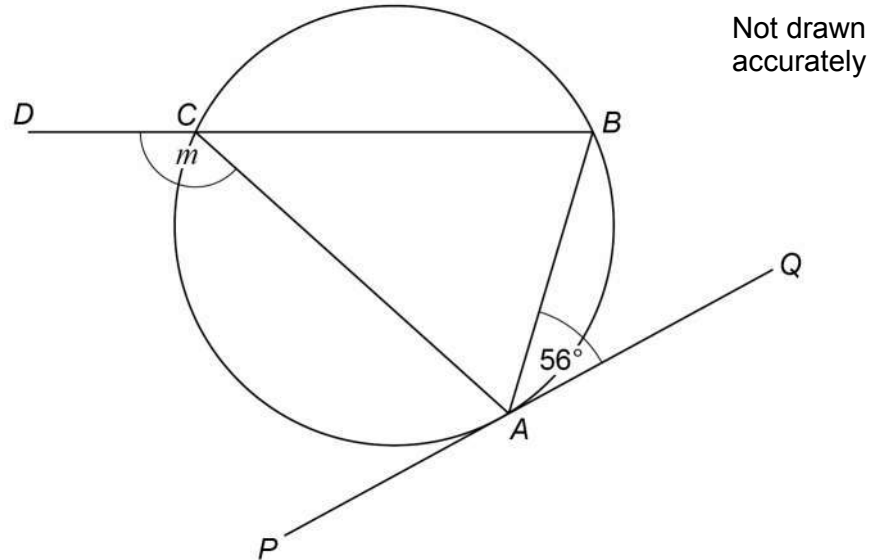


22

$A$ ,  $B$  and  $C$  are points on a circle.

$DCB$  is a straight line.

$PAQ$  is a tangent to the circle.



Sam is trying to work out the size of angle  $m$ .

Here is his working.

$$\text{angle } ACB = 56^\circ$$

angles in the same segment are equal

$$m = 180^\circ - 56^\circ \quad \text{angles at a point on a straight line add up to } 180^\circ$$

$$m = 124^\circ$$

Make a criticism of his working.

[1 mark]

Reason on the first line is incorrect

Should have stated the alternate segment theorem



23

A sequence of numbers is formed by the iterative process

$$u_{n+1} = \frac{3}{u_n + 1}, \quad u_1 = 4$$

Work out the values of  $u_2$  and  $u_3$ **[2 marks]**

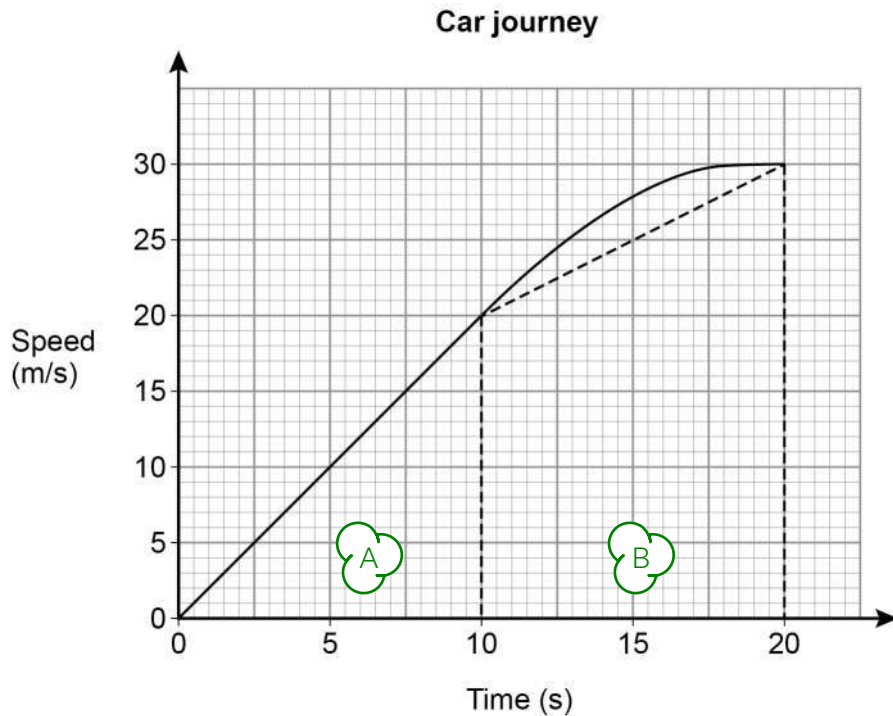
Enter 4 then press =. Enter 3/(ANS + 1)  
then press = to get  $u_2$ . Press = again to get  $u_3$

$$u_2 = \underline{\hspace{10em} 0.6 \hspace{10em}}$$

$$u_3 = \underline{\hspace{10em} 1.875 \hspace{10em}}$$

**Turn over for the next question****Turn over ►**

- 24 The speed-time graph shows 20 seconds of a car journey.  
Harry wants to estimate the distance the car travels in this time.  
He uses a triangle and a trapezium, as shown, to estimate the area under the graph.



- 24 (a) Complete Harry's method to estimate the distance the car travels. [3 marks]

$$\frac{1}{2} \times 10 \times 20 + \frac{1}{2} (20 + 30) \times 10$$

Area of triangle A. Area of triangle =  $\frac{1}{2} \times \text{base} \times \text{height}$

Area of trapezium B. Area of trapezium =  $\frac{1}{2} (a + b)h$ , where a and b are the parallel sides and h is the distance between them

Adding the area of shapes A and B works out an estimate of the total area under the graph, which is an estimate of the distance

Answer 350 m



24 (b) For this journey, which of these is true for Harry's method?

Tick **one** box.

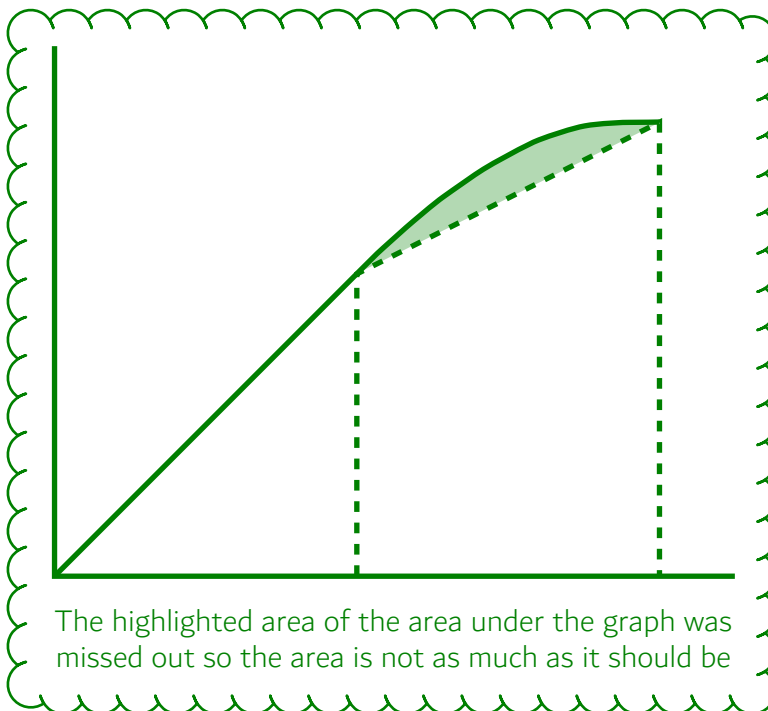
[1 mark]

It works out an overestimate of the distance

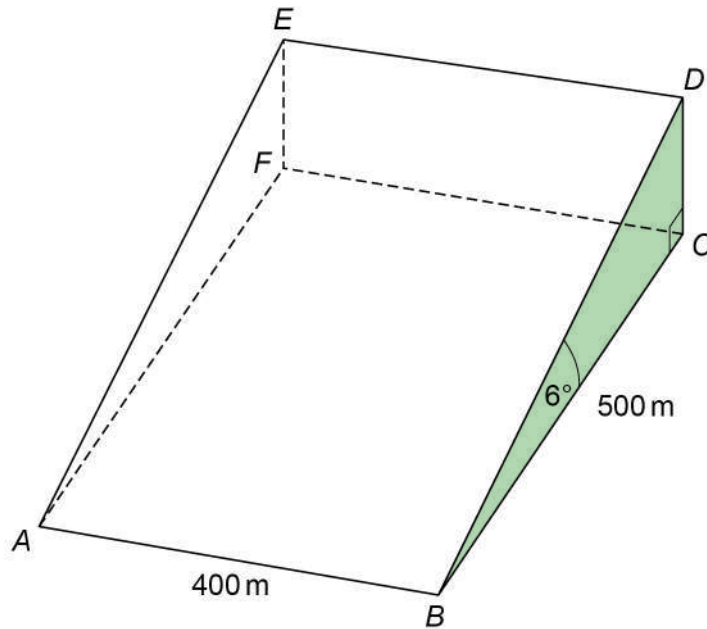
It works out an underestimate of the distance

It could work out an overestimate or an underestimate of the distance

Turn over for the next question



- 25  $ABCDEF$  is a triangular prism which represents part of a hill.  
 $ABCF$  is the horizontal rectangular base.  
 $D$  is vertically above  $C$ .



- 25 (a) Work out the height  $CD$ .

[2 marks]

S<sup>O</sup>H C<sup>A</sup>HT<sup>O</sup>A

Right angled trigonometry can be used on the highlighted triangle

$\tan 6 \times 500$

Store the exact answer by pressing STO then A as it is needed for the next question

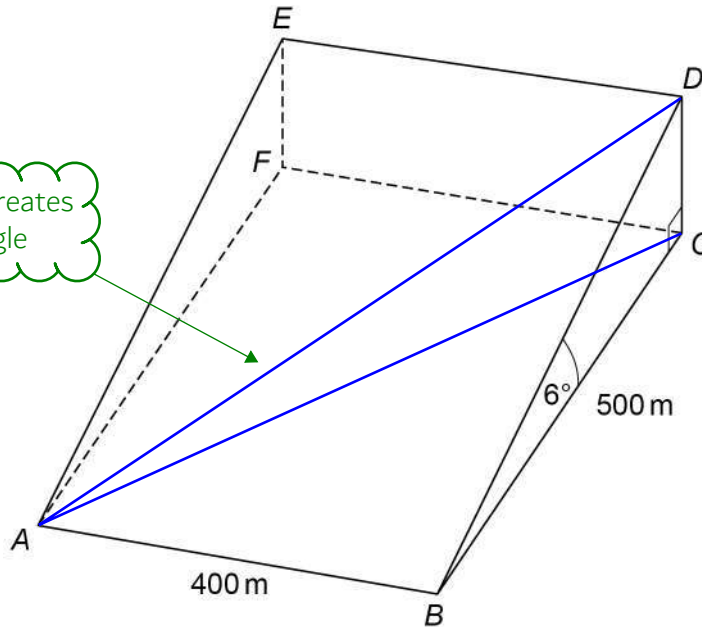
Answer 52.6 m

We have the adjacent and are looking for the opposite so both A and O are ticked, so the TOA formula triangle can be used. Opposite = (tan of the angle) x adjacent



25 (b) Jamil walks in a straight line from A to D.

Drawing on the angle creates a right angled triangle



Work out the size of angle DAC.

You **must** show your working.

[4 marks]

$$\begin{matrix} \text{O} & \text{A} & \text{O} \\ \text{S} & \text{H} & \text{T} \\ \text{C} & & \text{A} \end{matrix}$$

Right angled trigonometry can be used to find the angle. Ticking O as we have the opposite, side CD. Ticking A as we can find the adjacent, side AC

$$a^2 + b^2 = c^2$$

Stating Pythagoras' theorem, where c is the longest side and a and b are the shorter sides

$$\sqrt{a^2 + b^2} = c$$

Rearranging to make c the subject as AC is the longest side in right angled triangle ABC

$$\tan^{-1}\left(\frac{CD}{\sqrt{500^2 + 400^2}}\right)$$

From the formula triangle, (tan of the angle) = opposite/adjacent. So the angle =  $\tan^{-1}(\text{opposite/adjacent})$ . Using the value found in part (a) for the opposite, side CD, and substituting 500 for a and 400 for b in the rearranged Pythagoras' theorem to express the adjacent, side AC

Pressing ALPHA then A for side CD recovers the previously stored result

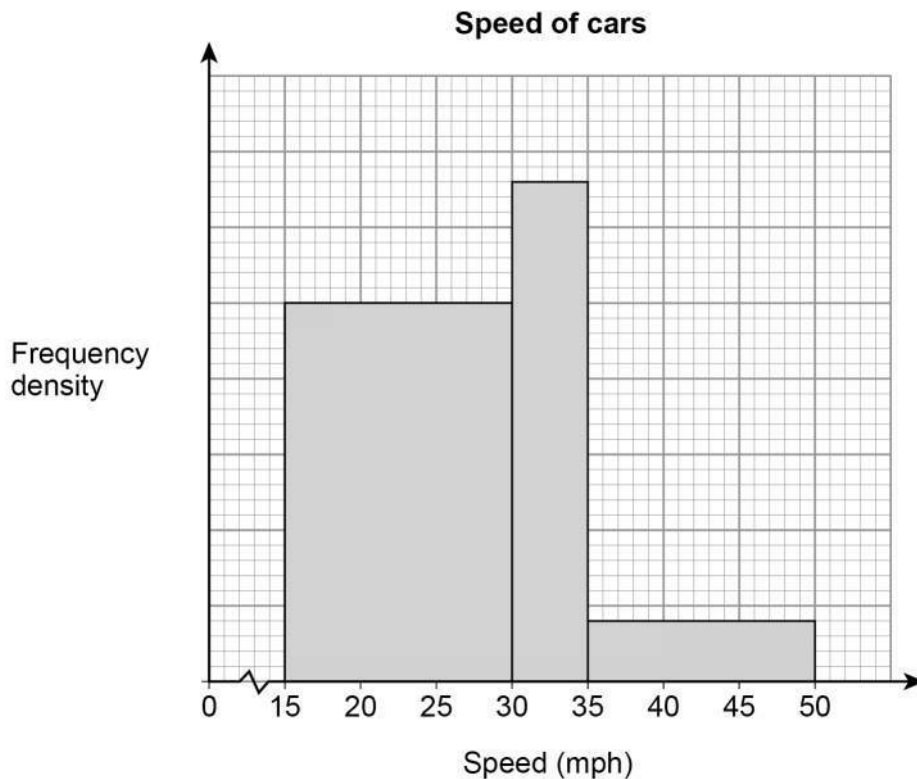
Answer 4.7 degrees

6

Turn over ►



- 26 The histogram shows information about the speed of cars as they pass a checkpoint. The scale on the frequency density axis is missing.



The histogram shows information about 480 cars.

- 26 (a) How many cars does the first bar represent?

[4 marks]

$$(30-15) \times 25x + (35-30) \times 33x + (50-35) \times 4x$$

Let  $x$  be the frequency density represented by one small box. Frequency = class width  $\times$  frequency density. Adding all of the frequencies for each bar together represents the total frequency

$$600x = 480$$

Simplifying the expression for the total frequency and setting it equal to the 480

$$(30-15) \times 25 \times \frac{480}{600}$$

Frequency = class width  $\times$  frequency density. Rearranged to find  $x$  and substituted it into the expression for the frequency of the first bar

Answer 300



26 (b) Cars with a speed greater than 40 mph are over the speed limit.

Use the histogram to estimate the number of cars that are over the speed limit.

[2 marks]

$$\frac{(50-40) \times 4 \times \frac{480}{600}}{}$$

Splitting the last bar gives a bar from 40mph to 50mph.  
Frequency = class width x frequency density. Substituting  
in the value of x to express the frequency density

Answer 32

Turn over for the next question





27

A bag contains 30 discs.

10 are red and 20 are blue.

One disc is taken out at random and replaced by **two** of the other colour.

Another disc is then taken out at random and replaced by **two** of the other colour.

Another disc is then taken out at random.

Work out the probability that all three discs taken out are **red**.

[3 marks]

$$\frac{10}{30} \times \frac{9}{31} \times \frac{8}{32}$$

Red AND red AND red. AND means to multiply the probabilities. There is one fewer red each time but one more in total each time, as one is taken out and two are added in

Answer \_\_\_\_\_

$$\frac{3}{124}$$

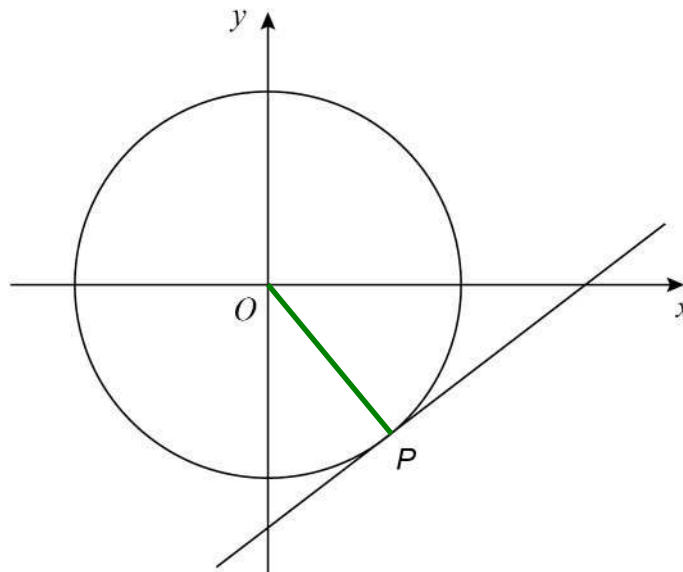


28

$P$  is a point on the circle with equation  $x^2 + y^2 = 80$

$P$  has  $x$ -coordinate 4 and is below the  $x$ -axis.

Not drawn  
accurately



Work out the equation of the tangent to the circle at  $P$ .

[5 marks]

$$y = \sqrt{80 - 4^2} = -8$$

Rearranged the equation of the circle to make  $y$  the subject and substituted in the  $x$ -coordinate of  $P$  to find the  $y$  coordinate of  $P$ . It cannot be 8 as it is below the  $x$ -axis so must be  $-8$

$$-1 \div \left( \frac{-8-0}{4-0} \right)$$

Gradient = (change in  $y$ ) / (change in  $x$ ). Expressing the gradient of the radius and doing  $-1$  divided by this to work out the gradient of the tangent, which is the negative reciprocal of the gradient of the radius as the radius and tangent are perpendicular

$$c = y - mx$$

$$= -8 - \frac{1}{2}(4)$$

The general equation of a straight line is  $y = mx + c$ , where  $m$  is the gradient. Rearranging to find  $c$  then substituting in the  $x$  and  $y$  coordinates of  $P$ , which are on the tangent, and  $1/2$  as the gradient

The general equation of a straight line is  $y = mx + c$ .  $m = 1/2$  and  $c = -10$

Answer

$$y = \frac{1}{2}x - 10$$

END OF QUESTIONS

