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.CG Maths.
Worked Solutions

H

Tuesday 2 November 2021 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start to write your answer.

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Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** the questions.

1 Calculate.

(a) $(6^2 + 5)^3$

Typing it into the calculator exactly as it is above

(a) ....68921............ **[1]**

(b) $\sqrt{\dfrac{8.4^2 - 1.9^2}{2.5 + 5.7}}$

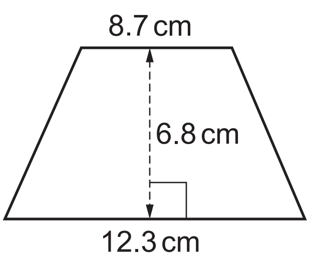
Write your answer correct to **3** significant figures.

Typing it into the calculator exactly as it is above gives 2.857382394

The 5 is the third significant figure. The 7 after this causes it to round up. Everything after the third significant figure is then set to 0 and ignored

(b) ....2.86............ **[3]**

2 Calculate the area of this trapezium.



Not to scale

$\frac{1}{2}(8.7+12.3)\times 6.8$ ← Area of trapezium = 1/2 x (a + b) x h, where a and b are the parallel sides and h is the perpendicular distance between them

............71.4............ cm^2 **[2]**

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3 Simplify.

$x^{12} \div x^4$

$a^x \div a^y = a^{x-y}$. This basically means that the powers can be subtracted. 12 - 4 = 8

x^8 .... [1]

4 Li throws two fair four-sided dice, each numbered 1, 2, 3 and 4.
Li multiplies together the two numbers that the dice land on to produce a score.

Find the probability that Li's score is a prime number.

1×2, 1×3, 2×1, 3×1

Listing out the possible outcomes which will give a prime number when multiplied. For example, getting a 1 on the first dice and a 2 on the second dice will give a score of 2 (which is prime) as 1 x 2 = 2

$\frac{1}{4} \times \frac{1}{4} \times 4$

1 AND 2 OR 1 AND 3 OR 2 AND 1 OR 3 AND 1. AND means to multiply, OR means to add. 1 out of the 4 outcomes is a 1 so the probability of getting a 1 is 1/4. The probability of any of the numbers is the same. As the probability of each pair of outcomes is the same (they are all 1/4 x 1/4), the probability of one of the pairs of outcomes can be multiplied by 4

$\frac{1}{4}$ .... [4]

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5 **(a)** Fountain A squirts water every 24 minutes.
Fountain B squirts water every 42 minutes.
They squirt water together at 15:19.

Find the next time they squirt water together.

$2^3 \times 3$
$2 \times 3 \times 7$

Expressing 24 and 42 as a product of prime factors

The calculator can be used to do this

$2^3 \times 3 \times 7 = 168$

Working out the lowest common multiple by multiplying the highest power of each prime factor of both numbers. This means that they both squirt water together after 168 minutes

15°19° + 0°168°

Adding the time taken for them both to squirt water together to the time they squirt water together works out the time they next squirt water together

Time can be put into the calculator in the form hh°mm°ss°, where hh is the hours, mm is the minutes and ss is the seconds

The answer of 18°7'0" means 18:07

Newer models of the Casio calculator can calculate the lowest common multiple of two numbers

(a) ....18:07........ **[4]**

(b) A school sends 60 students from Year 8 and 105 students from Year 9 to a museum.

The school divides these students into groups using the following rules.

- The groups must all be the same size.
- All students in any group must be from the same year.
- There should be as few groups as possible.

Find the size of each group and the total number of groups.

$2^2 \times 3 \times 5$
$3 \times 5 \times 7$

Expressing 60 and 105 as a product of prime factors

The calculator can be used to do this

$3 \times 5 = 15$

Working out the highest common factor by multiplying the lowest power of each prime factor of both numbers. This means that the greatest number of students in a group (which leads to the fewest number of groups) is 15

$\frac{60}{15} + \frac{105}{15}$

60/15 works out how many groups of Year 8 and 105/15 works out how many groups of Year 9 there are. Adding these together works out the total number of groups

Newer models of the Casio calculator can calculate the highest common factor of two numbers

Size of each group = ...15........

Total number of groups = ...11........ **[4]**

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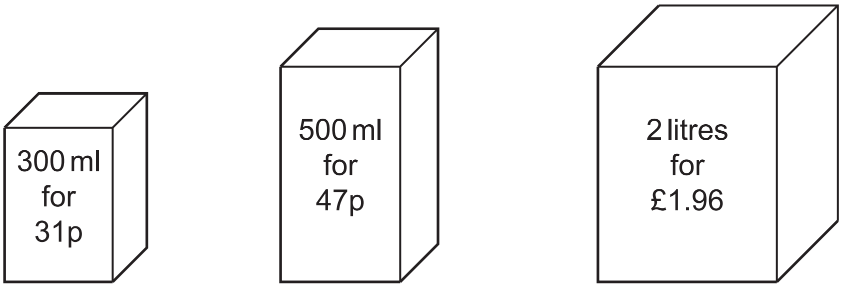
6 A shop sells the same milk in three different sized cartons.
The diagram shows the price of each carton.



(a) Which carton is the best value for money?
Show how you decide.

$\frac{300}{31} = 9.6...$

$\frac{500}{47} = 10.6...$

$\frac{2 \times 1000}{1.96 \times 100} = 10.2...$

Dividing the number of millilitres by the cost in pence works out how many millilitres each carton is per penny

There are 1000 millilitres in a litre so multiplying the 2 litres by 1000 converts it into millilitres. There are 100 pence in a pound so multiplying the £1.96 by 100 converts it into pence

500ml ....

The 500ml carton is the best value as it has the most millilitres per penny

....

.... **[3]**

(b) A student only buys milk on a Saturday morning.
They use 120 ml of milk each day.
Any unused milk has to be thrown away at the end of the following Friday.

Show that it is cheaper for the student to buy the milk they need in 300 ml cartons than in 500 ml cartons. **[3]**

$\frac{120 \times 7}{300} = 2.8$

From Saturday morning to the end of the following Friday is 7 days. So 120ml multiplied by the 7 days works out how much milk is used. Dividing this by the 300ml cartons works out how many 300ml cartons are needed

$3 \times 31 = 93$

There needs to be a whole number of cartons bought so the 2.8 is rounded up to 3. Multiplying this by the 31p cost of each carton works out how much the student would spend if buying 300ml cartons

$\frac{120 \times 7}{500} = 1.68$

$2 \times 47 = 94$

Doing the same method works out that the 500ml cartons would cost 94p for the milk needed, which is more than the 93p needed for 300ml cartons

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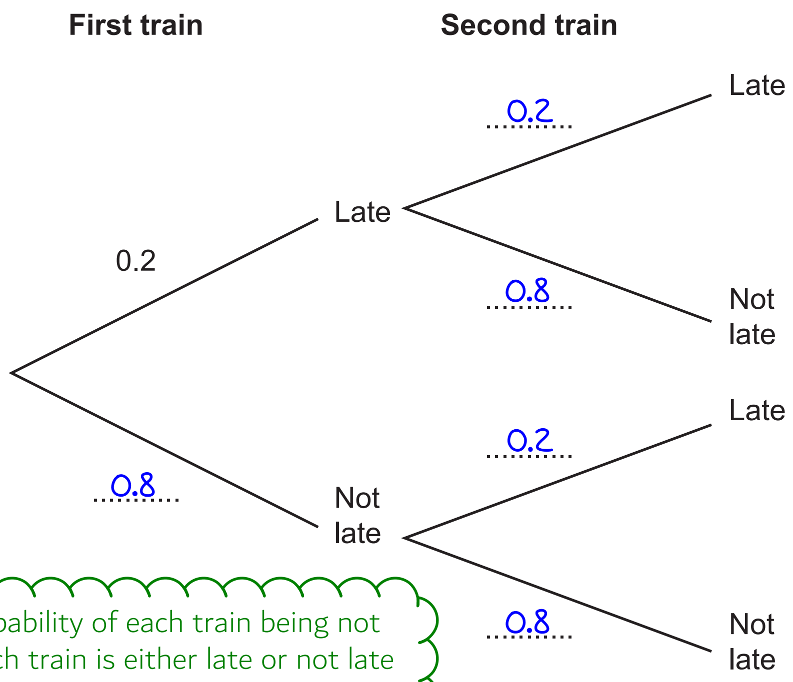
7 **(a)** Over a long period of time, it is found that the probability of a train from Bewford to London being late is 0.2.

(i) One morning there are two trains from Bewford to London.

Use the information to complete the tree diagram.



1 - 0.2 = 0.8

This works out the probability of each train being not late. It is certain that each train is either late or not late so the probabilities of each set of branches must add to 1

[2]

(ii) Work out the probability that both trains are **not late**.

0.8×0.8

Not late AND not late. Assuming the two events are independent, AND means to multiply the probabilities

(a)(ii) ....0.64........ **[2]**

(iii) Give a reason why the probabilities used in the tree diagram for the second train may **not** be reliable.

The first train being late may have an effect on the second train

.... **[1]**

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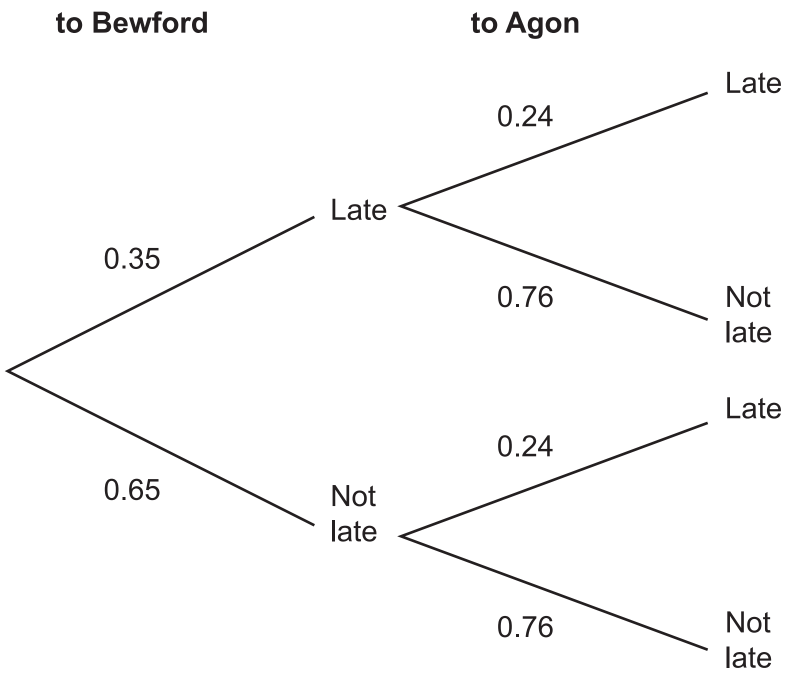
(b) Morgan takes a train from London to Bewford and then another train to Agon.
The tree diagram shows the probabilities of Morgan's trains being late or not late.



Morgan will **not catch** the train to Agon if the train to Bewford is late and the train to Agon is not late.

Work out the probability that Morgan will **catch** the train to Agon.

1−0.35×0.76

It is certain that Morgan will either catch the train to Agon or not catch the train to Agon. Therefore the probabilities must add to 1 and subtracting the probability of not catching the train to Agon from 1 leaves the probability of catching the train to Agon. The probability of not catching the train to Agon is found with 0.35 x 0.76 as this is the probability of the train to Bewford being late AND the train to Agon not being late. AND means to multiply the probabilities

(b) ....0.734........ **[3]**

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Turn over

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8 Jamie invests £6000 at a simple interest rate of r% each year.
After 6 years the value of their investment is £7170.

Find the value of r.

$$\frac{\frac{7170-6000}{6000}\times 100}{6}$$

£7170 - £6000 works out how much interest was gained. Putting this over the original £6000 expresses the interest as a fraction. Multiplying this fraction by 100 converts it into a percentage. As it is simple interest, the interest gained is the same each year so the percentage can be divided by 6 to give the interest rate each year

r = ...3.25........ **[4]**

9 The price of a plane ticket is increased by 15% to £1426.

Find the original price of the plane ticket.

$$\frac{1426}{100+15}\times 100$$

Reducing the £1426 by 15% does not work as the 15% is of the original price, not of the £1426. Let 100% be the original price. 100% + 15% expresses the percentage of the original price the ticket has increased to. Dividing the £1426 by this works out 1% of the original price. Multiplying this by 100 works out 100%, which is the original price

£ ....1240........ **[3]**

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10 Alex, Blake and Charlie play a computer game.

Alex goes first and scores *n* points.

- Blake scores 8 points less than 3 times the number of points scored by Alex.
- Charlie scores 25 more points than Blake.
- The three people score a total of 618 points.

Work out how many points they each score.
You must show your working.

$n + 3n - 8 + 3n - 8 + 25$ — Adding the expressions of the number of points each person scores gives the total number of points scored

(Alex's score: n; Blake's score: $3n - 8$; Charlie's score: $3n - 8 + 25$)

$7n + 9 = 618$ — Simplifying the expression of the total number of points scored by collecting like terms. This must be equal to the 618 points

$7n = 609$ — Subtracting 9 from both sides to get the n term on its own

$n = 87$ — Dividing both sides by 7 finds n, which is Alex's score

$87 \times 3 - 8 = 253$ — Blake scores 8 points less than 3 times the number of points scored by Alex

$253 + 25 = 278$ — Charlie scores 25 more points than Blake

Alex = ...87......

Blake = ...253......

Charlie = ...278...... **[7]**

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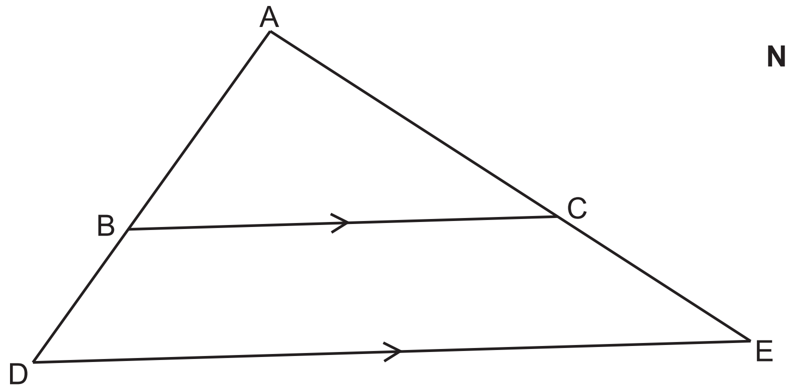
11 The diagram shows triangles ABC and ADE.



Not to scale

B lies on AD and C lies on AE.
BC is parallel to DE.

Complete these statements to show that triangles ABC and ADE are similar.

Angle ABC = angle ADE because they are corresponding angles.

Angle ACB = angle ....AED.... because ....they are corresponding angles....

Angle BAC is ....shared....

Triangles ABC and ADE are similar because ....they have the same angles....

.... **[3]**

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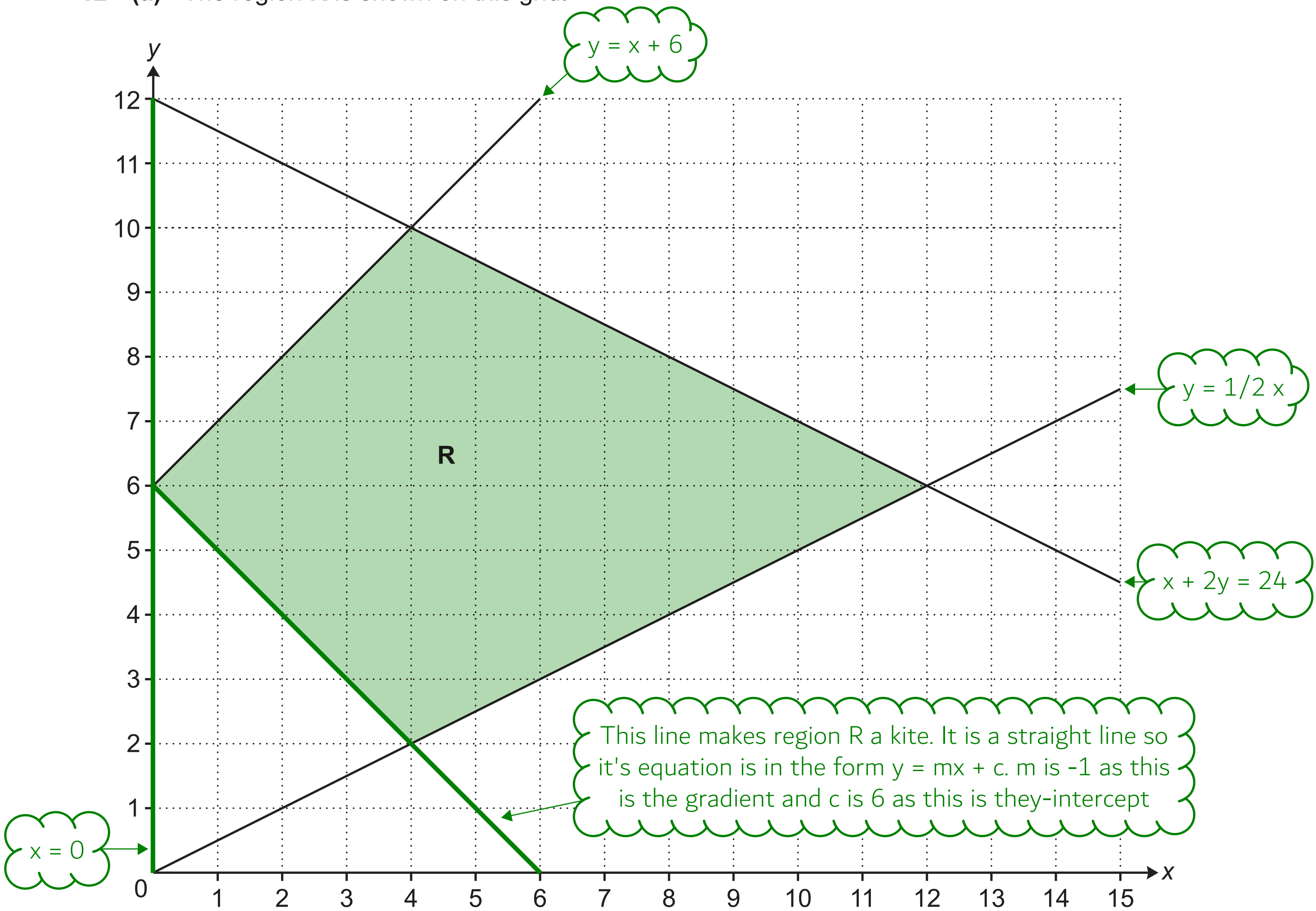
12 (a) The region **R** is shown on this grid.



Region **R** is defined by four inequalities.
One of the inequalities is $x \geqslant 0$.

Use the symbols $\leqslant$ and $\geqslant$ to complete the other three inequalities.

$x \geqslant 0$

y ....$\geqslant$.... $\frac{1}{2}x$ ← As the region is above the line

$x + 2y$ ....$\leqslant$.... 24

y ....$\leqslant$.... $x + 6$ ← As the region is below the lines **[2]**

(b) The inequality $x \geqslant 0$ is replaced by a new inequality.
Region **R** is then a kite.

Write down the new inequality.

The equation of the line is y = -x + 6. The equals can be replaced with greater than or equal to as the region is above

(b) ....$y \geqslant -x + 6$.... **[3]**

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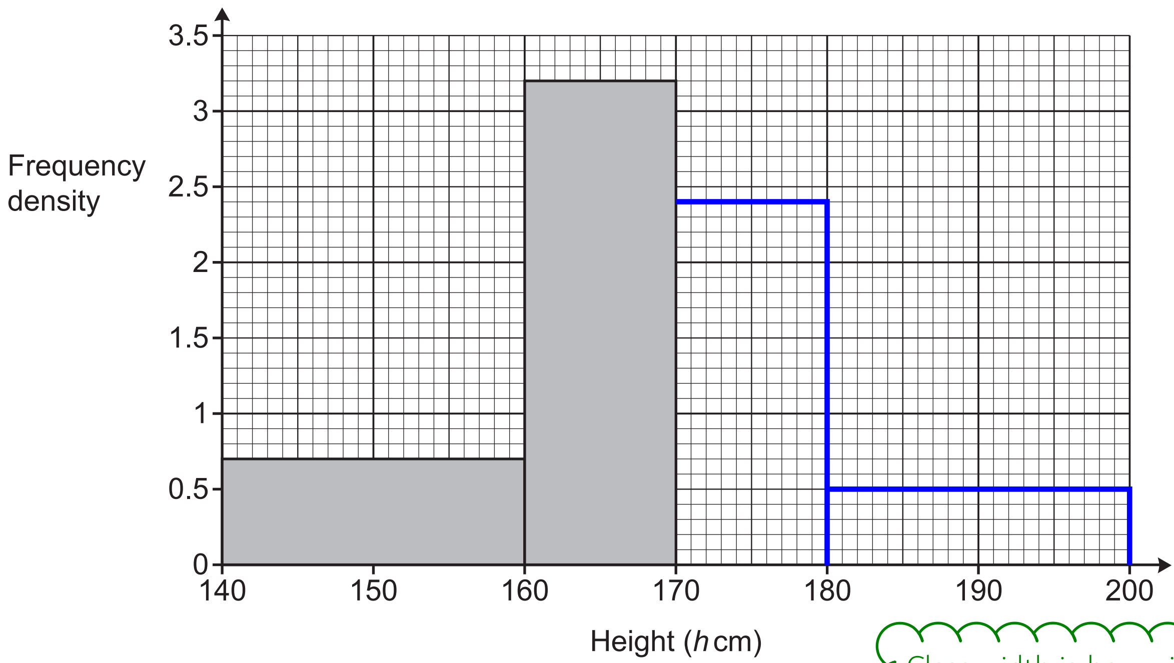
13 The height, h cm, of each member of a tennis club is recorded.
The histogram shows some of the results.



Class width is how wide the bar is on the histogram. It is found by subtracting the lower bound from the upper bound of each bar

40% of the members have a height in the interval $160 \leqslant h < 170$.
30% of the members have a height in the interval $170 \leqslant h < 180$.
100% of the members have a height in the interval $140 \leqslant h < 200$.

Complete the histogram for the intervals $170 \leqslant h < 180$ and $180 \leqslant h < 200$. **[6]**

$c^{F}d$ ← Writing the formula triangle for class width, frequency, frequency density. C stands for class width, F stands for frequency and d stands for frequency density

$(170-160)\times 3.2 = 32$ ← From the formula triangle, frequency = class width x frequency density. The class width of the second bar is (170 - 160) and its frequency density is 3.2. So the frequency of the second bar is 32

$\frac{32}{40}\times 100 = 80$ ← The frequency of the second bar is 40% of the members. So dividing it's frequency by 40 finds 1% of the members. Multiplying this by 100 works out 100%, the total number of members

$80\times\frac{30}{100} = 24$ ← Putting 30 over 100 converts 30% into a fraction, which when the total number of members is multiplied by it finds that 30% of the members is 24

$\frac{24}{180-170} = 2.4$ ← From the formula triangle, frequency density = frequency/(class width). The frequency of the third bar is 24 and it's class width is (180 - 170). So the frequency density of the third bar is 2.4

$\frac{80-(160-140)\times 0.7-32-24}{200-180} = 0.5$ ← Frequency = class width x frequency density. The class width of the first bar is (160 - 140) and it's frequency density is 0.7. Subtracting all of the other frequencies from the total number of members expresses the number of members in the fourth bar. Frequency density = frequency/(class width) so dividing this by (200 - 180) works out that the frequency density of the third bar is 0.5

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14 Find the coordinates of the turning point of the graph of $y = x^2 + 6x + 17$.

$y=(x+3)^2+17-3^2$

Completing the square. Halving the coefficient of x (which is 6) and putting the result (which is 3) in a bracket which x and squaring the bracket. Squaring the 3 and subtracting this from the 17

The turning point occurs when the bracket is equal to 0 as the smallest a squared value can be is 0. x = -3 for this to happen. When the bracket is equal to 0, y = 17 - 3^2 = 8

(..........-3.......... , ...8..........) **[4]**

15 Here are the first four terms of a quadratic sequence.

-1 3 13 29

The *n*th term is $an^2 + bn + c$.

Find the values of *a*, *b* and *c*.

4 10

Listing the differences of the first three terms. 3 - -1 = 4 and 13 - 3 = 10

6

The second difference (the difference of the differences) is 6 as 10 - 4 = 6

$3n^2$: 3 12

Halving the second difference works out that a = 3. Listing out the sequence of $3n^2$

-4 -9: $-5n+1$

Working out the sequence which must be added to the $3n^2$ to get the original sequence. -4 must be added to 3 to get -1 and -9 must be added to 12 to get 3. This is the sequence of -5n + 1 as it decreases by 5 each time and the 0th term would be 1

The sequence is $3n^2$ - 5n + 1

a = ...3..........

b = ...-5..........

c = ...1.......... **[4]**

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16 The formula

$P = 6800 \times 1.045^n$

is used to predict the population, P, of an island n years after 2018.

(a) Write down the population of the island in 2018.

2018 is 0 years after 2018 so n is 0. Anything to the power of 0 is 1 so $1.045^0 = 1$. 6800 x 1 = 6800

(a) ....6800........................ **[1]**

(b) Write down the percentage growth rate used in the formula.

The formula is basically a compound interest formula. To convert the decimal multiplier into a percentage it should be multiplied by 100. 1.045 x 100 = 104.5. So it increases to 104.5% each year, which is a 4.5% increase

(b) ....4.5........................ % **[1]**

(c) **(i)** Work out the population predicted by the formula for the year 2030.

$6800 \times 1.045^{2030-2018}$

2030 - 2018 expresses the difference between the two years and therefore how many years after 2018 the 2030 is, which is the value of n. Substituting this into the formula

The population should be a whole number so 11531.99... is rounded to the nearest whole number

(c)(i) ....11532........................ **[2]**

(ii) Give **one** reason why the answer to **(c)(i)** may **not** be reliable.

The rate may not continue

The population cannot continue to increase at the same rate forever. If it did, the population of the island would be about 89 trillion billion after 1000 years...

[1]

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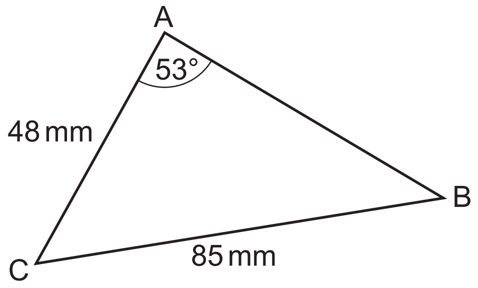
17 The diagram shows triangle ABC.



Not to scale

We do not know if it is a right angled triangle so non-right angled trigonometry will be needed. There is not enough information in the triangle to work out AB using either the sine rule or cosine rule straight away. So we need to work out another angle first

AC = 48 mm, BC = 85 mm and angle BAC = 53°.

Calculate length AB.
You must show your working.

$\frac{\sin A}{a} = \frac{\sin B}{b}$ ← There are opposite pairs of sides and angles so the sine rule can be used to work out angle B. Quoting the sine rule with the angles as numerators

$B = \sin^{-1}\left(\frac{b\sin A}{a}\right)$ ← Rearranged to make angle B the subject by multiplying both sides by side b and then doing the inverse sin of both sides

$= \sin^{-1}\left(\frac{48\sin 53}{85}\right)$ ← a is opposite A, b is opposite B and c is opposite C. Substituting in 48 for b, 53 for A and 85 for a

$= 26.8...$ ← Storing the exact value as B on the calculator

$C = 180 - 53 - B$ ← There are 180° in total in a triangle. So subtracting the other angles from 180 finds C

$= 100.1...$ ← Storing the exact value as C on the calculator

$\frac{a}{\sin A} = \frac{c}{\sin C}$ ← There are opposite pairs of sides and angles so the sine rule can be used to work out side c. Quoting the sine rule with the sides as numerators

$c = \frac{a\sin C}{\sin A}$ ← Rearranged to make c the subject by multiplying both sides by sinC

$= \frac{85\sin C}{\sin 53}$ ← Substituting in 85 for a, the exact stored value of C and 53 for A

........................104.8........................ mm **[6]**

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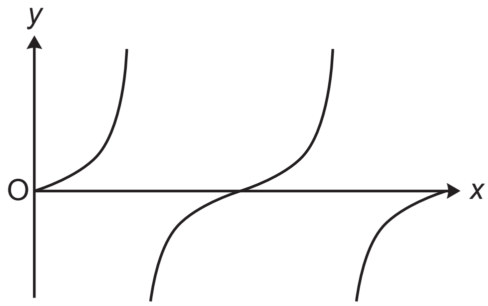
18 **(a)** For each graph below, select its possible equation from this list.

$y = x$ $\quad y = x^2$ $\quad y = \frac{1}{x}$

$y = \sin x$ $\quad y = \cos x$ $\quad y = \tan x$

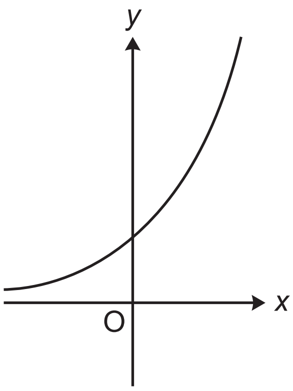
$y = 3^x$ $\quad y = \left(\frac{1}{3}\right)^x$

(i)



(a)(i) $y =$ ....tanx.......... **[1]**

(ii)



(ii) $y =$ ....3^x.......... **[1]**

These are both typical tan and exponential graphs so we could just memorise what they look like

Alternatively, table mode can be used on the calculator to give a table of values of each equation and we can roughly imagine if each one would look like these graphs

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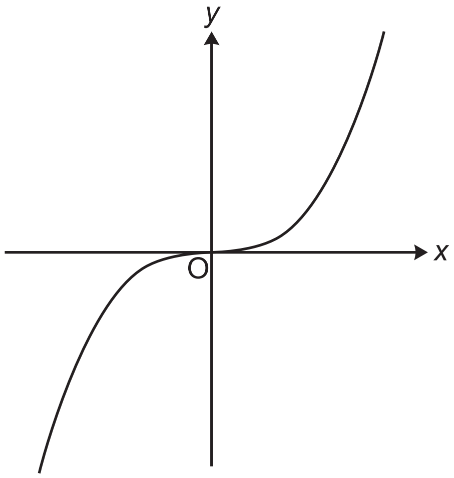
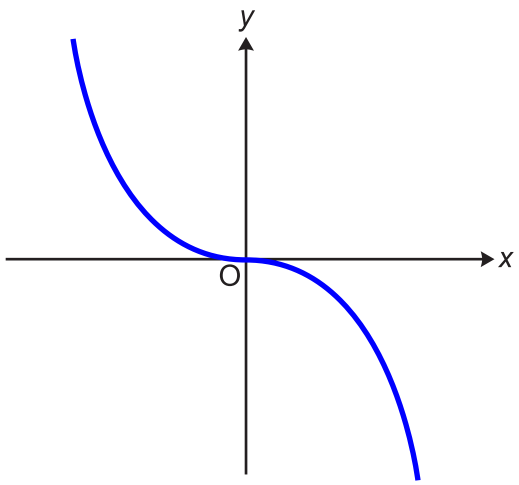
(b) Here is a sketch of $y = x^3$.



On the axes below, sketch the graphs of

(i) $y = -x^3$



The graph reflects in the x-axis

[1]

(ii) $y = x^3 - 8$, showing the values of any intercepts with the axes.

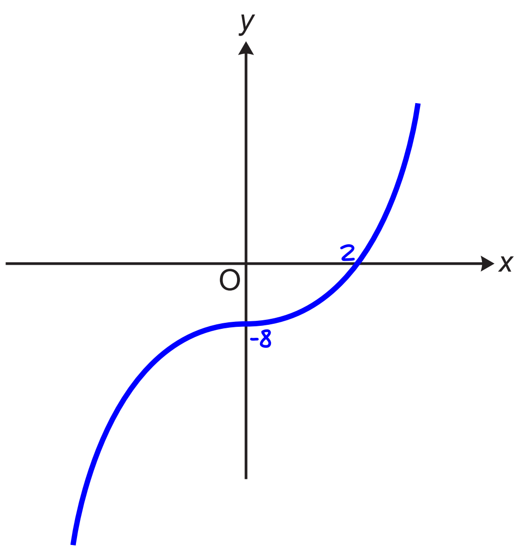
$0 = x^3 - 8$

$x^3 = 8$

$x = \sqrt[3]{8}$

$= 2$

Working out the x-intercept by setting y equal to 0 then rearranging to find x



The graph translates 8 down meaning the y-intercept will be -8 as it was 0 before

[3]

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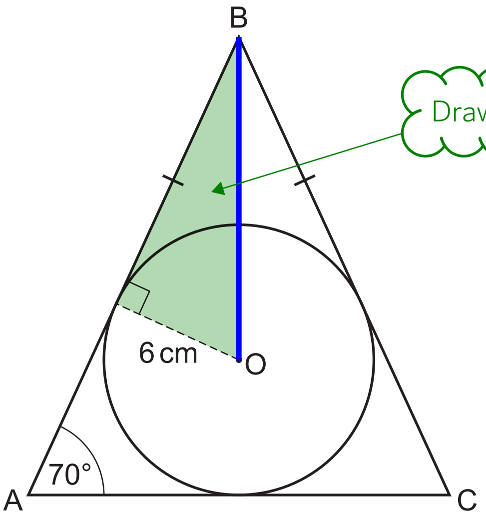
19 ABC is an isosceles triangle.
The sides of the triangle ABC are all tangents to a circle of radius 6 cm, centre O.



Drawing on length BO makes a right angled triangle

Not to scale

Angle BAC = 70° and BA = BC.

(a) Show that length BO is 17.54 cm, correct to **2** decimal places. **[4]**

$\frac{180-70\times2}{2}=20$

The base angles of an isosceles triangle are the same so angle ACB is also 70°. There are 180° in total in a triangle so subtracting 2 lots of the 70° angle leaves angle ABC. Isosceles triangles are symmetrical so both halves of the angle must be the same. So dividing angle ABC by 2 leaves angle ABO

$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

Right angled trigonometry can be used so quoting SOH CAH TOA as formula triangles. Ticking O as we have the opposite and H as we are looking for the hypotenuse

$\frac{6}{\sin20}=17.542...$

There are two ticks on the SOH formula triangle so this one can be used. Covering over H tells us that the hypotenuse = opposite/(sin of the angle). The opposite is 6 and the angle is 20

Storing the exact value as A on the calculator as it is needed for the next question

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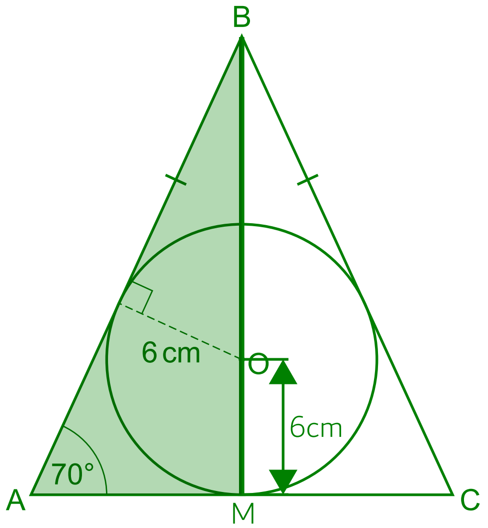
(b) Find the area of triangle ABC.
You must show your working.



Area of triangle = 1/2 x base x height. So the base and height need to be found. Drawing on the height from B to M forms the green right angled triangle. BM is 6cm longer than BO, which was found in the previous question, as OM is a radius and the radius of the circle is 6cm

17.542... + 6 = 23.5... ← So adding 6 to side BO works out the height BM

Storing the exact value as B on the calculator

S O H C A H T O A ← Right angled trigonometry can be used to work out side AM, which is half of the base. Ticking O as we have the opposite and A as we are trying to find the adjacent

$\frac{23.5...}{\tan 70} \times 23.5...$ ← There are two ticks on TOA so this formula triangle can be used. From the formula triangle, adjacent = opposite/(tan of the angle). The opposite is 23.5... and the angle is 70. Half of the base, side AM, is expressed then this can be multiplied by the height BM to work out the area of the triangle

Using the exact value stored as B on the calculator for 23.5...

........................201.7........................ cm^2 **[5]**

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20 Solve algebraically.

$$y = x + 3$$
$$(x - 3)^2 + y^2 = 50$$

You must show your working.

$(x-3)^2 + (x+3)^2$ ← Substituting (x + 3) for y in the left side of the second equation

$x^2 - 6x + 9 + x^2 + 6x + 9 = 50$ ← Expanding each square bracket by squaring the first term, doubling the product of both terms, and squaring the last term. Now including the right side of the second equation

$2x^2 = 50 - 9 - 9$ ← Collecting like terms cancels out the x terms. So as there is only one power of x, the equation can be solved by rearranging. Subtracting both 9s from both sides to get the x^2 term on its own

$x^2 = \frac{32}{2}$ ← Dividing both sides by 2 to get x^2 on its own

$x = \sqrt{16}$ ← Square rooting both sides to get x on its own

$= \pm 4$ ← Remembering that when square rooting there is also a negative value

$y = \pm 4 + 3$ ← Substituting the value of x back into the first equation to find y

$x =$ ......4...... $y =$ ......7......

$x =$ ......-4...... $y =$ ......-1...... **[5]**

4 + 3 = 7
-4 + 3 = -1

END OF QUESTION PAPER

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