

Please write clearly in block capitals.

Centre number Candidate number

Surname _____

Forename(s) _____

Candidate signature _____

GCSE MATHEMATICS

H

Higher Tier Paper 3 Calculator

Tuesday 13 June 2017

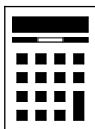
Morning

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
TOTAL	



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided

1 $\mathbf{a} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$
Circle the vector $2\mathbf{a} + \mathbf{b}$

$$n \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} nx \\ ny \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} w \\ z \end{bmatrix} = \begin{bmatrix} x+w \\ y+z \end{bmatrix}$$

[1 mark]

$$\begin{pmatrix} -5 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -11 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -5 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} -11 \\ -1 \end{pmatrix}$$

Must be -5 in the x-direction and -3 in the y-direction.

$$\begin{aligned} 2 \times -4 &= -8 \\ -8 + 3 &= -5 \\ 2 \times -1 &= -2 \\ -2 + -1 &= -3 \end{aligned}$$

- 2 Which of these values of n makes 2.7×10^n a cube number?
Circle your answer.

[1 mark]

0

1

2

3

2.7×10^1 gives 27, which is 3^3 .

The cube root of 2.7×10^n does not give whole numbers for the other values.

- 3 Rearrange $2x = \frac{y}{w}$ to make w the subject.
Circle your answer.

[1 mark]

$$w = \frac{2y}{x}$$

$$w = \frac{2x}{y}$$

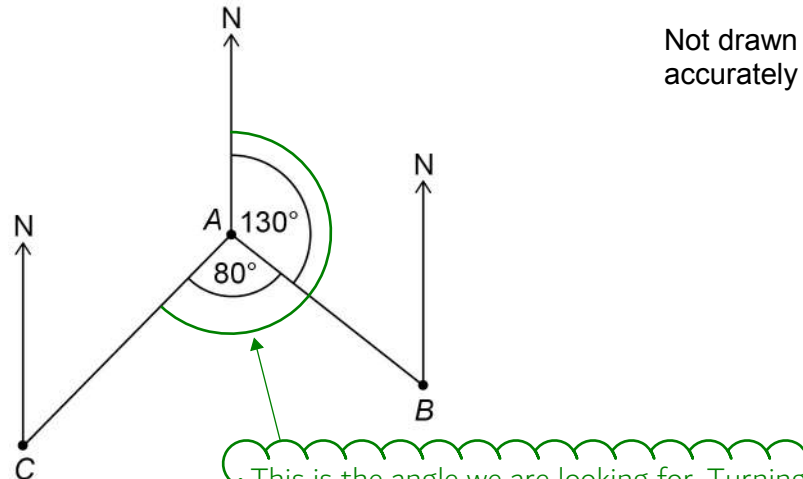
$$w = \frac{y}{2x}$$

$$w = \frac{x}{2y}$$

Multiply both sides by w to get $2xw = y$.
Divide both sides by $2x$ to get w on its own.



4



Work out the bearing of C from A.

Circle your answer.

$$130 + 80 = 210$$

[1 mark]

030°

130°

150°

210°

Turn over for the next question

Turn over ►



- 5 A coin lands on Tails 200 times.
The relative frequency of Tails is 0.4

Work out the number of times the coin was thrown.

Let x be the number of times the coin was thrown.
 $0.4x = 200$

[2 marks]

$$x = \frac{200}{0.4}$$

Answer

500

- 6 How are the whole number solutions to A and B different?

A Solve $3 \leq 3x < 18$

B Solve $3 < 3x \leq 18$

All whole numbers which
are greater than or equal
to 1 but less than 6.

[2 marks]

A: $1 \leq x < 6$

1, 2, 3, 4, 5

B: $1 < x \leq 6$

2, 3, 4, 5, 6

Dividing $3x$ by 3 gets x so we
have to divide all sides by 3.
 $3/3 = 1$ and $18/3 = 6$

All whole numbers which
are greater than 1 but
less than or equal to 6.



7 (a) The length of a pipe is 6 metres to the nearest metre.

Complete the error interval for the length of the pipe.

[2 marks]

The resolution is 1m as it is to the nearest metre. Halving this then adding and subtracting this from 6 gives the upper and lower bound. $6 + 1/2 = 6.5$. $6 - 1/2 = 5.5$

Answer 5.5 m \leq length < 6.5 m

7 (b) The length of a different pipe is 4 metres to the nearest metre.

Olly says,

“The total length of the two pipes is 11 metres to the nearest metre.”

Give an example to show that he could be correct.

[2 marks]

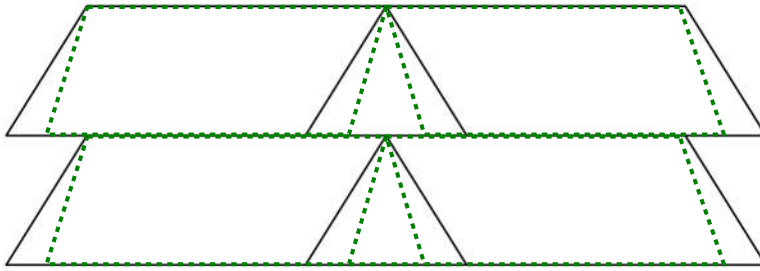
$$6.4 + 4.4 = 10.8$$

This rounds to 11.

The first pipe can be higher than 6 but has to be less than 6.5.
This pipe can be higher than 4 but has to be less than 4.5.



- 8 This shape is made from two triangles and four congruent parallelograms.



Not drawn
accurately

For each statement, tick the correct box.

- 8 (a) The triangles are equilateral.

Must be true

Could be true

Must be false

The parallelograms could be like this. The triangles aren't equilateral. But they could be equilateral as they are in the original diagram.

[1 mark]

- 8 (b) The triangles are congruent.

Must be true

Could be true

Must be false

All three sides in both triangles must be the same as they are shared with the parallelograms. Regardless of how the parallelograms are drawn, the triangles will be identical so must be congruent.

[1 mark]



9 There are 720 boys and 700 girls in a school.

The probability that a boy chosen at random studies French is $\frac{2}{3}$

The probability that a girl chosen at random studies French is $\frac{3}{5}$

9 (a) Work out the number of students in the school who study French.

[3 marks]

$$\frac{2}{3} \times 720 + \frac{3}{5} \times 700$$

2/3 of the 720 boys and
3/5 of the 700 girls.

Answer

900

9 (b) Work out the probability that a student chosen at random from the whole school does **not** study French.

[2 marks]

$$720 + 700 = 1420$$

$$1420 - 900 = 520$$

Total number of
students in the school.

520 students do not
study French.

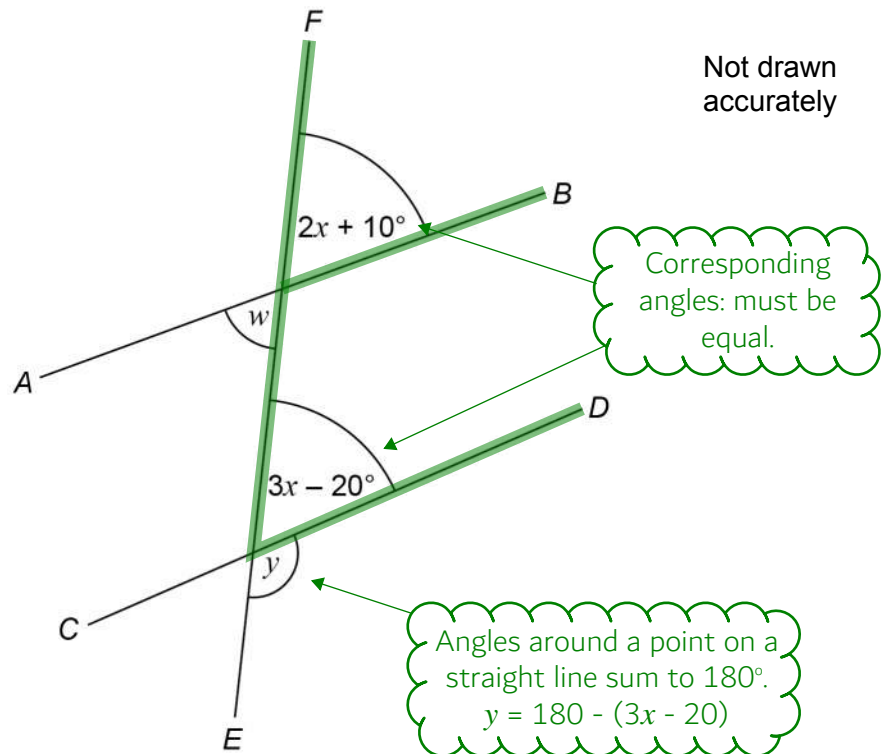
Answer

$\frac{520}{1420}$

520 out of 1420 students
do not study French.



- 10 AB , CD and EF are straight lines.



- 10 (a) Ava assumes that AB and CD are parallel.

What answer should she get for the size of angle y ?

[4 marks]

$$2x + 10 = 3x - 20$$

$$30 = x$$

Subtracting $2x$ from both sides to get all the x terms on the same side. Adding 20 to both sides to get the x terms on their own.

$$3 \times 30 - 20 = 70$$

$$180 - 70$$

Substituting 30 for x in $3x - 20$ then subtracting it from 180 as it lies on a straight line with y

Answer 110 degrees



10 (b)

In fact,

 AB and CD are **not** parallelangle w is 60° What effect does this have on the size of angle y ?

Tick a box.

 y is bigger y is now 125, which is bigger than 110 y is the same y is smaller

Show working to support your answer.

[3 marks]

$$2x + 10 = 60$$

Opposite angles are equal so w must equal $2x + 10$

$$2x = 50$$

Subtracting 10 from both sides to get the x terms on their own. Then dividing both sides by 2 to find x .

$$x = 25$$

$$3 \times 25 - 20 = 55$$

Substituting 25 for x in $3x - 20$ then subtracting it from 180 as it lies on a straight line with y

$$180 - 55 = 125$$

Turn over for the next question

Turn over ►



- 11 Purple paint is made by mixing red paint and blue paint in the ratio 5 : 2
Yan has 30 litres of red paint and 9 litres of blue paint.

What is the **maximum** amount of purple paint he can make?

[3 marks]

$$5r = 30 \quad r = 6 \quad 2r = 12$$

Assuming that all the red paint is used (represented by 5 parts in the ratio), then 12L of blue paint would be needed and there isn't this much.

$$2q = 9 \quad q = 4.5 \quad 5q = 22.5$$

$$9 + 22.5$$

Assuming that all the blue paint is used (represented by 2 parts in the ratio), then 22.5L of red paint would be needed. Adding this to the 9L gives us the total volume of the purple paint.

Answer 31.5 litres

- 12 $(ar^b)^4 = 16r^{20}$ where a and b are positive integers.

Work out a and b

Laws of indices:

$$(a^x)^y = a^{xy}$$

[2 marks]

$$a^4 = 16$$

$$r^{b \times 4} = r^{20}$$

$$a = \sqrt[4]{16} = 2$$

These can be dealt with separately.

$$b \times 4 = 20$$

$$b = \frac{20}{4}$$

$a =$ 2 $b =$ 5



13

In a class of 28 students

the mean height of the 12 boys is 1.58 metres

the mean height of all 28 students is 1.52 metres.

Work out the mean height of the girls.

[4 marks]

$$\text{Mean for the girls} = \frac{\text{total height of the girls}}{\text{number of the girls}}$$

$$\text{total height of the girls} = \text{total height of the students} - \text{total height of the boys}$$

$$\text{total height} = \text{mean} \times \text{number}$$

$$\text{number of the girls} = \text{number of students} - \text{number of boys}$$

$$\frac{28 \times 1.52 - 12 \times 1.58}{28 - 12}$$

Answer 1.475 metres

14

 $xy = c$ where c is a constant.

Circle the correct statement.

Increasing x must decrease y in order
for it to equal to a constant value.

[1 mark]

 y is directly proportional to x y is directly proportional to $\frac{1}{x}$ These all mean the same thing.
Increasing x will increase y . y is inversely proportional to $\frac{1}{x}$ x is directly proportional to y

Turn over for the next question

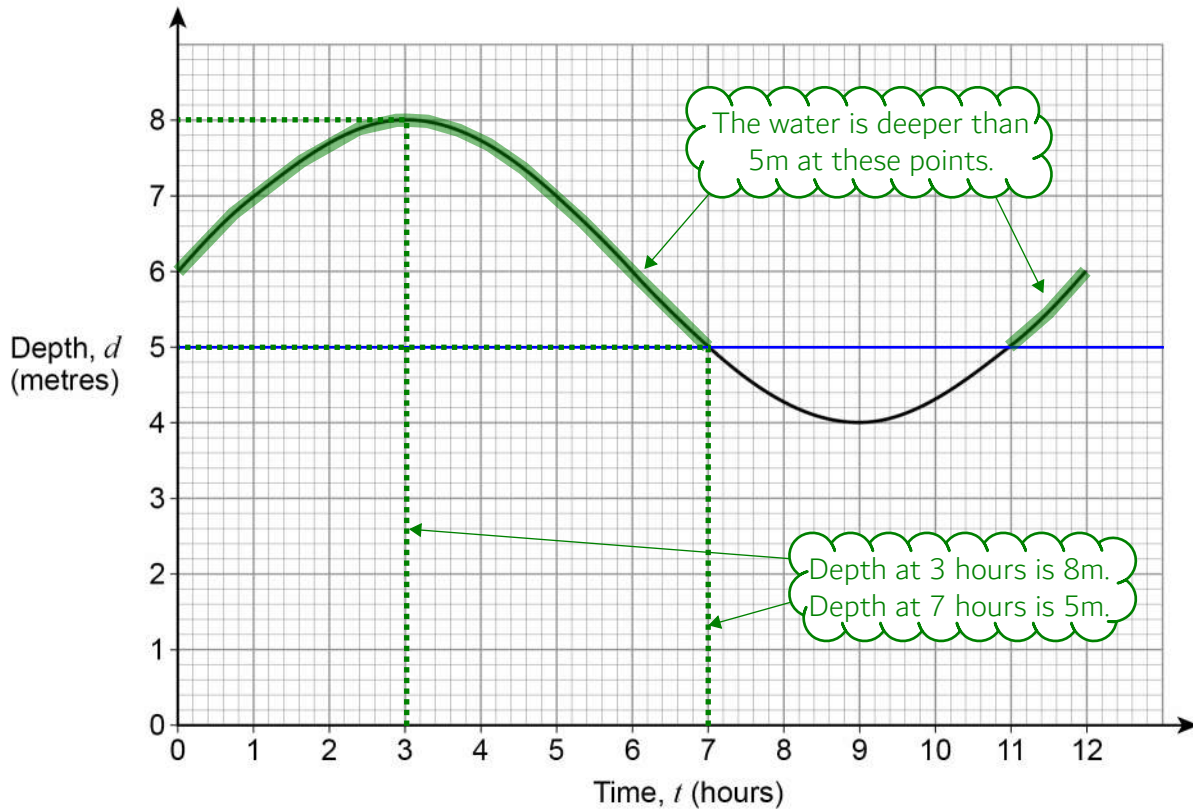
Turn over ►



15 The graph shows the depth of water in a harbour for 12 hours.

d is the depth of water in a harbour in metres

t is the number of hours after 9 am



15 (a) For how many of the 12 hours is the depth more than 5 metres?

[1 mark]

Answer _____

8

12 noon is represented by $t = 3$ as it is 3 hours after 9am. 4pm is when $t = 7$.

15 (b) By how much does the depth change between 12 noon and 4 pm?

[1 mark]

Answer _____

3

metres

8 - 5



- 16** The value of a new car is £18 000
The value of the car decreases by
25% in the first year
12% in each of the next 4 years.

Work out the value of the car after 5 years.

[3 marks]

$$18000 \times 0.75 \times 0.88^4$$

$100\% - 25\% = 75\%$
 $75\% \text{ of } 18000 = 18000 \times 0.75$

 $100\% - 12\% = 88\%$
 Reducing by 88% four times can be calculated
 by multiplying by 0.88 four times; this is
 multiplying by 0.88^4 .

Answer £ 8095.89

Turn over for the next question



17

Liam drives his car.

He drives the first 9 miles in 9 minutes.

1 mile per minute is 60 miles per hour.

He then drives at an average speed of 70 miles per hour for 1 hour 36 minutes.

He finds this information about his car.

Average speed	Miles travelled per gallon
65 miles per hour or less	50
More than 65 miles per hour	40

Use the information to show that his car uses less than 3 gallons of petrol for the drive.

[5 marks]

$$d = st = 70 \times \left(1 + \frac{36}{60}\right) = 112$$

112 miles travelled.

1 hour add 36 out of 60 minutes. This converts the time into hours.

$$\frac{9}{50} + \frac{112}{40} = 2.98$$

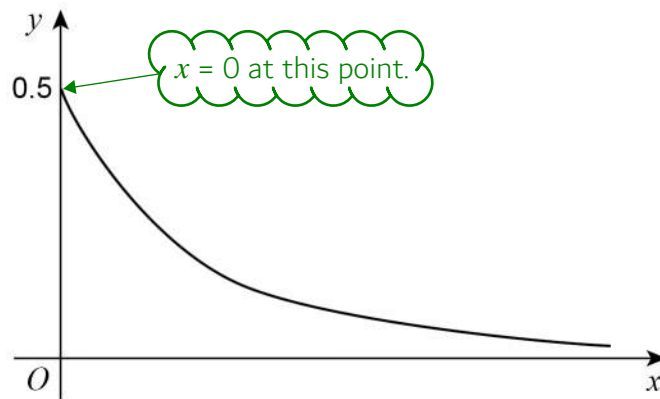
2.98 is less than 3 gallons.

If 50 miles can be done with one gallon, 9/50 gallons would be used for 9 miles.

This expresses how many lots of 40 miles were done and hence how many gallons were used.



- 18 Nick sketches the graph of $y = 0.5^x$ for $x \geq 0$



Make **one** criticism of his sketch.

[1 mark]

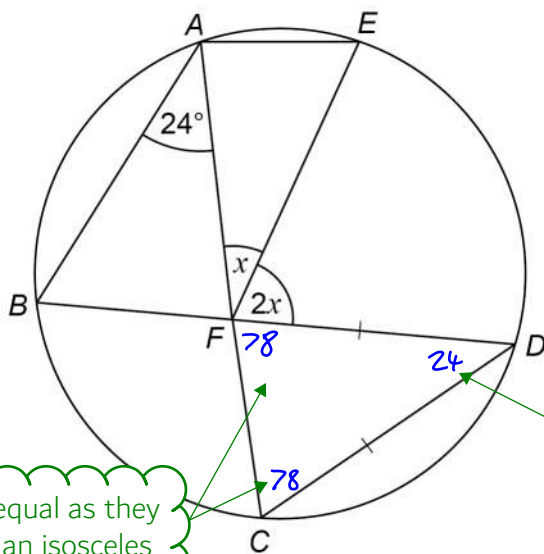
$$0.5^0 = 1$$

Turn over for the next question

Turn over ►



19 A, B, C, D and E are points on a circle.
 BFD and AFC are straight lines.
 DC = DF



Not drawn accurately

These angles must be equal as they are the base angles of an isosceles triangle (two of the sides are equal).

Circle theorem: angles in the same segment are equal.

Work out the size of angle x .
 You **must** show your working which may be on the diagram.

[4 marks]

$$\frac{180 - 24}{2} = 78$$

$$x + 2x + 78 = 180$$

$$3x = 180 - 78$$

x and $2x$ lie on straight line AFC. Angles around a point on a straight line are 180° .

$$(180 - 78) / 3 = 34$$

Answer 34 degrees



20 This sign shows when a lift is safe to use.

Total mass of people must be 450 kg or less

Ben and some other people are in the lift.

Their total mass is 525 kg to the nearest 5 kg

Ben gets out.

He has a mass of 78 kg to the nearest kg

Is the lift now safe to use?

You **must** show your working.

This is acceptable as the mass can be equal to 450kg.

[4 marks]

$$527.5 - 77.5 = 450$$

In the worst-case scenario, the total mass of the people in the lift would be as high as possible and Ben having the lowest mass possible. This leads to the greatest possible mass being left in the lift.

$$525 + 5/2 = 527.5$$

$$78 - 1/2 = 77.5$$

Adding or subtracting half of the resolution of the measurement calculates the bounds.

Answer _____

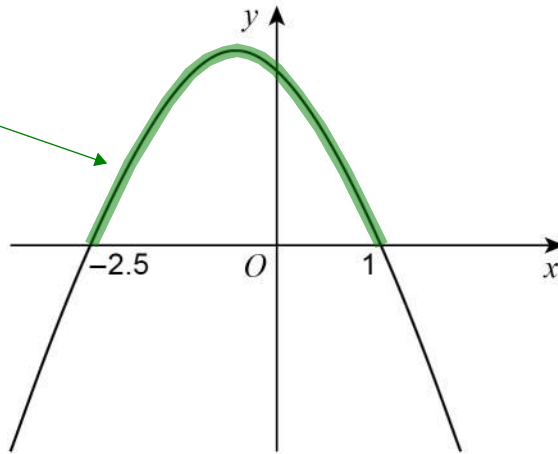
Yes

Turn over for the next question



- 21 Here is a sketch of $y = f(x)$ where $f(x)$ is a quadratic function.
The graph intersects the x -axis where $x = -2.5$ and $x = 1$

This section of the curve is greater than 0.



Not drawn accurately

Circle the solution of $f(x) > 0$

[1 mark]

$$x < -2.5 \text{ or } x > 1$$

$$x > -2.5 \text{ or } x > 1$$

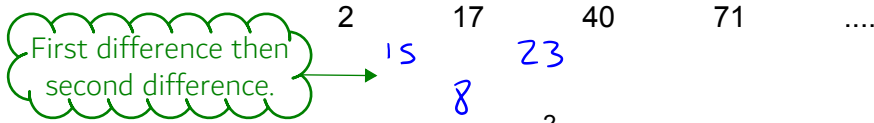
$$-2.5 < x < 1$$

$$x > -2.5 \text{ or } x < 1$$



22

Work out an expression for the n th term of the quadratic sequence



Give your answer in the form $an^2 + bn + c$ where a , b and c are constants.

[3 marks]

4 16

-2 1

The sequence of $4n^2$.

Half the second difference to get the coefficient of n^2 .

Working out the linear sequence which needs to be added to get the original sequence. It must involve $3n$ as it goes up by 3. The 0th term would be -5 so it must be $3n - 5$

Answer $4n^2 + 3n - 5$

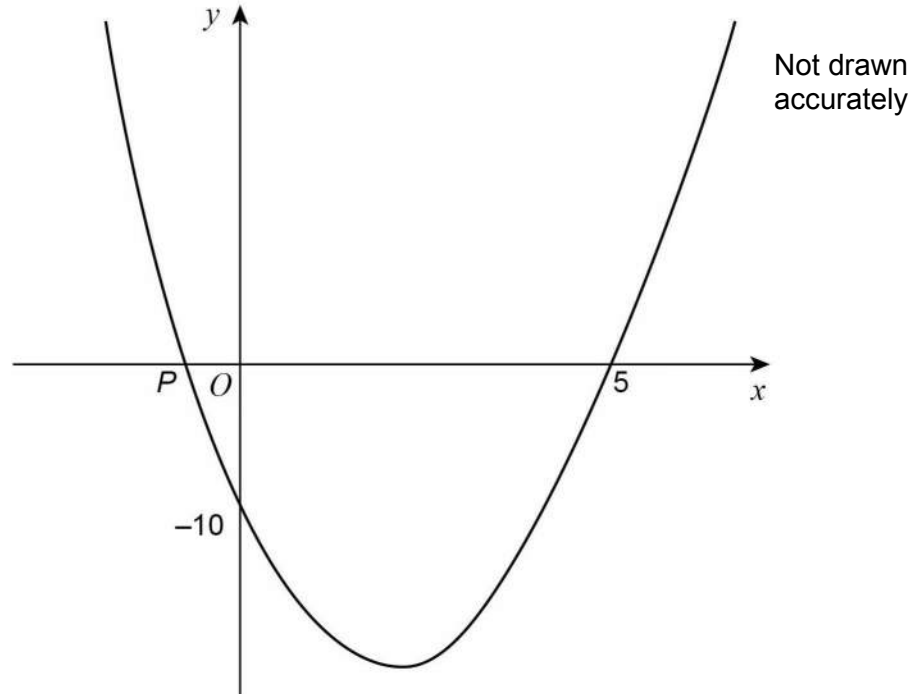
Turn over for the next question



23

Here is a sketch of $y = x^2 + bx + c$

The curve intersects

the x -axis at $(5, 0)$ and point P the y -axis at $(0, -10)$ Work out the x -coordinate of the turning point of the graph.**[4 marks]**

The turning point can be found by completing the square. It is also half way between the solutions as quadratics are symmetrical. Either way, we need to find the equation.

$$y = x^2 + bx - 10$$

Substituting in $(0, -10)$ gives: $-10 = 0^2 + b \times 0 + c$, $-10 = c$

$$b = \frac{y - x^2 + 10}{x} = \frac{0 - 5^2 + 10}{5} = -3$$

Rearranged to make b the subject and substituting $(5, 0)$

$$y = \left(x - \frac{3}{2}\right)^2 + d$$

Completing the square on the equation $y = x^2 - 3x - 10$. d does not need to be evaluated to find the x -coordinate of the turning point. The minimum point occurs when $x = 3/2$ as it makes the bracket equal 0 and 0^2 is the lowest possible square.

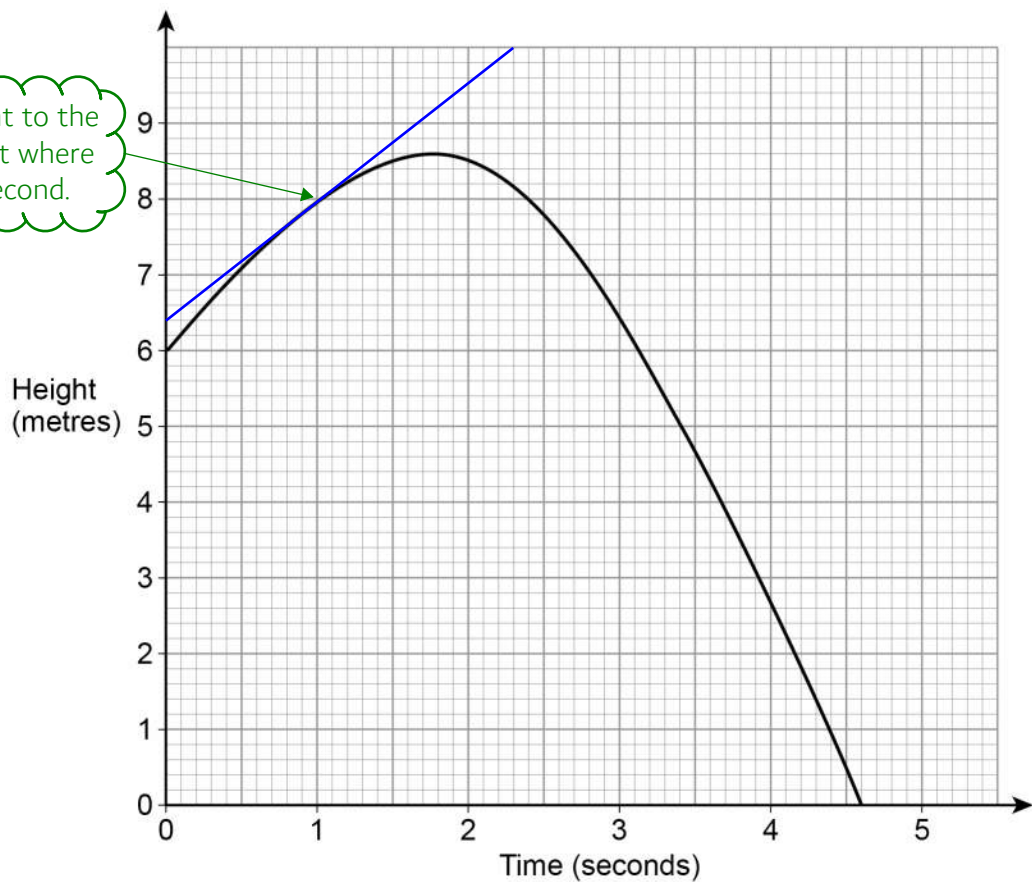
Answer

$$\frac{3}{2}$$


24

A ball is thrown from a point 6 metres above the ground.

The graph shows the height of the ball above the ground, in metres.



Drawing a tangent to the curve at the point where the time is 1 second.

Estimate the speed of the ball, in m/s, after 1 second.

You **must** show your working.

$$\frac{10 - 6.4}{2.3 - 0}$$

[2 marks]

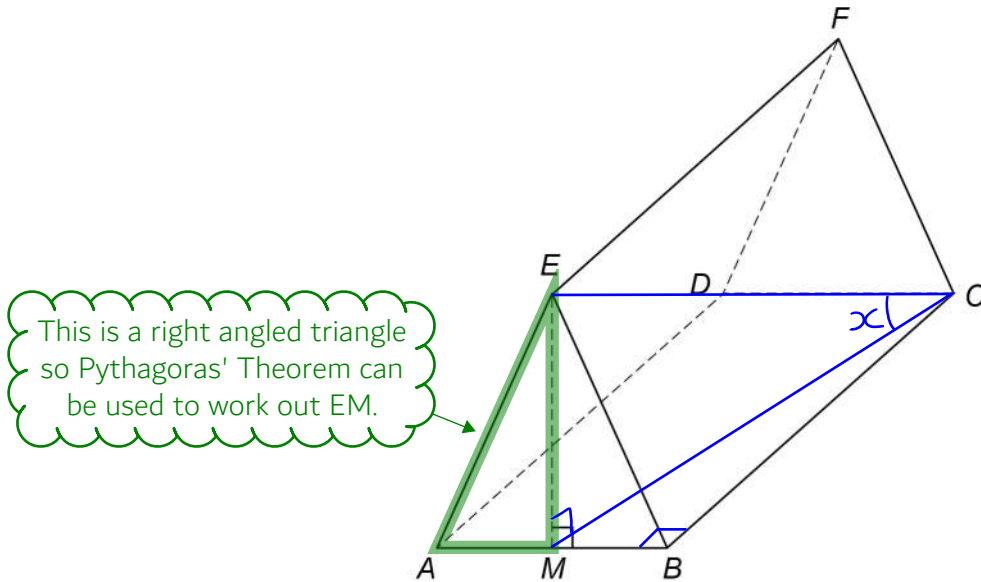
Calculating the gradient of the tangent using (change in y)/(change in x)

Answer 1.6 m/s

Turn over ►



- 25** Rectangle $ABCD$ is the horizontal base of a triangular prism $ABCDEF$.
 $AE = BE$
 E is vertically above M , the midpoint of AB .
 $AB = 16$ cm $AE = 17$ cm $BC = 30$ cm



This is a right angled triangle
so Pythagoras' Theorem can
be used to work out EM .

- 25 (a)** Show that $EM = 15$ cm

[2 marks]

$$\sqrt{17^2 - 8^2} = 15$$

$$a^2 + b^2 = c^2$$

Rearranged to make a the subject
then substituted c (the longest side)
for 17 and b for half of 16 (side AM).



25 (b) Work out the size of angle ECM .

[4 marks]

$$\begin{array}{ccccccc} \text{S} & \text{O} & & \text{H} & \text{C} & & \text{A} & \text{H} & \text{T} & \text{O} & \text{A} \\ \hline & & & & & & & & & & \\ \hline \tan^{-1} & \left(\frac{15}{\sqrt{8^2 + 30^2}} \right) & & & & & & & & & \end{array}$$

Right angled trigonometry can be used on triangle ECM to find the angle ECM . Listing SOH CAH TOA as formula triangles and ticking what sides we have. EM is the opposite and MC is the adjacent, which can be found using Pythagoras' Theorem on triangle ECB .

There are two ticks on TOA so this formula triangle can be used. Covering over T, tan of the angle, tells us that $(\tan \text{ of the angle}) = \text{opposite/adjacent}$. Rearranging to make the angle the subject gives $\text{angle} = \tan^{-1}(\text{opposite/adjacent})$. The opposite, side EM, is 15cm as was shown in the previous question. The adjacent, side MC, is found using Pythagoras' Theorem on triangle ECB .

$$\begin{aligned} a^2 + b^2 &= c^2 \\ c &= \sqrt{a^2 + b^2} \end{aligned}$$

a is 8cm as MB is half of AB. b is 30cm as it is side BC.

Answer 25.8 degrees

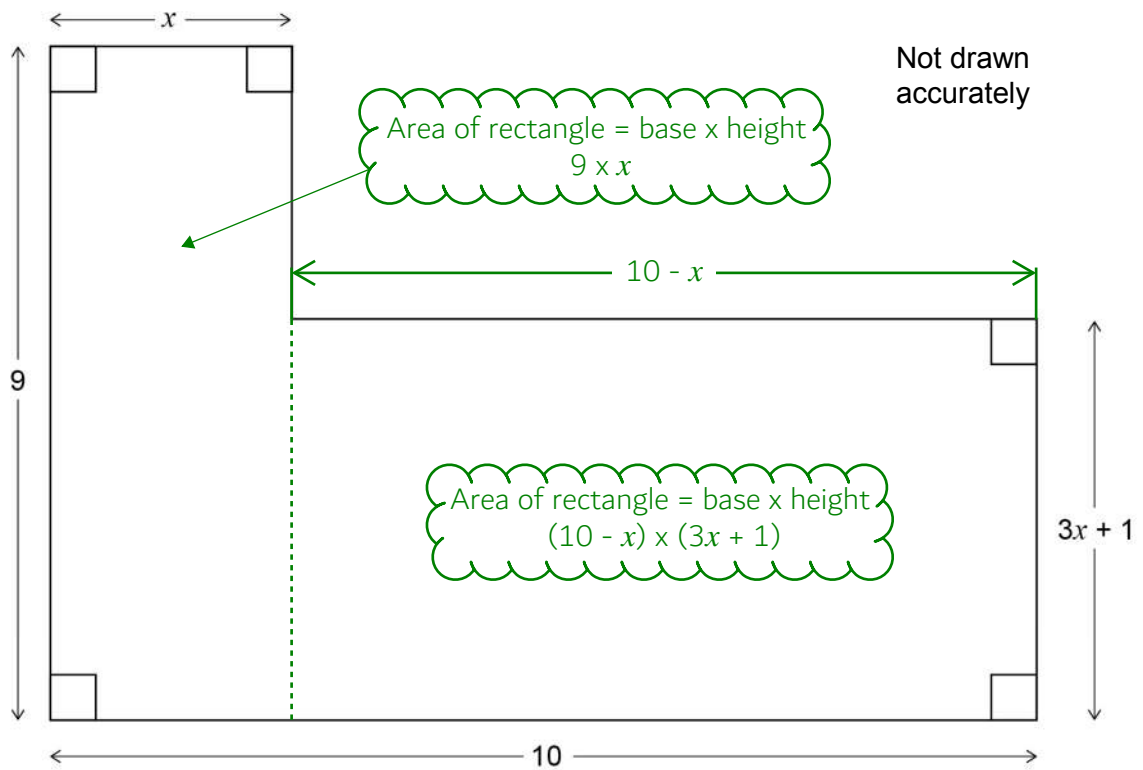
Turn over for the next question



26

Here is an L-shape.

All dimensions are in centimetres.



The area of the L-shape is 65 cm^2

Work out the value of x .

Adding the area of the
rectangles together gives 65.

[6 marks]

$$9x + (10 - x)(3x + 1) = 65$$

Expanding the brackets.

$$9x + 30x + 10 - 3x^2 - x = 65$$

Collecting like terms and subtracting 65 from
both sides to bring it into the quadratic form.

$$-3x^2 + 38x - 55 = 0$$

Solving using the quadratic formula.
 $a = -3$, $b = 38$ and $c = -55$

$$x = \frac{-38 \pm \sqrt{38^2 - 4 \times -3 \times -55}}{2 \times -3}$$

$10 - x$ gives a negative length
and this is not possible.

$$x = \frac{5}{3}$$

$$x = 11$$

Answer

$$\frac{5}{3}$$

Turn over for the next question

6

Turn over ►



27

Prove that $x^2 + x + 1$ is always positive.

[3 marks]

$$\begin{aligned} & \left(x + \frac{1}{2}\right)^2 + 1 - \left(\frac{1}{2}\right)^2 \\ & \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \end{aligned}$$

Completing the square finds the turning point (in this case the minimum point) of a quadratic.

Minimum value is $\frac{3}{4}$

The lowest the squared bracket can be is 0 as any positive or negative number squared gives a positive number. $0 + 3/4 = 3/4$

END OF QUESTIONS

