AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

GCSE MATHEMATICS

H

Higher Tier   Paper 2 Calculator

Thursday 6 June 2019   Morning   Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Do not write outside the box

Answer **all** questions in the spaces provided

1 Circle the point that lies on the curve $y = x^2 - 4x + 1$

[1 mark]

$(-1)^2 - 4(-1) + 1 = 6$

(–1, 4) (–1, –4) (–1, –2) (–1, 6)

Substituting in the x-coordinate (which is the same for all options) into the equation to work out which y-coordinate is correct

2 The height of a tree is 12 metres, correct to the nearest metre.

Circle the error interval.

[1 mark]

11.5 m $\leqslant$ height $<$ 12.5 m	11.5 m $\leqslant$ height $\leqslant$ 12.5 m
11.5 m $<$ height $\leqslant$ 12.5 m	11.5 m $<$ height $<$ 12.5 m

The height can be equal to 11.5m as this rounds up to 12m but can't be equal to 12.5m as this rounds up to 13m

Do not write outside the box

3 $2a$ is five times bigger than b.

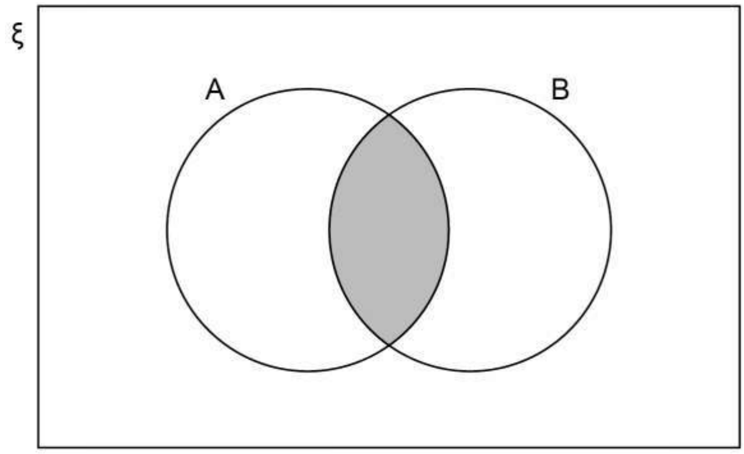
Circle the ratio $a : b$

[1 mark]

10 : 1 1 : 10 5 : 2 2 : 5

2a = 5b, so a could be 5 and b could be 2

4



Which of these represents the shaded region?

Circle your answer.

[1 mark]

$A \cup B$ $(A \cap B)'$ $A \cap B$ $A' \cup B'$

The shaded region is the intersection of A and B

Turn over for the next question

4

Do not write outside the box

5 Using ruler and compasses, show the region inside the grid that is

less than 4 cm from *A*

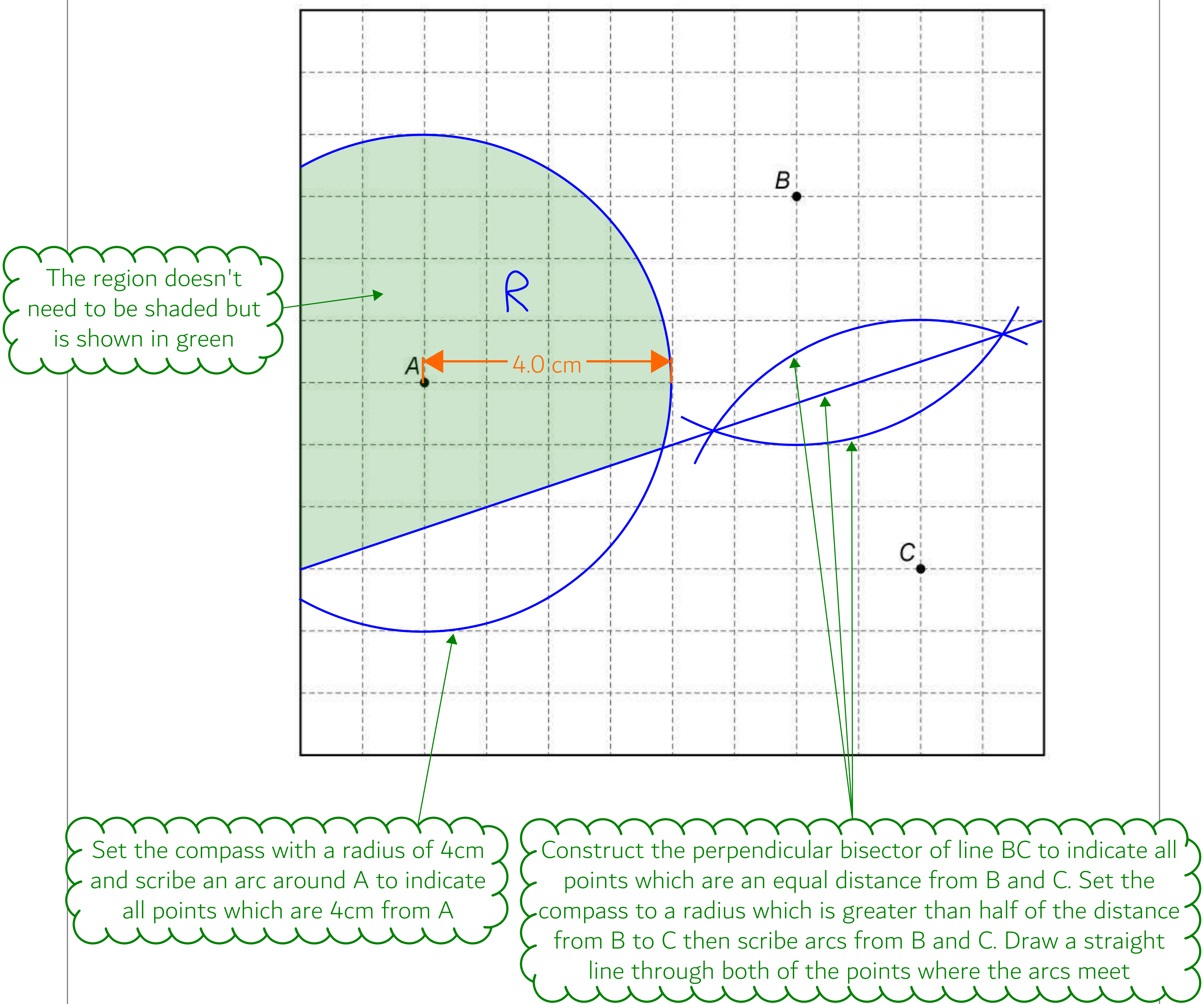
and

nearer to *B* than to *C*.

Label the region R.

Show all your construction lines.

[3 marks]



Do not write outside the box

6 Beth drives 200 miles in 4 hours.

She drives the first 18 miles at an average speed of 36 mph

Work out her average speed for the rest of the journey.

[3 marks]

$s \; d \; t$

This is the formula triangle for speed, distance and time

$$\frac{200-18}{4-\frac{18}{36}}$$

From the formula triangle, speed = distance/time. 18 miles has been done in the first part of the journey and there are 200 miles in total so the distance for the rest of the journey is 200 - 18. The whole journey takes 4 hours so subtracting the time for the first part of the journey leaves the time for the rest of the journey. From the formula triangle, time = distance/speed so 18/36 works out the time for the first part of the journey

Answer 52 mph

Turn over for the next question

6

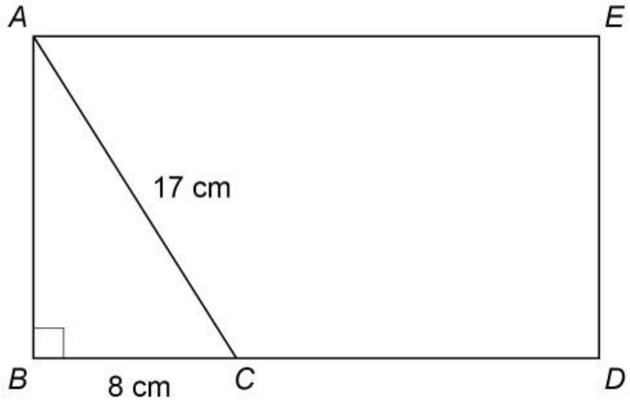
Do not write outside the box

7 The diagram shows rectangle *ABDE* and right-angled triangle *ABC*.

AC = 17 cm

BC = 8 cm

Not drawn accurately



BC : *CD* = 1 : 2

Work out the area of rectangle *ABDE*.

[4 marks]

$a^2+b^2=c^2$

Pythagoras' Theorem can be used to work out side AB as there are two sides in the right-angled triangle ABC

$a=\sqrt{c^2-b^2}$

Subtracting b^2 then square rooting both sides to make a (which represents side AB) the subject

$\sqrt{17^2-8^2}\times(8\times3)$

Substituting 17cm for c (as c is the longest side) and 8cm for b in the equation above to find side AB. This gives the width of the rectangle. BC is represented by 1 part of the ratio and there are 3 parts in total in the ratio so multiplying 8 by 3 works out the length BD. Area of rectangle = length x width so both of these are multiplied together

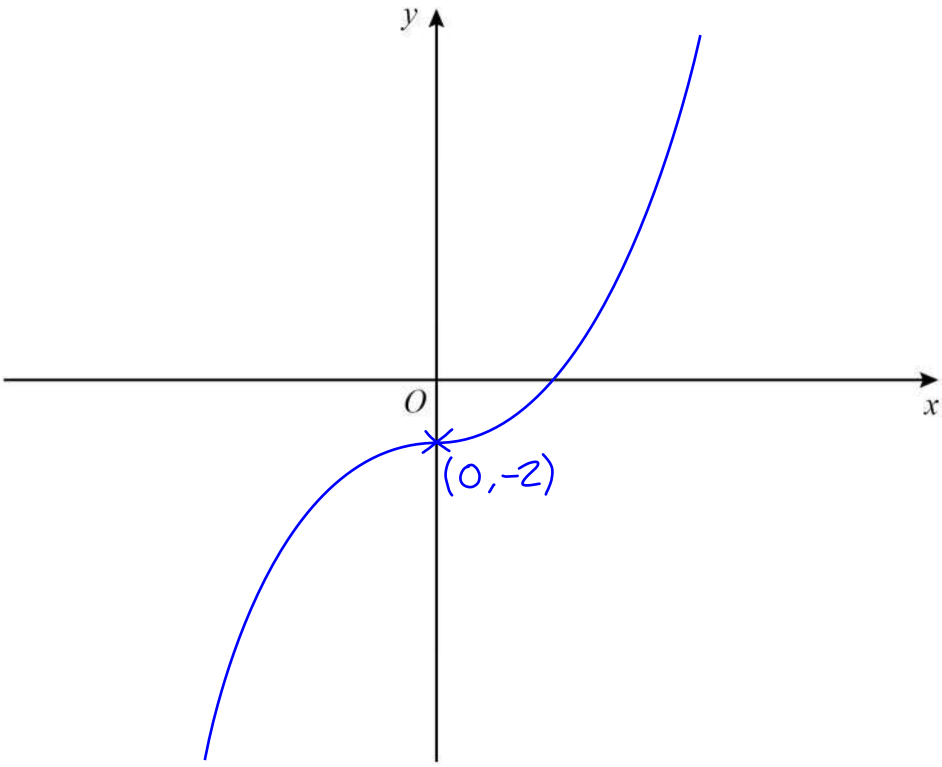
Answer 360 cm^2

Do not write outside the box

8 On the axes, sketch the curve $y = x^3 - 2$

You **must** show the coordinates of the y-intercept.

[2 marks]



$y = x^3$ is a typical graph. Subtracting 2 translates the graph downward 2 in the y direction

We could use the calculator to create a table of values, roughly plot the points on the graph then join them up with a curve.

Press Menu then 3 to go to table mode. Set $f(x) = x^3 - 2$, ignore $g(x)$, start: -5, end: 5, step: 1

Turn over for the next question

6

9 In a sport, injury time is added time played at the end of a match.

The table shows the injury time, t (minutes) played in 380 matches.

Injury time, t (minutes)	Frequency	
$0 < t \leqslant 2$	59	59
$2 < t \leqslant 4$	158	217 ← $\boxed{59 + 158}$
$4 < t \leqslant 6$	106	
$6 < t \leqslant 8$	45	
$8 < t \leqslant 10$	12	

9 (a) Circle the **two** words that describe the data.

[1 mark]

continuous discrete grouped ungrouped

Time is continuous as it could be any value. It isn't discrete, which means it can only be certain values. The time is grouped into intervals ($0 < t \leqslant 2$, for example)

9 (b) Which class interval contains the median?

You **must** show your working.

[2 marks]

$\frac{380+1}{2} = 190.5$

Using the formula $(n + 1)/2$, where n is the number of data points, works out which value is the median. So the median will be halfway between the 190th and 191st value. The first 59 values are in the first category. The first 217 values are in the first two categories. As 217 is the first cumulative frequency above 190.5, the median must be in the second category

Answer 2 $< t \leqslant$ 4

Do not write outside the box

9 (c) What percentage of the matches had **more than** 6 minutes of injury time?

[2 marks]

$\frac{45+12}{380} \times 100$

Both the $6 < t \leqslant 8$ and $8 < t \leqslant 10$ are more than 6 minutes. The total frequency for both of these categories is found by 45 + 12. Expressing this as a fraction of the total number of games then multiplying by 100 to convert the fraction into a percentage

Answer 15 %

10 x is an integer.

$$-4 < x \leqslant 2$$

and

$$2 \leqslant x + 3 < 9$$

Work out all the possible values of x.

[3 marks]

$-1 \leq x < 6$

Subtracting 3 from all sides of the second inequality gets x on its own in the middle

The smallest integer which satisfies both inequalities is -1 as $-1 \leqslant x$ and -4 is less than this. The largest integer which satisfies both inequalities is 2 as $x \leqslant 2$ and 9 is greater than this. Listing these and all integers in between

Answer $-1, 0, 1, 2$

8

Turn over ▶

Do not write outside the box

11 Joe and Kyle share an amount of money in the ratio $7 : n$

Joe gets 35% of the money.

Work out the value of n.

[2 marks]

$\frac{100-35}{5}$

100 - 35 works out the percentage which Kyle gets. 35 is divided by 5 to get 7 so the percentage for Kyle needs to be divided by 5 too

Answer 13

12 A biased coin is thrown 250 times.

The relative frequency of Heads is worked out after every 50 throws.

Total number of throws	50	100	150	200	250
Relative frequency	0.4	0.29	0.4	0.32	0.3

Circle the best estimate of the probability of Heads.

[1 mark]

0.3   0.32   0.342   0.4

The more times it is thrown, the more likely the relative frequency will be an accurate probability

Do not write outside the box

13 The amounts spent on clothes by 40 boys and 40 girls in one month were recorded.

The table shows information about the amounts spent by the boys.

Amount, x (£)	**Midpoint**	**Number of boys**	
$0 \leqslant x < 20$	10	22	220
$20 \leqslant x < 40$	30	9	270
$40 \leqslant x < 60$	50	6	300
$60 \leqslant x < 80$	70	3	210
		Total = 40	1000

Each category has a range of 20. 20/2 = 10 so adding 10 to each of the smallest values works out the midpoint

Midpoint x frequency (number of boys) gives the total for each category. Adding these all together gets the total amount spent

The mean for the girls was £35

Estimate the mean for the girls as a percentage of the mean for the boys.

[5 marks]

$$\frac{35}{1000 \div 40} \times 100$$

Mean = total/number. The total is 1000 and the number is 40. So 1000 ÷ 40 works out the mean for the boys. 35/(mean for the boys) expresses the mean for the girls as a fraction of the mean for the boys and multiplying it by 100 converts it into a percentage

Answer 140 %

8

Turn over ▶

Do not write outside the box

14 Ali and Mel are making 3-digit codes.

The digit 0 is **not** used.

Ali only uses odd digits.

Mel only uses even digits.

14 (a) Ali can make x more codes than Mel.

Assume that digits **cannot** be repeated.

Work out the value of x.

[3 marks]

$(5 \times 4 \times 3) - (4 \times 3 \times 2)$

x is the difference between the number of codes Ali can make and Mel can make

Ali can use the digits 1, 3, 5, 7 and 9. There are 5 possibilities for the first digit, 4 for the second (as digits cannot be repeated) and 3 for the third. Using the product rule for counting, multiplying together the number of possibilities for each digit gives the overall total amount of possibilities

Mel can use the digits 2, 4, 6 and 8. There are 4 possibilities for the first digit, 3 for the second (as digits cannot be repeated) and 2 for the third. Using the product rule for counting, multiplying together the number of possibilities for each digit gives the overall total amount of possibilities

Answer 36

14 (b) In fact, digits **can** be repeated.

What does this tell you about the actual value of x?

Tick **one** box.

This uses the same calculation before to calculate x except the number of possibilities for the second and third digits is the same as the first digit

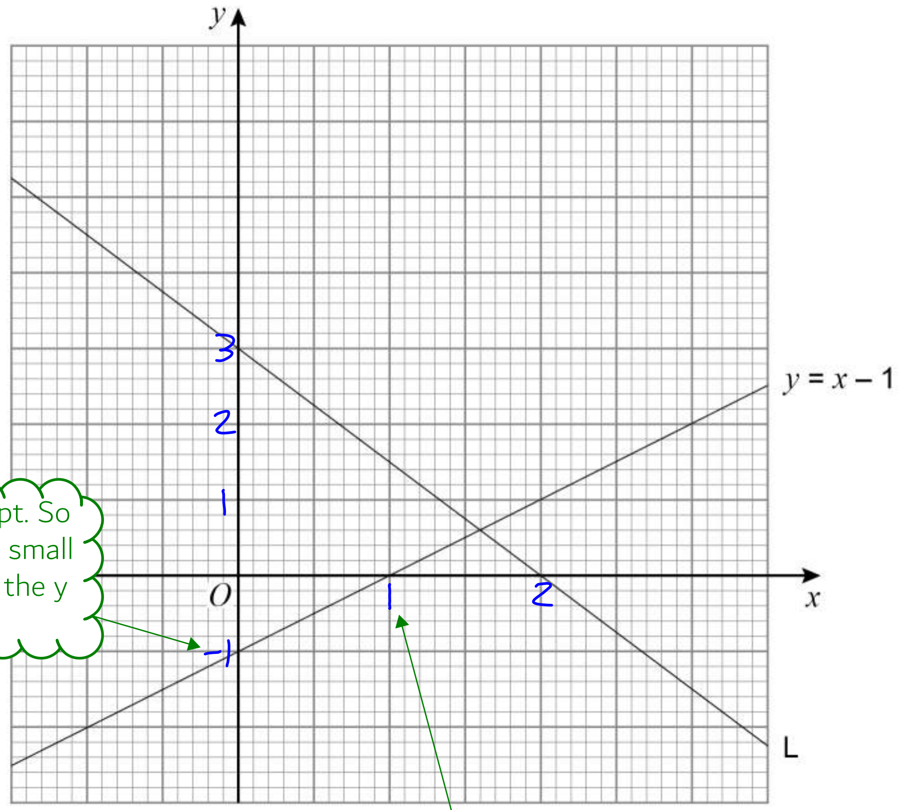
$\boxed{5 \times 5 \times 5 - 4 \times 4 \times 4 = 61}$

[1 mark]

- [x] It is bigger than my answer to part (a)
- [ ] It is smaller than my answer to part (a)
- [ ] It is the same as my answer to part (a)

15 Here is line L and the graph of $y = x - 1$

The scales of the axes are not shown.



x = 0 on the y-intercept. So y = 0 - 1 = -1. Every 5 small boxes represents 1 in the y direction

Work out the equation of line L.

[4 marks]

$0 = x - 1$

$x = 1$

y = 0 on the x-intercept. Substituting this into y = x - 1 and rearranging to find x shows that the line crosses the x-axis at 1. Every 10 small boxes represents 1 in the x direction

The general equation of a straight line is y = mx + c, where m is the gradient and c is the y-intercept.

The gradient is (change in y)/(change in x). From the point (0 , 3) to (2 , 0), y has gone downward 3 so the change in y is -3. The change in x is 2.

By filling in the scale on the graph, the y-intercept is 3

Answer $y = \frac{-3}{2}x + 3$

8

Turn over ▶

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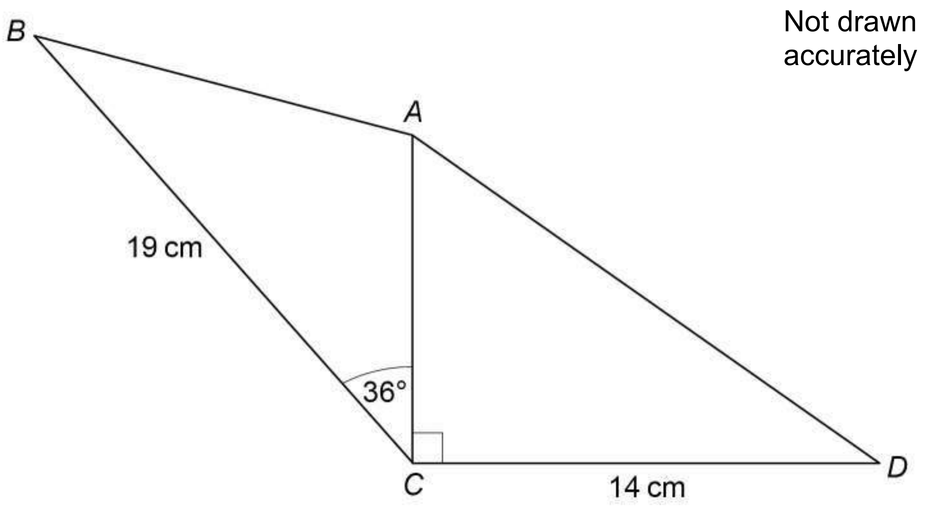
Do not write outside the box

16 *ABC* and *ACD* are triangles.



The area of *ACD* is 80.5 cm^2

Work out the area of *ABC*.

Give your answer to 3 significant figures.

[4 marks]

$\frac{1}{2}bh = A$

$h = \frac{A}{\frac{1}{2}b}$

Area of triangle = 1/2 x base x height. Rearranging the formula to make the height CA the subject

$\frac{1}{2} \times 19 \times \boxed{\frac{80.5}{\frac{1}{2}\times 14}} \times \sin 36$

Area of triangle = 1/2 $ab\sin C$. C is the angle between the sides a and b

Substituting the height CA for b

Answer 64.2 cm^2

Do not write outside the box

17 $m = \dfrac{p - 2b}{2}$

p = 68.3 correct to 1 decimal place.

b = 8.7 correct to 1 decimal place.

Work out the lower bound for m.

[3 marks]

$$\frac{\left(68.3 - \frac{0.1}{2}\right) - 2\left(8.7 + \frac{0.1}{2}\right)}{2}$$

The resolution (how much it goes up by) of 1 decimal place is 0.1. Dividing this by 2 and subtracting it from 68.3 works out the lower bound for p and adding it to 8.7 works out the upper bound for b. p needs to be as small as possible and b needs to be as large as possible (as it is being subtracted) to get the lower bound for m

Answer 25.375

Turn over for the next question

7

Do not write outside the box

18 In a bag there are blue discs, green discs and white discs.

There are four times as many blue discs as green discs.

number of blue discs : number of white discs = 3 : 5

One disc is selected at random.

Work out the probability that the disc is either blue or white.

[3 marks]

B	G	W
4	1	
3		5
12	3	20

Writing the ratios of blue to green (which is 4:1 as there are are four times as many blue discs as green discs) and blue to white. Both ratios have blue in common. A common multiple of 4 and 3 is 12. Multiplying all sides of the first ratio by 3 and all sides of the second ratio by 4 combines the ratios together as it makes the number of parts representing blue the same

(Total number of counters which are either blue or white)/(total number of counters) expresses the probability as a fraction

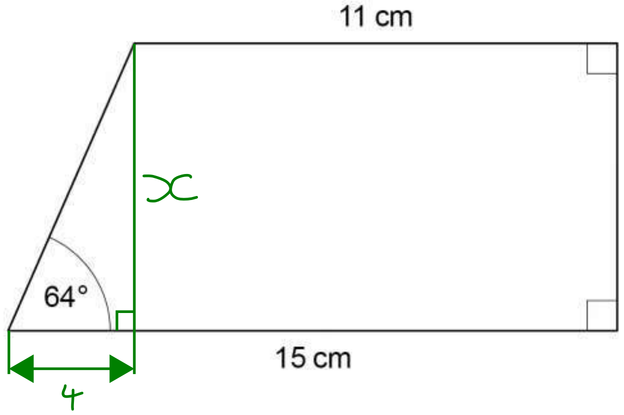
$\boxed{(12 + 20)/(12 + 3 + 20)}$

Answer $\frac{32}{35}$

Do not write outside the box

19 Work out the area of the trapezium.

Not drawn accurately



[4 marks]

S O H C A H T O A

$\frac{1}{2}(15+11)\times(\tan 64 \times 4)$

Right-angled trigonometry can be used to work out x. Listing out SOH CAH TOA as formula triangles then ticking O and A (as we have the adjacent and are finding the opposite) tells us that we can use the tan formula as it has two ticks. Covering O (as we are trying to find the opposite) tells us that the opposite = (tan of the angle) x adjacent

Area of trapezium = 1/2 (a + b)h, where a and b are the parallel sides and h is the distance between them

Answer 106.6 cm^2

Turn over for the next question

7

Do not write outside the box

20 Expressions for consecutive triangular numbers are

$$\frac{n(n+1)}{2} \quad \text{and} \quad \frac{(n+1)(n+2)}{2}$$

Prove that the sum of two consecutive triangular numbers is always a square number.

[4 marks]

$n^2 + n + n^2 + 2n + n + 2$

The denominators of the fractions are the same so the numerators can be added to express the sum. Expanding the brackets. Only the numerator is shown here

$\frac{2n^2 + 4n + 2}{2}$

Collecting like terms and simplifying the numerator. Now showing the denominator

$n^2 + 2n + 1$

Dividing all the terms on the numerator by 2 to cancel out the denominator

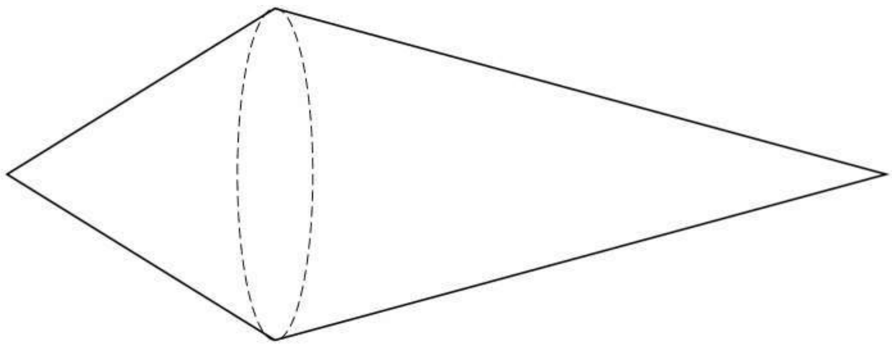
$(n+1)^2$

Factorising to express as a bracket squared (which is a square number). Two numbers which add to 2 and multiply to 1 are 1 and 1. Putting these in brackets with n but as both brackets are the same it can be squared instead

Do not write outside the box

21 A solid shape is made by joining two cones.

Each cone has the same radius.



One cone has slant height = 2 × radius

The other cone has slant height = 3 × radius

The total surface area of the shape is 57.8π cm^2

Curved surface area of a cone = $\pi r l$ where r is the radius and l is the slant height

Work out the radius.

[3 marks]

$\pi \times r \times 2r + \pi \times r \times 3r$

Expressing the total surface area of the shape in terms of the radius, r

$5\pi r^2 = 57.8\pi$

Simplifying the expression of the surface area in terms of the radius, r, and setting it equal to the given surface area

$r = \sqrt{\frac{57.8\pi}{5\pi}}$

Rearranged to make the radius, r, the subject

Answer 3.4 cm

7

Turn over ▶

Do not write outside the box

22 Show that $(5\sqrt{3} - \sqrt{12})^2$ simplifies to an integer.

[3 marks]

$75 - 60 + 12$

27

Expanding the square bracket using 'square the first term, double the product of the two terms, square the last term'.

$(5\sqrt{3})^2 = 5 \times 5 \times \sqrt{3} \times \sqrt{3} = 25 \times 3 = 75$

$2 \times 5\sqrt{3} \times -\sqrt{12} = -10\sqrt{36} = -10 \times 6 = -60$

$(-\sqrt{12})^2 = 12$

23 A and B are similar cuboids.

surface area of A : surface area of B = 16 : 25

Work out volume of A : volume of B

Circle your answer.

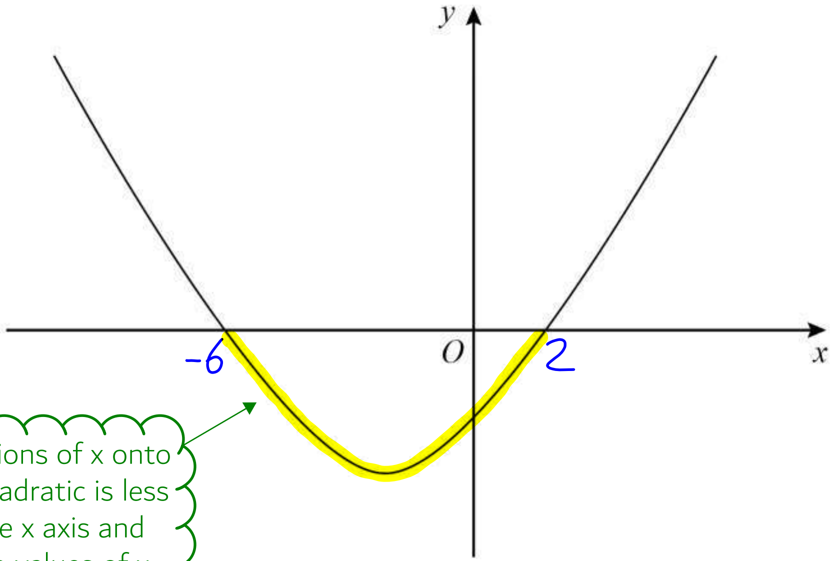
[1 mark]

4 : 5   16 : 25   64 : 125   256 : 625

Surface area is a squared dimension. Square rooting both sides of the ratio to get the ratio of the lengths gives 4 : 5. Volume is a cubed dimension so cubing both sides of the ratio gives 64 : 125

Do not write outside the box

24 Here is a sketch of the curve $y = x^2 + 4x - 12$



Marking the solutions of x onto the graph. The quadratic is less than 0 below the x axis and between the two values of x

Work out the values of x for which $x^2 + 4x - 12 < 0$

Give your answer as an inequality.

[3 marks]

$(x+6)(x-2)=0$

Factorising to solve when the quadratic is equal to 0. Two numbers which multiply to get -12 and add to get 4 are 6 and -2. Put these in brackets with x. Either x + 6 = 0 or x - 2 = 0. x = -6 or x = 2

Answer $-6<x<2$

7

Turn over ►

25 A sample of 50 eggs is taken from Farm A.

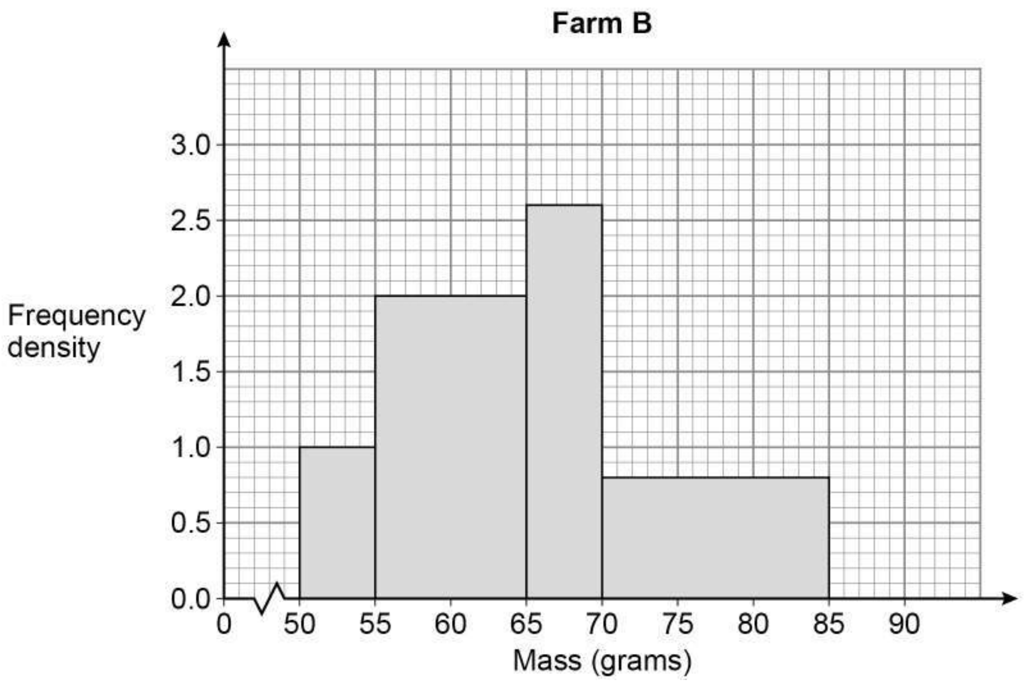
The table shows information about the masses of the eggs from Farm A.

Farm A

Mass, m (grams)	**Frequency**
$53 < m \leqslant 58$	8
$58 < m \leqslant 63$	19
$63 < m \leqslant 68$	15
$68 < m \leqslant 73$	8

A sample of 50 eggs is taken from Farm B.

The histogram shows information about the masses of the eggs from Farm B.



Do not write outside the box

For medium eggs, $53 \text{ g} < \text{mass} \leqslant 63 \text{ g}$

The Farm A sample has more medium eggs than the Farm B sample.

Using the table and the histogram, estimate how many more.

You **must** show your working.

[4 marks]

$(8+19)-(2\times 1+8\times 2)$

Subtracting the estimate of the number of medium eggs from Farm B from the number of medium eggs from Farm A works out the difference, and therefore how many more

From the table for Farm A, both the first two categories have masses which are classed as medium eggs

Frequency = class width x frequency density. Splitting the first bar from 53 to 55 gives a class width of 2 and frequency density of 1. Splitting the second bar from 55 to 63 gives a class width of 8 and a frequency density of 2

Answer 9

Turn over for the next question

4

Do not write outside the box

26 $(x + 5)(x + 2)(x + a) \equiv x^3 + bx^2 + cx - 30$

Work out the values of the integers a, b and c.

[3 marks]

$x^2 + 2x + 5x + 10$ Expanding the first two brackets

$(x^2 + 7x + 10)(x + a)$ Collecting like terms and simplifying. Putting back into the left side of the original identity

$x^3 + ax^2 + 7x^2 + 7ax + 10x + 10a$ Expanding the brackets

$x^3 + (7 + a)x^2 + (7a + 10)x + 10a$ Writing in the same form as the right side of the identity

$10a = -30$ Equating coefficients of the left and right sides of the identity

$a =$ -3 $\boxed{-30/10 = -3}$

$b =$ 4 Equating coefficients of the left and right sides of the identity. 7 + a = b so 7 - 3 = b

$c =$ -11 Equating coefficients of the left and right sides of the identity. 7a + 10 = c so 7(-3) + 10 = c

Do not write outside the box

27 $\quad f(x) = \dfrac{2x}{5} - 1$

Work out the value of $\quad f^{-1}(3) + f(-0.5)$

[5 marks]

$x = \dfrac{2y}{5} - 1$

Swapping f(x) with x and x with y. Inverse function is basically when the x and y-axis switch

$\dfrac{5(x+1)}{2} = f^{-1}(x)$

Rearranged to make y the subject by adding 1, multiplying by 5 then dividing by 2 on both sides. Replacing y back with $f^{-1}(x)$

$\dfrac{5(3+1)}{2} + \left(\dfrac{2(-0.5)}{5} - 1\right)$

Substituting 3 for x in $f^{-1}(x)$ and -0.5 for x in f(x)

Answer 8.8

END OF QUESTIONS

8