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Worked Solutions

H

Tuesday 7 June 2022 – Morning

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- geometrical instruments
- tracing paper

Do not use:
- a calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

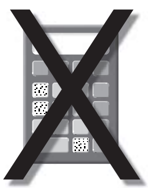
First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space, use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method even if your answer is wrong.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.



Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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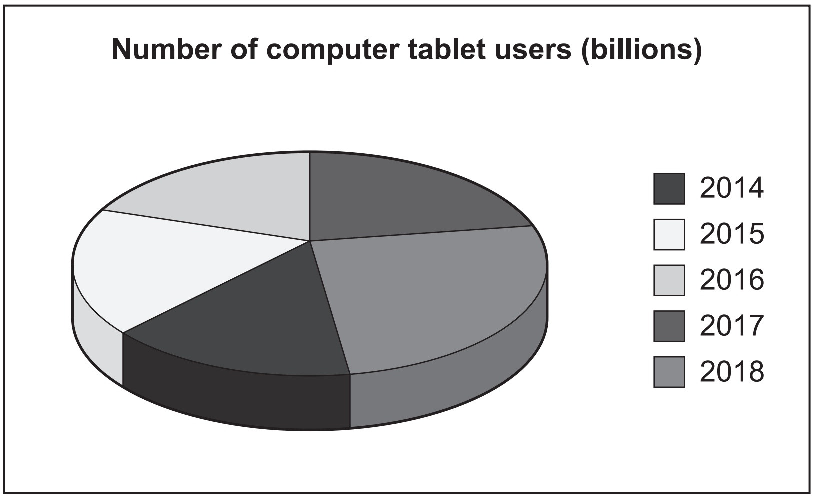
Answer **all** the questions.

1 Two pupils are given data that shows the estimated number of computer tablet users worldwide from 2014 to 2018.

(a) Li creates this pie chart to show the data.



Write down two reasons why Li's pie chart is not suitable to represent the data.

1 Number of computer tablet users is not given

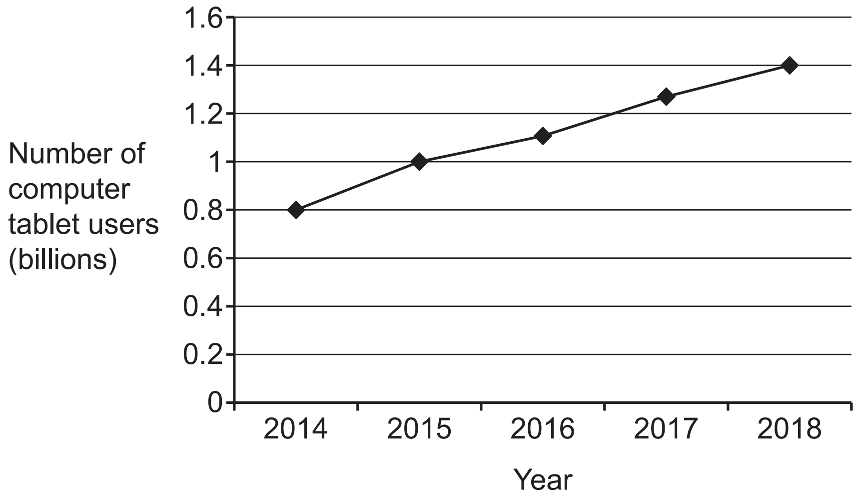
This is the title of the chart but the numbers are not given

2 It is 3D

Pie charts should be 2D so that the angles can be easily measured

[2]

(b) Amaya creates this line graph to show the same data.



Work out the percentage increase in the number of computer tablet users from 2014 to 2018.

$1.4 - 0.8 = 0.6$

It increased from 0.8 billion to 1.4 billion. Subtracting 0.8 from 1.4 works out that it increased by 0.6 billion

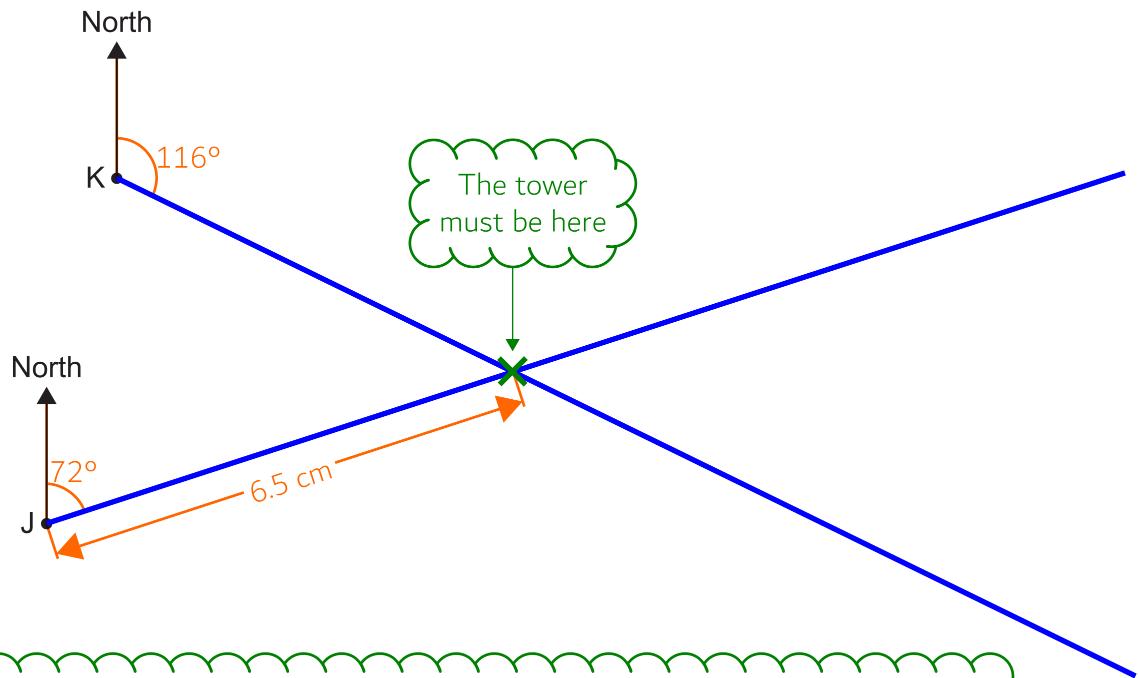
$\frac{0.6}{0.8} = \frac{6}{8} = \frac{3}{4}$

Giving the 0.6 billion as a fraction of the original 0.8 billion. The billions cancel out leaving 0.6/0.8. Multiplying both the numerator and denominator by 10 gets rid of the decimals and gives 6/8. Simplifying by dividing both the numerator and denominator by 2 gives 3/4, which is 75%

(b) ....75.......... % **[4]**

2 The scale diagram below shows the position of two castles, J and K.

Scale: 1 cm represents 2 km



Measuring the bearings using a protractor and drawing lines which represent all points on each bearing. The tower must be where the lines cross as this is on both bearings at the same time

The bearing of a tower from castle J is 072°.
The bearing of the tower from castle K is 116°.

Use construction to find the distance from castle J to the tower.
Give your answer to the nearest 0.1 km.

6.5×2 ← From the diagram, the distance from the castle J to the tower is 6.5cm. Multiplying this by the 2km each centimetre represents works out the actual distance

.................13................. km **[4]**

3 Dinosaurs first appeared on Earth 2.4×10^8 years ago.
Dinosaurs became extinct on Earth 7×10^7 years ago.

(a) Explain why it is appropriate to use standard form for these numbers.

There would be a lot of zeros

Numbers with lots of zeros are harder to read and write. Using standard form is useful for very large and small numbers

[1]

(b) Use the given information to work out how long dinosaurs existed on Earth.
Give your answer in standard form.

$2.4 - 0.7 = 1.7$

Subtracting the two numbers works out the difference between when dinosaurs first appeared and when they became extinct and therefore how long dinosaurs existed on Earth. Converting 7×10^7 to 0.7×10^8 by dividing the 7 by 10 and adding 1 to the power of 10 so that the two numbers can be subtracted easier and it will give an answer in standard form. $2.4 \times 10^8 - 0.7 \times 10^8$

(b) 1.7×10^8 **[3]**

4 **(a)** Complete this statement by writing the missing power in the box.

$$784 = 2^{\boxed{4}} \times 7^2$$

[1]

49
× 2
98
× 2
196
× 2
392
× 2
784

$7^2 = 7 \times 7 = 49$. This needs to be multiplied by 2 4 times to get 784 so needs to be multiplied by 2^4

(b) Use your answer to part **(a)** to find the value of $\sqrt{784}$.

$2^2 \times 7$

Halving the powers of the 2^4 and 7^2 does the square root

$2^2 = 2 \times 2 = 4$. $4 \times 7 = 28$

(b) ....28.......... **[2]**

5 Recipes measure small quantities in teaspoons and tablespoons.
3 teaspoons is equivalent to 1 tablespoon.

A cake recipe uses $\frac{3}{4}$ of a teaspoon of salt and 1 tablespoon of baking powder.

The ratio of salt to baking powder used in the recipe can be written in the form 1 : *n*.

Find the value of *n*.

$\frac{3}{4} : 3$ ← The tablespoon can be replaced with 3 teaspoons. Writing the number of teaspoons of the salt and baking powder as a ratio

$3 : 12$ ← Eliminating the fractions from the ratio by multiplying both sides by 4

$12 \div 3$ ← Dividing both sides of the ratio by 3 gets 1 on the left and 4 on the right. So n is worth 4

n = ...4........................ **[3]**

6 Morgan is playing a computer game.
They can score 0, 1, 2 or 3 points on each turn.
They record their scores for 100 turns.
The table shows the relative frequencies of their scores.

Score	0	1	2	3
Relative frequency	0.08	0.42	0.38	0.12

(a) Complete the table. **[2]**

1.00
−0.08
−0.42
−0.38
0.12

All the relative frequencies must add up to 1 as it was always 0, 1, 2 or 3.
Subtracting the other relative frequencies from 1 leaves the relative frequency for 3

(b) Morgan says

I scored more than 160 points **in total** in my 100 turns.

Is Morgan correct?
Show how you decide.

Multiplying the relative frequencies by 100 works out that there were 8 scores of 0, 42 scores of 1, 38 scores of 2 and 12 scores of 3

$0 \times 8 = 0$

Multiplying the score of 0 by the 8 times it was scored works out that the total score for the 0s is 0

$1 \times 42 = 42$

Multiplying the score of 1 by the 42 times it was scored works out that the total score for the 1s is 42

$38 \times 2 = 76$

Multiplying the score of 2 by the 38 times it was scored works out that the total score for the 2s is 76

$12 \times 3 = 36$

Multiplying the score of 3 by the 12 times it was scored works out that the total score for the 3s is 36

$42 + 76 + 36 = 154$

Adding the total score for the 1s, the total score for the 2s and the total score for the 3s works out that Morgan scored 154 in total

No

154 is not more than 160

.... **[4]**

7 (a) A car accelerates at 4.06 m/s^2 for 10.1 seconds from an initial velocity of 2.93 m/s.

Harper rounds each value to 1 significant figure.
Harper uses the rounded values and the formula

$$s = ut + \frac{1}{2}at^2$$

to estimate the distance travelled in the 10.1 seconds.
Harper's answer is 430 metres.

Using Harper's method, show that their answer is wrong. **[4]**

$3 \times 10 + \frac{1}{2} \times 4 \times 10^2$

s is the distance, u is the initial velocity, t is the time and a is the acceleration. Rounding each amount to 1 significant figure then substituting them into the formula. For the 4.06, the first significant figure is the 4. The 0 after this causes it to round down and stay as a 4. Then everything after it is set to 0 and is ignored. So the acceleration to 1 significant figure is 4 m/s^2

230

Using Harper's method shows that the distance should be 230m, not 430m

(b) Rearrange this formula to make *t* the subject.

$$s = \frac{1}{2}at^2$$

Dividing both sides by 1/2 a then square rooting both sides makes t the subject

(b) $\sqrt{\dfrac{S}{\frac{1}{2}a}} = t$ **[3]**

8 A bag only contains red marbles, blue marbles and yellow marbles.

- The probability of picking a red marble is $\frac{2}{5}$.
- There are nine yellow marbles.
- The probability of picking a blue marble is three times as likely as picking a yellow marble.

Work out the **total** number of marbles in the bag.
You must show your working.

$\frac{5}{5} - \frac{2}{5} = \frac{3}{5}$ ← There is 1 lot of marbles, which can be expressed as 5/5. Subtracting the 2/5 of the marbles which are red leaves 3/5 which are blue or yellow

9×3 ← There must be 3 times as many blue marbles as yellow marbles. So multiplying the 9 yellow marbles by 3 works out that there are 27 blue marbles

$27 + 9$ ← Adding the 9 yellow marbles to the 27 blue marbles works out that there are 36 marbles which are blue or yellow

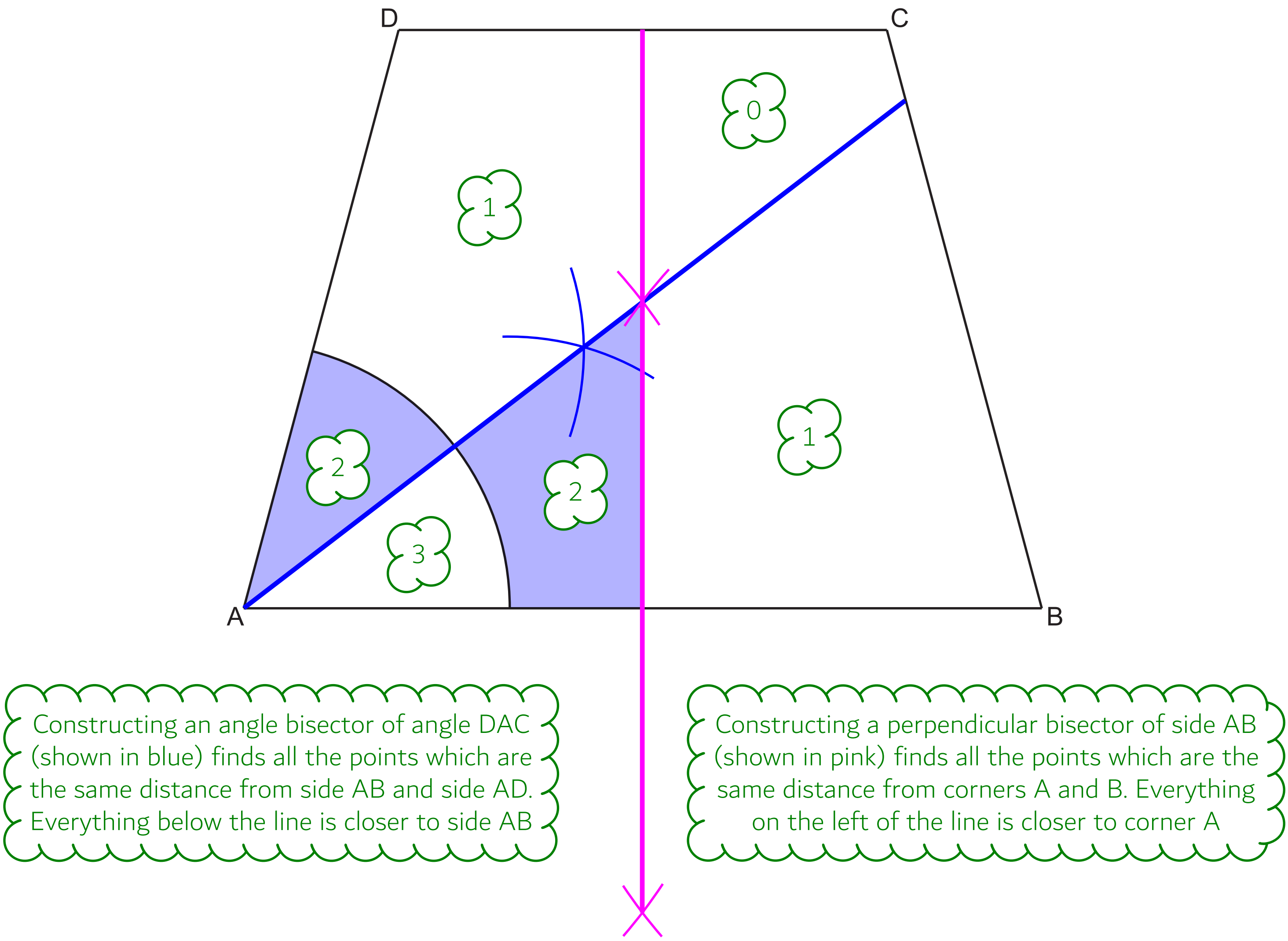
$36 \div 3$ ← 3/5 of the marbles must be 36 marbles. Dividing the 36 by 3 works out that 1/5 of the marbles is 12

12×5 ← Multiplying 1/5 of the marbles by 5 works out 5/5, which is the total number of marbles

........................60........................ **[5]**

9 The diagram shows the scale drawing of a sandpit, ABCD.
It also shows the arc of all points in the sandpit that are 80 cm from corner A.

Scale: 1 cm represents 20 cm



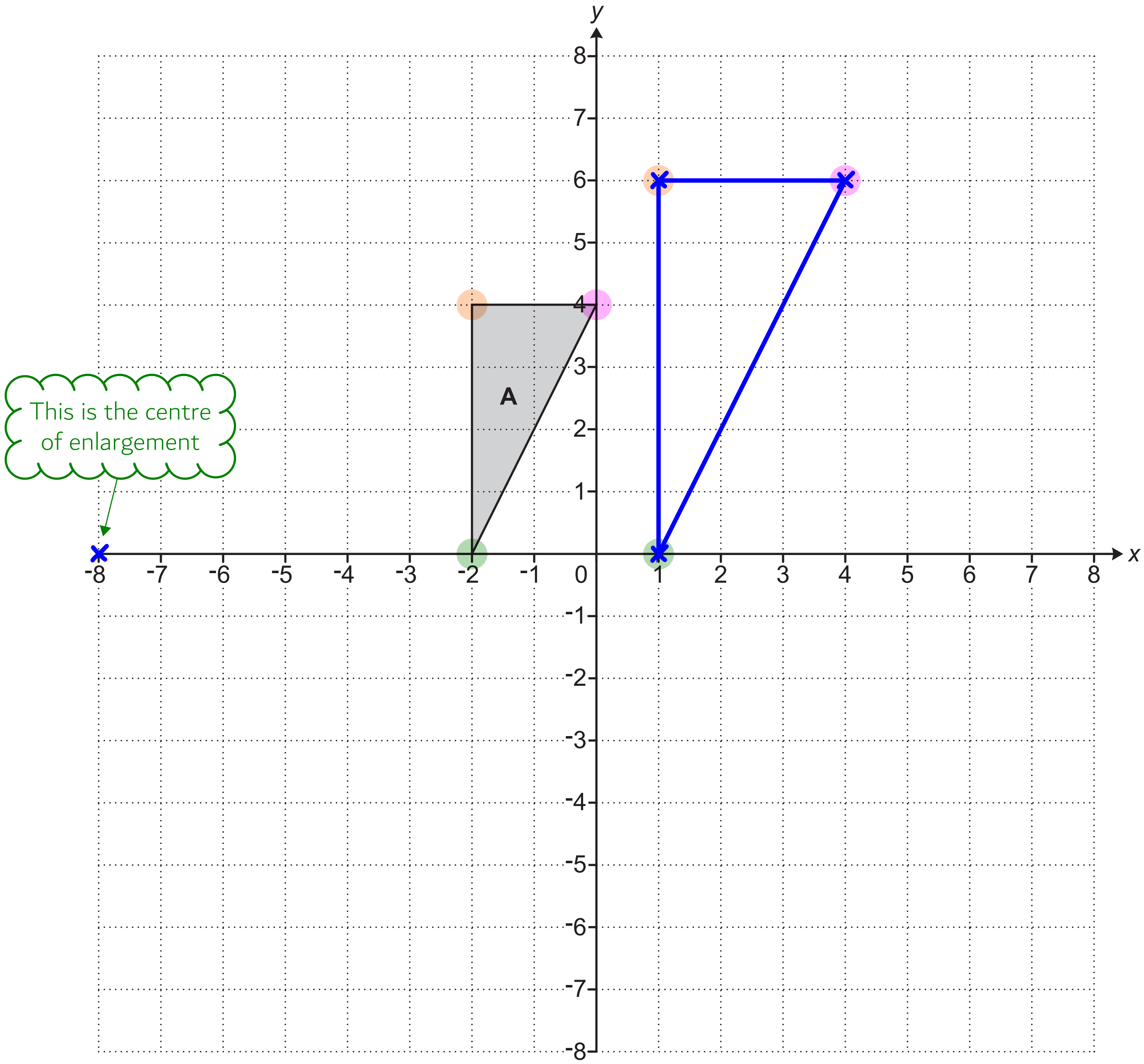
A game is played by throwing a ball into the sandpit.
Points may be scored when the ball lands in the sandpit.

- 1 point if the ball lands within 80 cm of corner A,
  and
- 1 point if the ball is closer to side AB than side AD,
  and
- 1 point if the ball is closer to corner A than corner B.

By completing the construction, find and shade the regions where 2 points can be scored.
Show all your construction lines. **[6]**

The number of points for each region is indicated

10 **(a)** Enlarge triangle **A** with scale factor 1.5 and centre of enlargement (-8, 0).



$\begin{pmatrix}6\\0\end{pmatrix} \times 1.5 = \begin{pmatrix}9\\0\end{pmatrix}$ ← From the centre of enlargement to the corner highlighted in green it is 6 to the right. Expressing this as a column vector then multiplying it by 1.5 to give the new vector. Doing this new vector from the centre of enlargement

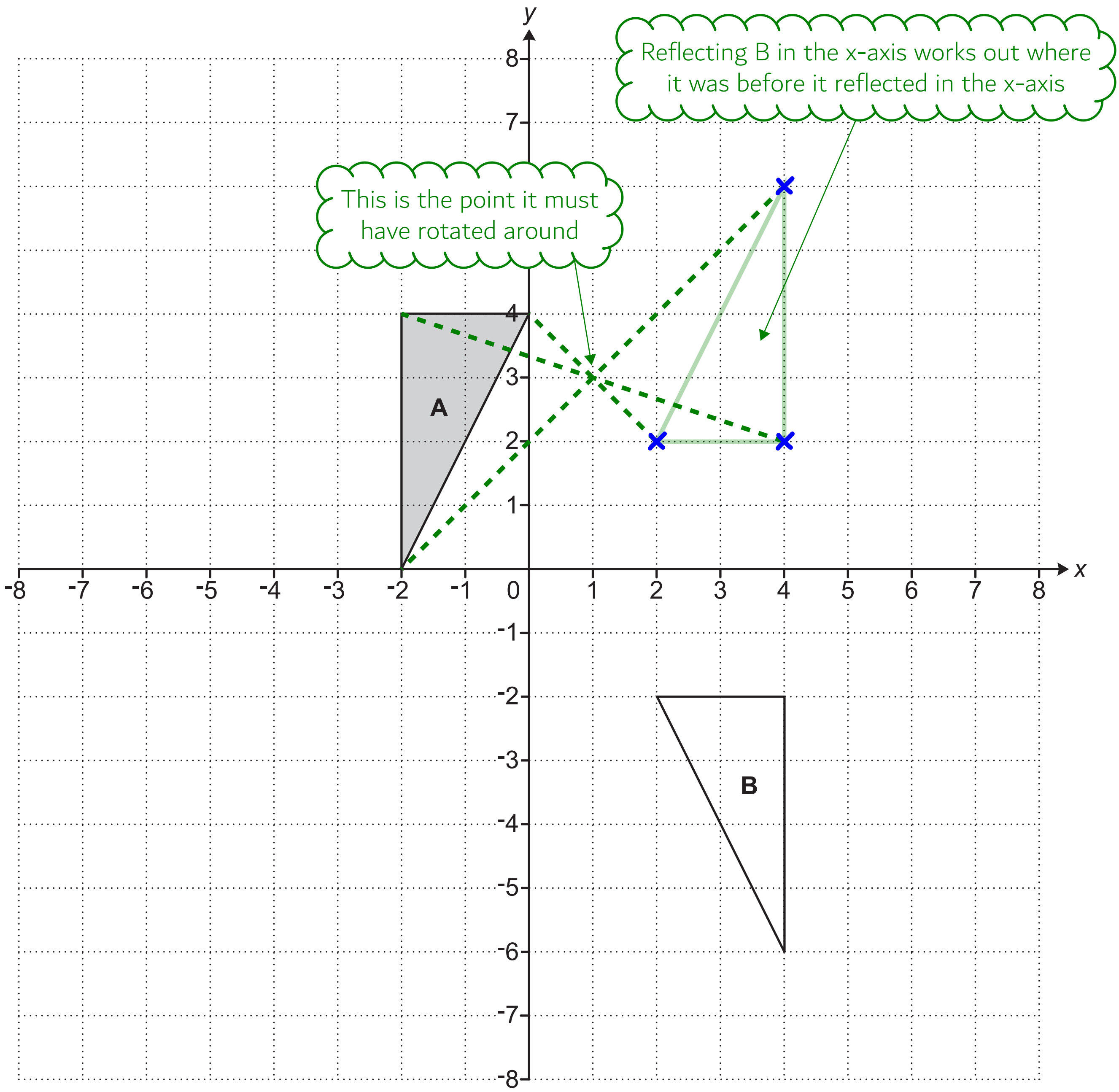
[3]

$\begin{pmatrix}6\\4\end{pmatrix} \times 1.5 = \begin{pmatrix}9\\6\end{pmatrix}$ ← Doing a similar method for the corner highlighted in orange

$\begin{pmatrix}8\\4\end{pmatrix} \times 1.5 = \begin{pmatrix}12\\6\end{pmatrix}$ ← Doing a similar method for the corner highlighted in pink

1.5 is 1 and a half so to multiply by 1.5, find half and add it on

(b) Triangle **A** and triangle **B** are shown on the coordinate grid below.



Triangle **A** is mapped onto triangle **B** using a combination of two transformations:

- a transformation T, followed by
- a reflection in the *x*-axis.

Describe fully transformation T.

It could be a rotation as it is the same shape but is upside down. Half a turn is 180°. To work out the point it is rotating about: for a 180° rotation it is where all the lines going from the same corners meet

Rotation, 180°, about (1 , 3) ....

.... **[4]**

11 y is inversely proportional to x^2.
$y = 9$ when $x = 2$.

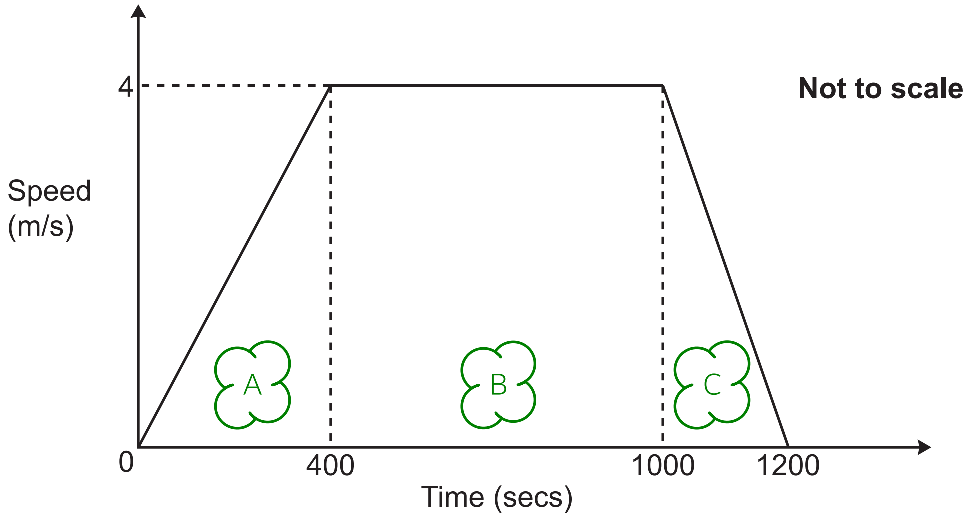
Find the value of y when $x = 10$.

$y \propto \frac{1}{x^2}$ ← Writing out the statement using the proportional symbol. Inversely proportional to means that the right side is '1 over'

$9 \div 5^2$ ← x has been multiplied by 5 from 2 to 10. Therefore y must be divided by 5^2 as inverse proportion means that the opposite happens to the other side and x is squared

$y =$ ....0.36.......... **[3]**

12 An athlete goes for a training run.
The graph shows their speed as they run.



(a) Write down the athlete's acceleration between 400 seconds and 1000 seconds.

The speed is not changing

(a) ......0...... m/s^2 **[1]**

(b) Work out the athlete's average speed, in m/s, during the 1200 seconds.
You must show your working.

Distance on a speed-time graph can be found by finding the area under the line

$\frac{1}{2} \times 400 \times 4 = 800$ ← Area of triangle A is 800. Area of triangle = 1/2 x base x height

$600 \times 4 = 2400$ ← Area of rectangle B is 2400. Area of rectangle = length x width

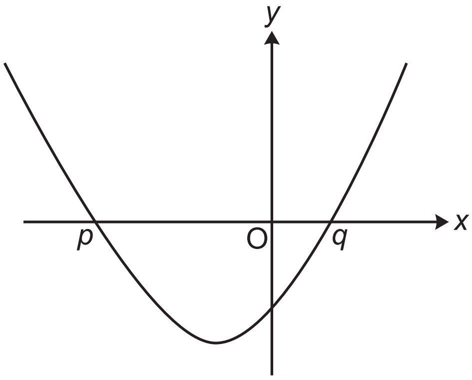
$\frac{1}{2} \times 200 \times 4 = 400$ ← Area of triangle C is 400. Area of triangle = 1/2 x base x height

$800 + 2400 + 400$ ← Adding the area of triangle A, rectangle B and triangle C works out the area under the line and therefore that the distance is 3600m

$3600 \div 1200$ ← m/s means to divide the distance in metres by the time in seconds

(b) ......3...... m/s **[5]**

13 The graph of $y = x^2 + 6x - 2$ is shown below.
The roots of the equation $x^2 + 6x - 2 = 0$ are at p and q.



(a) **(i)** Calculate y when $x = 1$.

Substituting 1 for x in the equation.
$y = 1^2 + 6(1) - 2 = 1 + 6 - 2 = 5$

(a)(i) $y =$ 5 **[1]**

(ii) Without solving the equation, explain why q must lie between 0 and 1.

When x = 0, y < 0. When x = 1, y = 5. The curve must cross the x-axis between 0 and 1

It is a continuous curve and it goes from less than 0 to more than 0 so must be 0 at some point between **[2]**

(iii) Explain why using a method of iteration is not the most appropriate way of finding a solution to this equation.

It gives an estimate

Instead it would be better to solve using the quadratic formula **[1]**

(b) The exact value of q is $\frac{-6+\sqrt{44}}{2}$.

Write $\frac{-6+\sqrt{44}}{2}$ in the form $a + \sqrt{b}$.

$\sqrt{4} \times \sqrt{11}$ ← Simplifying $\sqrt{44}$ by using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ to split it into two square roots multiplied together, one of which is the square root of a square number

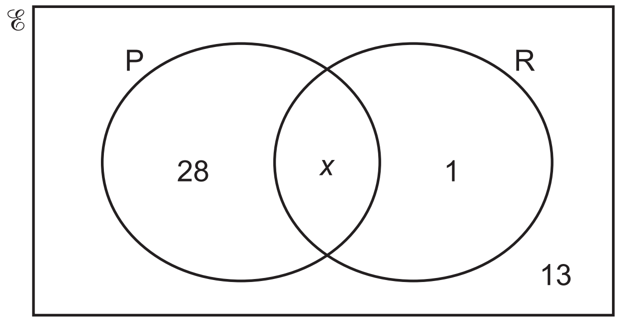
$2\sqrt{11}$ ← $\sqrt{4} = 2$

Dividing both -6 and $2\sqrt{11}$ by 2 gives this

(b) $-3+\sqrt{11}$ **[3]**

14 In a survey about music, some students were asked whether they like pop (P) and whether they like rap (R).

The Venn diagram shows some of the results.
x students liked both types of music.



(a) The ratio of the number of students who liked pop to the number who liked rap was 5 : 2.

Work out the **total** number of students in the survey.

$2(28+x)=5(x+1)$ ← 5 x 2 = 2 x 5, so multiplying the number of pop by 2 and the number of rap by 5 will make them equal. 28 + x is the number of pop and x + 1 is the number of rap

$28 \times 2 = 56$

$56+2x=5x+5$ ← Expanding the brackets

$56=3x+5$ ← Subtracting 2x from both sides gets all the x on the same side

$51=3x$ ← Subtracting 5 from both sides gets the x term on its own

$51 \div 3 = 17$ ← Dividing both sides by 3 gets x on its own and finds that x is 17

$28+17+1+13=59$ ← Adding all the numbers in the Venn diagram works out the total number of students in the survey

(a) ....59............ **[4]**

(b) One of the students is selected at random.

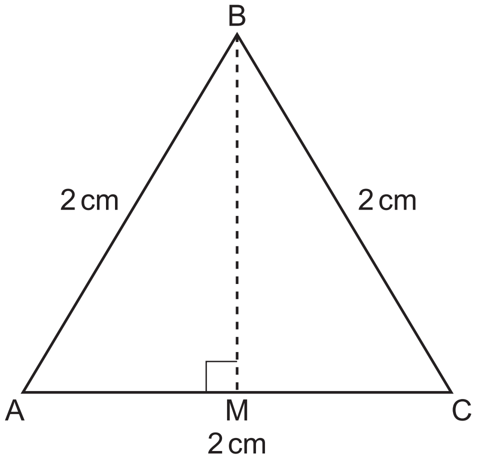
Find the probability that this student does **not** like rap given that they like pop.

$28+17=45$ ← Adding the 17 who like both pop and rap to the 28 who like just pop works out that 45 in total liked pop

28 out of the 45 who like pop do not like rap →

(b) ....$\frac{28}{45}$............ **[2]**

15 ABC is an equilateral triangle of side length 2 cm.
M is the midpoint of AC.



Not to scale

Using this diagram, show that $\tan 30° = \frac{1}{\sqrt{3}}$. **[4]**

$\angle ABM = 30°$ ← Line BM is a line of symmetry. It cuts angle ABC in half. The angle in an equilateral triangle is 60° as there are 180° in total in a triangle, which when divided by the 3 angles finds that each is 60°. Dividing the angle ABC by 2 gives 30°

$AM = 1$ ← M is the midpoint of AC so AM must be half of AC

$a^2 + b^2 = c^2$ ← Pythagoras' Theorem can be used to work out the missing side in right-angled triangle ABM

$1^2 + BM^2 = 2^2$ ← Substituting AM (which is 1cm) for a, BM for b and AB for c (as it is the longest side)

$BM^2 = 4 - 1$ ← $2^2 = 2 \times 2 = 4$. $1^2 = 1 \times 1 = 1$. Subtracting 1 from both sides gets BM^2 on its own

$BM = \sqrt{3}$ ← 4 - 1 = 3. Square rooting both sides finds side BM

T O A ← Writing the formula triangle for tan using right angled trigonometry

$\tan 30 = \frac{1}{\sqrt{3}}$ ← From the formula triangle, tan of the angle = opposite/adjacent. The angle ABM is 30°, the opposite to this is AM (which is 1cm) and the adjacent to this is BM (which is $\sqrt{3}$cm)

16 Work out $0.\dot{6} \times 0.\dot{5}\dot{4}$ giving your answer as a fraction in its simplest form.
You must show your working.

$x = 0.\dot{6}$ ← Let x be the first recurring decimal

$10x = 6.\dot{6}$ ← There is 1 recurring digit so multiplying by 10 once creates a different decimal with the same recurring digit in the same decimal place

$9x = 6$ ← Subtracting x from 10x gives this. It eliminates the recurring digit

$x = \frac{6}{9}$ ← Dividing both sides by 9 expresses x as a fraction

$y = 0.\dot{5}\dot{4}$ ← Let y be the second recurring decimal

$100y = 54.\dot{5}\dot{4}$ ← There are 2 recurring digits so multiplying by 10 twice creates a different decimal with the same recurring digits in the same decimal places

$99y = 54$ ← Subtracting y from 100y gives this. It eliminates the recurring digits

$y = \frac{54}{99}$ ← Dividing both sides by 99 expresses y as a fraction

$\frac{6}{9} \times \frac{54}{99}$ ← Multiplying the recurring decimals as fractions

$\frac{2}{1} \times \frac{6}{33}$ ← Simplifying the multiplication by dividing both the 9 and 54 by 9 and the 6 and 99 by 3

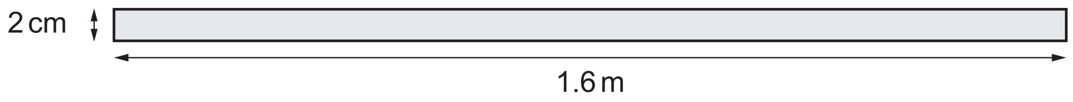
$\frac{12}{33}$ ← Multiplying the fractions by multiplying the numerators and multiplying the denominators. 2 x 6 = 12 and 1 x 33 = 33

Simplifying the fraction by dividing both the numerator and denominator by 3 →

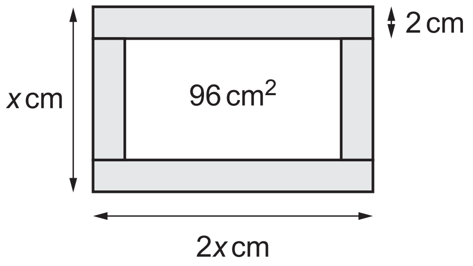
$\frac{4}{11}$ .... **[5]**

17 Charlie is making some wooden frames.
Charlie has a strip of wood 1.6 m long and 2 cm wide.

Not to scale



Each frame will be made from four pieces of wood cut from the strip to form a rectangle, as shown below.



Not to scale

The width of each frame is x cm.
The length of each frame is $2x$ cm.
The area enclosed by each frame must be 96 cm^2.

Work out the maximum number of frames Charlie can make from the 1.6 m length of wood.
You must show your working.

$(2x-4)(x-4)$

Expressing the area enclosed by each frame in terms of x. Area of rectangle = length x width. Length is 2x subtract 2 lots of the 2cm width of each frame, which gives 2x - 4. Width is x subtract 2 lots of the 2cm width of each frame, which gives x - 4

$2x^2-8x-4x+16=96$

Expanding the brackets and setting the expression of the area enclosed by each frame equal to the actual area of 96cm^2

$2x^2-12x-80=0$

Collecting like terms and subtracting 96 from both sides to bring it into the quadratic form

$x^2-6x-40=0$

Dividing all terms on both sides by 2 to simplify the quadratic

1, 40
2, 20
4, 10

Listing out the factor pairs of 40 until a pair is found which add to -6 when one of the pair is negative (which needs to be so that they multiply to -40)

$(x-10)(x+4)=0$

4 and -1 multiply to -40 and add to -6 so putting these in brackets with x factorises it

Method continues on the next page

.................................3................................. **[6]**

$x = 10$

One of the two brackets must equal to 0 in order to multiply to 0. If x - 10 = 0, x = 10. If x + 4 = 0, x = -4, but this solution is ignored as x is a length and cannot be negative

$2 \times 10 \times 2 + 2 \times 10 - 8$

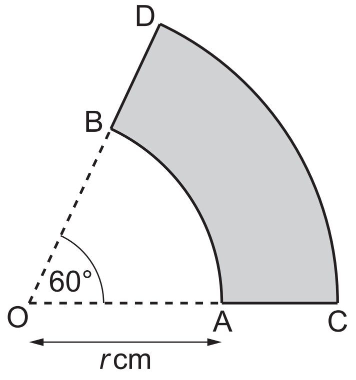
The length of strip used for each frame is 2 lots of 2x and 2 lots of x subtract 4 lots of the 2cm width of each frame. Substituting 10 for x in this finds that 52cm of strip is used for each frame

$52 \overline{)1\,{}^{1}6\,{}^{16}0}$ = 0 0 3 r4

52, 104, 156

There are 100cm in 1m so the 1.6m of strip is 160cm. Dividing this by the 52cm of strip used for each frame works out that 3 frames can be made with 4cm of strip left over

18 The diagram shows a shaded shape made by removing sector OAB from sector OCD.
Both sectors have an angle of 60°.
The radius, OA, of the smaller sector is r cm.
The ratio of radius OA to radius OC is 2 : 3.



Not to scale

Work out, in terms of π and r, the **total** length of arc AB and arc CD.
Give your answer in its simplest form.
You must show your working.

$\frac{60}{360} = \frac{6}{36} = \frac{1}{6}$ ← Expressing the fraction of the whole circle each sector is and simplifying it by dividing both the numerator and denominator by the same amount. There are 360° in total around the centre of a circle and each sector has 60° out of this

$\frac{1}{6} \times \pi \times r \times 2$ ← This expresses the length of arc AB. Circumference = π x diameter. The diameter is double the radius so is r x 2. Doing 1/6 of the circumference as this is the fraction of the whole circle the sector is

$\frac{2}{6}\pi r$ ← Simplifying the expression of arc AB by multiplying the 1/6 by 2

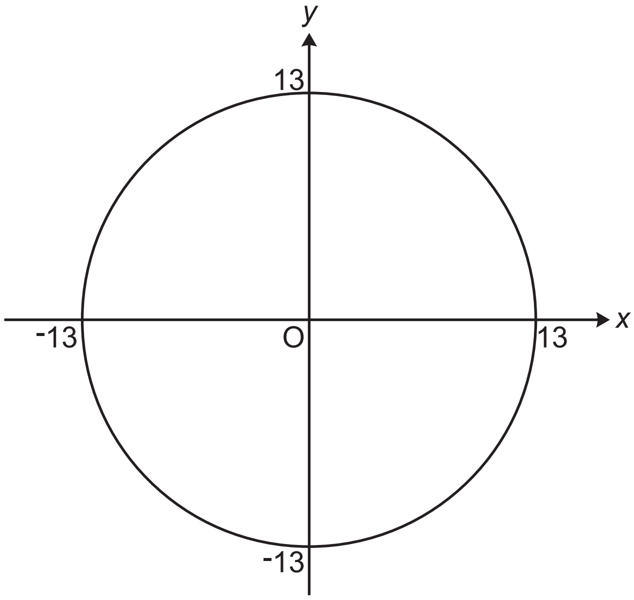
$\frac{1}{6} \times \pi \times \frac{3}{2} \times \pi \times r \times 2$ ← This expresses the length of arc CD. Circumference = π x diameter. The diameter is double the radius. The radius is 3/2 r because of the ratio. Doing 1/6 of the circumference as this is the fraction of the whole circle the sector is

$\frac{3}{6}\pi r + \frac{2}{6}\pi r$ ← Simplifying the expression of arc CD by multiplying the 3/2 by 2 to get 3 then multiplying this by the 1/6. Then adding the expression for the length of arc AB to get the total length of arc AB and arc CD

3/6 + 2/6 = 5/6

$\frac{5}{6}\pi r$ .... cm **[5]**

19 The graph below shows a circle with centre (0, 0) and equation $x^2 + y^2 = 169$.



(a) Show that the point (-12, 5) lies on the circumference of the circle. **[2]**

$(-12)^2 + 5^2$
$144 + 25 = 169$

Substituting the x and y-coordinate into the left side of the equation then showing that this is equal to 169 so satisfies the equation

(b) Find the equation of the tangent to the circle at the point (-12, 5), giving your answer in the form $y = mx + c$.

$\frac{5}{-12}$ ← Gradient of the radius to point (-12 , 5). The centre of the circle is (0 , 0). Gradient = (change in y)/(change in x). Change in y is 5 - 0, which is 5. Change in x is -12 - 0, which is -12

$y = \frac{12}{5}x + c$ ← The gradient of the tangent is the negative reciprocal of the gradient of the radius as they are perpendicular, so is 12/5. Substituting this into the y = mx + c, where m is the gradient

$c = 5 - \frac{12}{5}(-12)$ ← Rearranged to find c by subtracting 12/5 x from both sides. Then substituted in the x and y-coordinate of the point (-12 , 5) as this must satisfy the equation of the tangent

$= \frac{25}{5} + \frac{144}{5}$ ← -12/5 x -12 = 144/5. Considering 5 as the fraction 5/1. Multiplying the numerator and denominator by 5 to make it have the same denominator as the 144/5

Substituting the value of c back into the equation of the tangent

(b) $y = \frac{12}{5}x + \frac{169}{5}$ .... **[5]**

END OF QUESTION PAPER