

Please write clearly in block capitals.

Centre number

Candidate number

Surname \_\_\_\_\_

Forename(s) \_\_\_\_\_

Candidate signature \_\_\_\_\_

I declare this is my own work.

# GCSE MATHEMATICS

# H

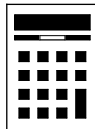
Higher Tier      Paper 2 Calculator

Time allowed: 1 hour 30 minutes

### Materials

For this paper you must have:

- a calculator
- mathematical instruments.



### Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24	
<b>TOTAL</b>	

### Advice

In all calculations, show clearly how you work out your answer.



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to [curtis@cgmaths.co.uk](mailto:curtis@cgmaths.co.uk)

Answer **all** questions in the spaces provided.

1 Circle the factor of  $x^2 - 5x$

[1 mark]

$x - 1$                        $-5x$                        $x - 5$                        $5x$

When factorised it is  $x(x - 5)$  so both  $x$  and  $(x - 5)$  are factors

2  $A$  is half of  $B$ .

Work out the ratio  $A : B$

Circle your answer.

[1 mark]

$1 : 2$                        $2 : 1$                        $1 : 3$                        $3 : 1$

There is 1 part for  $A$  and 2 parts for  $B$  and 1 is half of 2

3 The first three terms of a geometric progression are  $\frac{2}{3}$     $\frac{4}{9}$     $\frac{8}{27}$

Circle the fourth term.

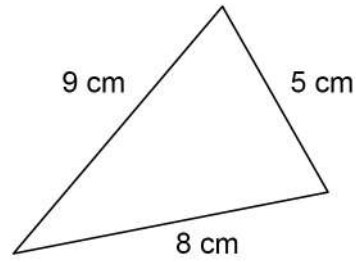
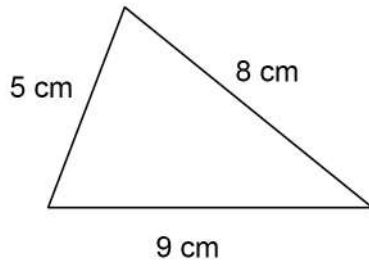
[1 mark]

$\frac{10}{81}$                        $\frac{14}{81}$                        $\frac{16}{81}$                        $\frac{32}{81}$

It multiplies by  $\frac{2}{3}$  between each term



4

Not drawn  
accurately

Circle the reason why these triangles are congruent.

[1 mark]

ASA

RHS

SAS

SSS

All three sides are the same in both triangles

5

Solve  $10x = 62.4 - 3x$ 

[2 marks]

$$13x = 62.4$$

Adding 3x to both sides to get all the x on the same side

$$x = \frac{62.4}{13}$$

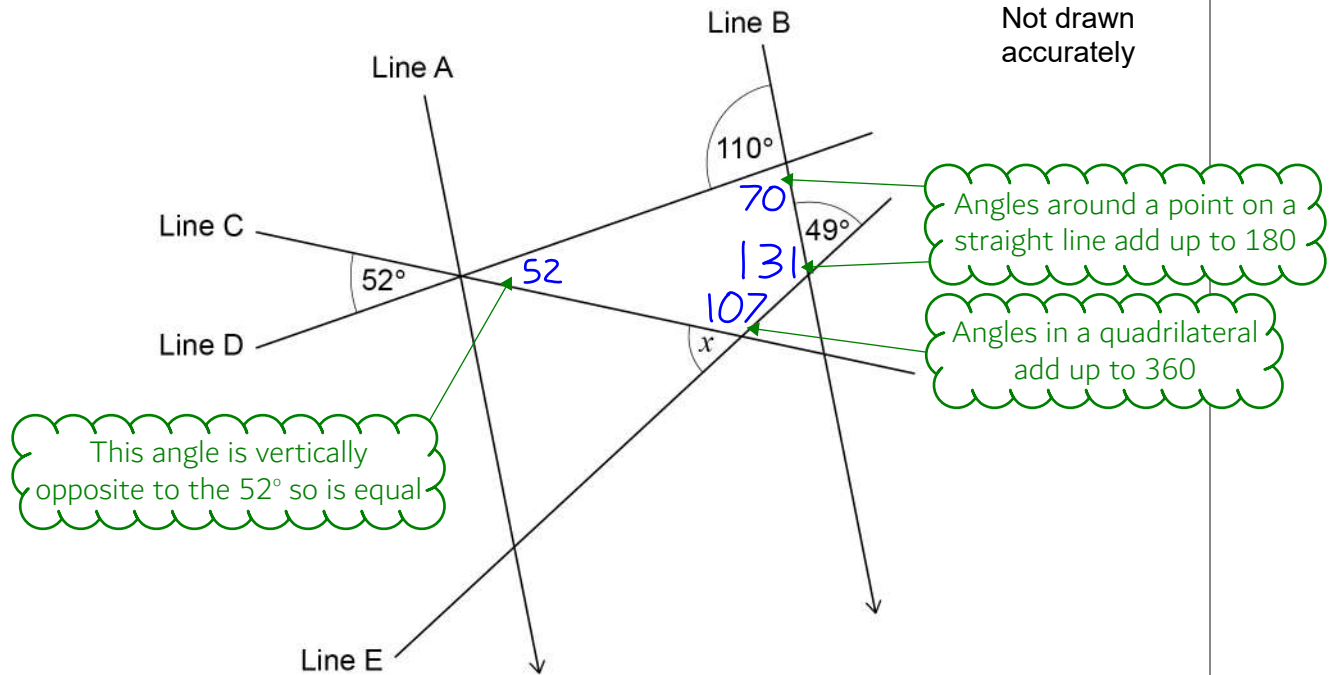
Dividing both sides by 13 to get x on its own

$$x = 4.8$$

Turn over ►



- 6 Lines A, B, C, D and E intersect as shown.  
Lines A and B are parallel.



Work out the size of angle  $x$ .

[3 marks]

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Answer 73 degrees

Angles around a point on a  
straight line add up to 180



- 7 102 boys and 85 girls took a test.  
The table shows information about the mean marks.

	Boys	Girls
Number of students	102	85
Mean mark	68.5	72.4

The pass mark for the test was 70

Was the mean mark for **all** of these students greater than the pass mark?

You **must** show your working.

[3 marks]

$m^t n$

A formula triangle for mean. m: mean. t: total. n: number

$$\frac{102 \times 68.5 + 85 \times 72.4}{102 + 85}$$

Multiplying the number of boys by their mean mark works out the total for the boys. Multiplying the number of girls by their mean mark works out the total for the girls. Adding both of these totals gives the overall total for all of the students. Dividing this by the number of students gives the mean for all the students

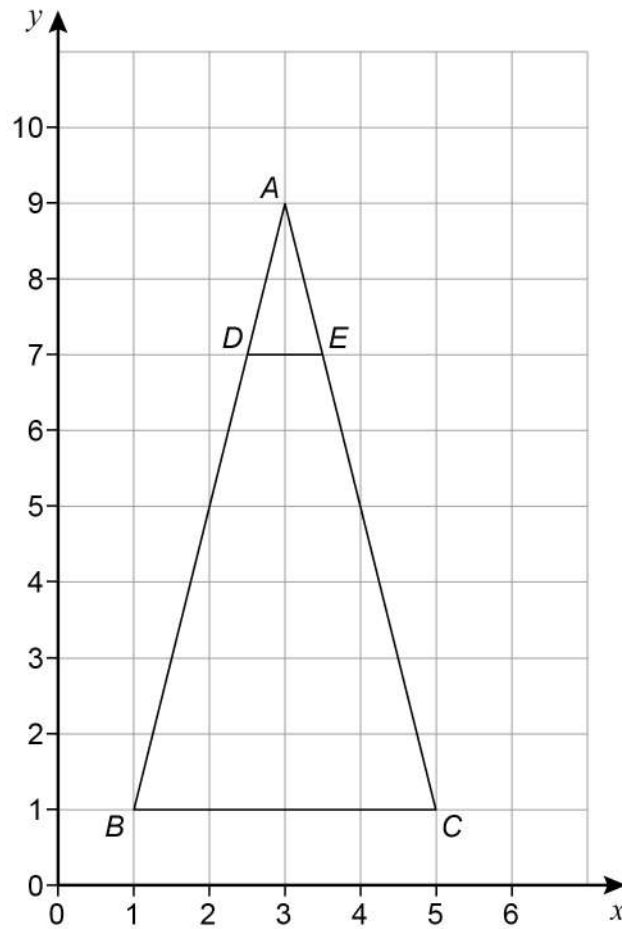
70.27

Yes

The mean mark for all of the students was greater than the pass mark of 70



8



Describe fully the **single** transformation that maps triangle  $ABC$  to triangle  $ADE$ .

[3 marks]

Enlargement by scale factor  $1/4$  from  $(3, 9)$

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9

A ball contains  $5000 \text{ cm}^3$  of air.

More air is pumped into the ball at a rate of  $160 \text{ cm}^3$  per second.

The ball is full of air when it becomes a sphere with radius  $15 \text{ cm}$



$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3 \quad \text{where } r \text{ is the radius}$$

Does it take **less than** 1 minute to fill the ball?

You **must** show your working.

[4 marks]

 $s^d t$ 

Quoting the distance, speed, time formula triangle as the volume is basically distance, the rate it is pumped is basically speed and we are calculating time

$$\frac{\frac{4}{3} \pi \times 15^3 - 5000}{160}$$

$\frac{4}{3} \pi \times 15^3$  works out the volume of the full sphere. Subtracting the  $5000 \text{ cm}^3$  which is already in the ball leaves the volume needed to fill the ball, which can be thought of as the distance. Time = distance/speed so the volume needed to fill the ball is divided by 160, which is the rate the air is pumped into the ball and can be thought of as the speed

57.1

Yes

57.1 seconds is less than 1 minute, which is 60 seconds





10

 $p$  is a positive number. $n$  is a negative number.

For each statement, tick the correct box.

**[4 marks]**

	Always true	Sometimes true	Never true
$p + n$ is positive	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
$p - n$ is positive	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$p^2 + n^2$ is positive	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$p^3 \div n^3$ is positive	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>

$p + n$  could be positive as  $2 + -1 = 1$  but it could also be not positive as  $1 + -2 = -1$ .  
 $p - n$  is always positive as subtracting a negative is a double negative so it becomes a positive and a positive add a positive must be positive.  $p^2 + n^2$  must be positive as  $p^2$  and  $n^2$  are positive, as squaring means to multiply by itself and a positive multiplied by positive is positive and a negative multiplied by a negative is double negative so becomes a positive and positive add a positive must be positive.  $p^3 \div n^3$  must always be negative as a positive cubed is positive and a negative cubed is negative and dividing a positive by a negative gives a negative

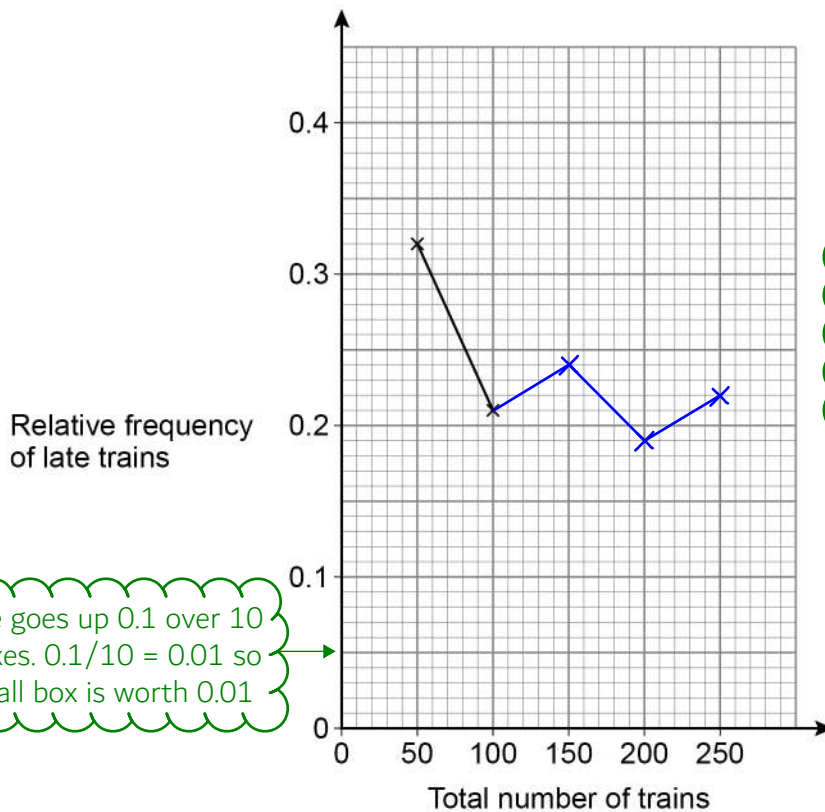


- 11 250 trains arrived at a station.  
The number of trains that were late was recorded after every 50 trains.  
The table shows some information about the results.

<b>Total number of trains</b>	50	100	150	200	250
<b>Total number of late trains</b>	16	21	36	38	55
<b>Relative frequency of late trains</b>	0.32	0.21			

- 11 (a) Complete the relative frequency graph.

[3 marks]



$$\begin{aligned} 36/150 &= 0.24 \\ 38/200 &= 0.19 \\ 55/250 &= 0.22 \end{aligned}$$

Expressing the number of late trains as a fraction of the total number of trains gives the relative frequency. Converting these to decimals so they can be plotted

The scale goes up 0.1 over 10 small boxes.  $0.1/10 = 0.01$  so each small box is worth 0.01

- 11 (b) Write down the best estimate of the probability that a train arriving at the station is late.

[1 mark]

Answer 0.22

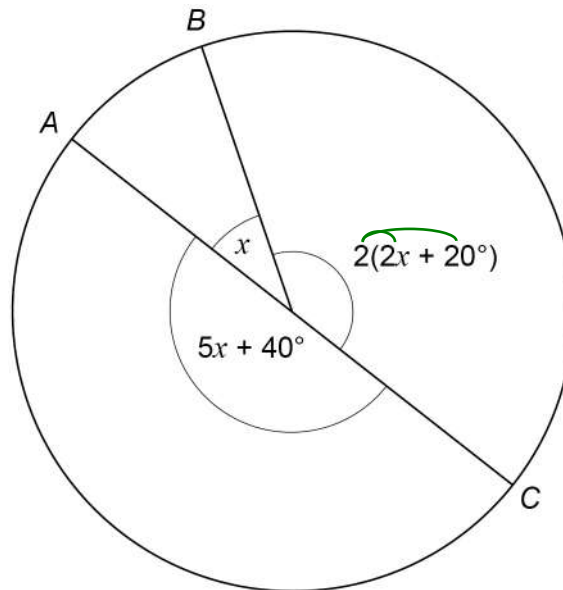
Each relative frequency is an estimate of the probability but the best one is the one based on the most trains



12

$A$ ,  $B$  and  $C$  are three points on a circle.  
The radii from  $A$ ,  $B$  and  $C$  are shown.

Not drawn  
accurately



Is  $AC$  a diameter of the circle?

You **must** show your working.

[3 marks]

$$x + 4x + 40$$

Adding together both of the angles above  $AC$  and expanding  $2(2x + 20)$

$$5x + 40$$

Simplifying by collecting like terms

Yes

The angles above line  $AC$  are the same as the angle below it meaning both must be  $180^\circ$  in order to add up to  $360^\circ$ . Therefore  $AC$  must be a diameter as the radii from  $A$  and  $C$  are  $180^\circ$  apart and therefore form a straight line going through the centre



13

A straight line

has gradient 6

and

passes through the point (3, 19)

Work out the equation of the line.

Give your answer in the form  $y = mx + c$ **[3 marks]**

$$c = 19 - 6 \times 3$$

Rearranged to make c the subject by subtracting mx from both sides to give  $c = y - mx$ . Then substituted in the point and the gradient. Substituted y for 19, m for 6 as m is the gradient and x for 3

m is 6 and c is 1

Answer

$$y = 6x + 1$$

Turn over for the next question

6

Turn over ►



14 The population of butterflies in a park is 4200

14 (a) Assume that the population increases by 12% each day.

Show that after 20 days the population would be greater than 40 000

[2 marks]

$$4200 \times \left(\frac{100+12}{100}\right)^{20}$$

100 + 12 works out the percentage it rises to each day. Dividing this by 100 converts it into a multiplier. Raising it to the power of 20 as it needs to be multiplied by 20 times. Multiplying 4200 by this increases it by 12% 20 times

$$40514$$

Rounded to the nearest whole number. It is greater than 40000

14 (b) In fact, the population  
increases by 13% each day for 19 days  
then  
**decreases** by 8% for 1 day.

After the 20 days, is the actual population greater than 40 000 ?

Tick a box.

Yes

No

Show working to support your answer.

[2 marks]

$$4200 \times \left(\frac{100+13}{100}\right)^{19} \times \left(\frac{100-8}{100}\right)$$

Same method as above but using an increase of 13% 19 times and then a decrease of 8% once

$$39403$$

Rounded to the nearest whole number. It is less than 40000



- 14 (c) The expected number of visitors to the park each day depends on the temperature.

Temperature	Expected number of visitors each day
Less than 21°C	700
21°C or more	900

On each of the 30 days in June

the park is open

the probability that the temperature is less than 21°C is 0.4

Work out the **total** number of expected visitors to the park in June.

[3 marks]

$$0.4 \times 30 \times 700 + (1 - 0.4) \times 30 \times 900$$

The total number of people expected on the days less than 21°C. Multiplying the probability of the temperature being less than 21°C by the number of days in June works out an estimate of how many of the days will be less than 21°C. There are 700 visitors on each of these days

The total number of people expected on the days 21°C or more. It is certain that it is either less than 21°C or 21°C or more so the probabilities have to add up to 1. So subtracting the probability of it being less than 21°C from 1 works out the probability of it being 21°C or more. Multiplying the probability of the temperature being 21°C or more by the number of days in June works out an estimate of how many of the days will be 21°C or more. There are 900 visitors on each of these days

Adding the total number of people expected on the days less than 21°C and the total number of people expected on the days 21°C or more gives the total number of expected visitors

Answer 24600



15  $L$  is directly proportional to  $D^2$

$$L = 85 \text{ when } D = 10$$

15 (a) Work out an equation connecting  $L$  and  $D$ .

[3 marks]

$$L = kD^2$$

$L = kD^2$  will be true whatever  $D^2$  is multiplied by. So multiplying it by  $k$ , which represents the number it is multiplied by

$$k = \frac{85}{10^2}$$

Rearranged the equation to make  $k$  the subject by dividing both sides by  $D^2$ . Then substituting  $L$  for 85 and  $D$  for 10 to work out  $k$

Substituting  $k$  for 0.85 in the original equation

Answer

$$L = 0.85D^2$$

15 (b) Work out the value of  $L$  when  $D = 5$

[2 marks]

$$0.85 \times 5^2$$

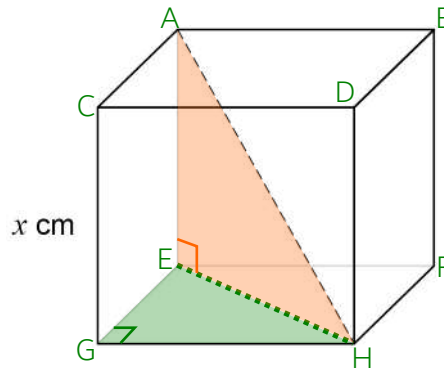
Substituting  $D$  for 5 in the equation found in part (a)

Answer

$$21.25$$



- 16** Here is a cube with edge length  $x$  cm  
One diagonal is shown.



- 16 (a)** Circle the length, in centimetres, of the diagonal.

[1 mark]

$\sqrt{3}x$

$\sqrt[3]{3x^2}$

$\sqrt{x^3}$

$\sqrt[3]{3}x$

See next page

- 16 (b)** The total length, in centimetres, of the edges of the cube is a multiple of 18  
Circle the correct statement.

[1 mark]

$x$  is a  
whole number

$x$  is not a  
whole number

$x$  might be a  
whole number

There are 12 edges on the cube. Not all multiples of 18 are multiples of 12 so dividing the multiple of 18 might give a decimal. But there are common multiples of 18 and 12 so it could be a whole number

Turn over for the next question

Turn over ►





Question number	Additional page, if required. Write the question numbers in the left-hand margin.
16a	$a^2 + b^2 = c^2$ <p>The diagonal AH is in the orange right angled triangle so can be found using Pythagoras' Theorem, where c is the longest side and a and b are the shorter sides. But first length EH needs to be found and this can be found by using Pythagoras' Theorem in the green triangle</p>
	$c = \sqrt{a^2 + b^2}$ <p>Both AH and EH are the longest side in their right angled triangle so making c the subject by square rooting both sides</p>
	$\sqrt{x^2 + x^2} = \sqrt{2x^2}$ <p>Finding EH by substituting in the sides of x and x. It is a cube so all of its edges are the same so must be x</p>
	$\sqrt{2x^2 + x^2}$ <p>Finding AH by substituting in the sides of x and EH</p>
	$\sqrt{3x^2}$ <p>The square root and the square cancel out leaving <math>2x^2 + x^2</math> within the square root. This is <math>3x^2</math></p> <p>This simplifies to <math>\sqrt{3}x</math></p>



- 17 20 people were asked which device they used more often, laptop or phone.  
The table shows the results.

	Laptop	Phone
Male	2	9
Female	4	5

- 17 (a) One male and one female are chosen at random.

Work out the probability that **exactly** one of them said laptop.

[3 marks]

$$\frac{2}{11} \times \frac{5}{9} + \frac{9}{11} \times \frac{4}{9}$$

Male laptop AND female phone OR male phone AND female laptop.  
AND means to multiply the probabilities. OR means to add the probabilities. There are 11 males in total and 9 females in total

Answer  $\frac{46}{99}$

- 17 (b) Two males are chosen at random.

Work out the probability that they **both** said phone.

[2 marks]

$$\frac{9}{11} \times \frac{8}{10}$$

Male phone AND male phone. AND means to multiply the probabilities. 9 out of the 11 males chose phone for the first pick. On the second pick there is one fewer male in total and one fewer who chose phone so there is now 8 out of 10

Answer  $\frac{36}{55}$



18 On the grid, identify the region represented by

$$x \leq 5$$

$$y \leq 4$$

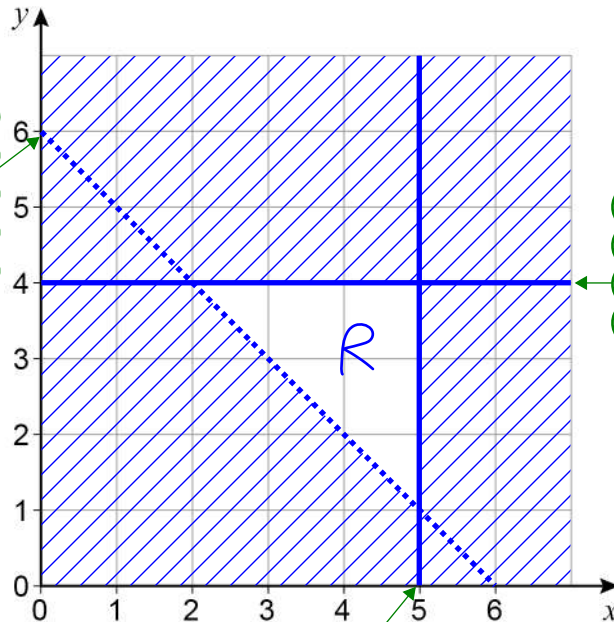
$$x + y > 6$$

Rearranged to make y the subject

$$y > 6 - x$$

Label the region R.

[3 marks]



The line of  $y = 6 - x$ . It is dashed as  $y$  cannot be equal to  $6 - x$ . Crossing out everything below the line as the region is above the line

The line of  $y = 4$ . It is solid as  $y$  can be equal to 4. Crossing out everything above the line as the region is below the line

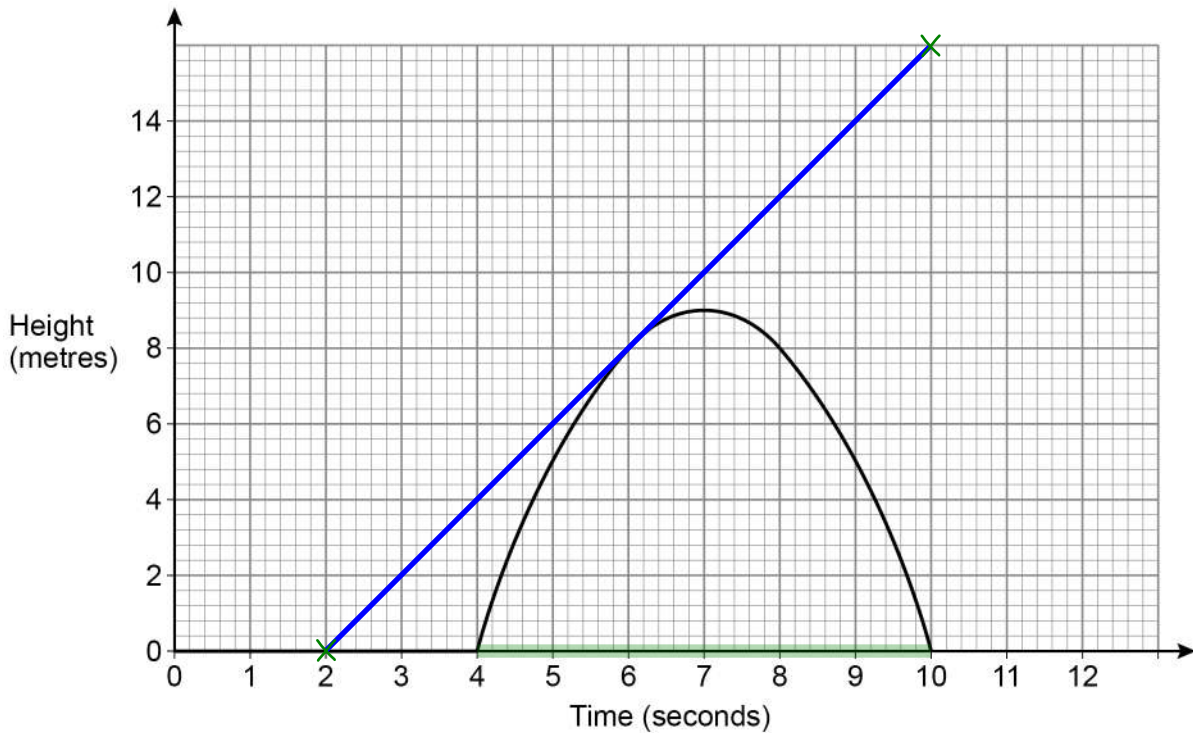
The line of  $x = 5$ . It is solid as  $x$  can be equal to 5. Crossing out everything to the right of the line as the region is on the left of the line

Turn over for the next question

Turn over ►



- 19 The graph shows the height above ground of a toy rocket for 10 seconds.



- 19 (a) For how long is the rocket in the air?  
Circle your answer.

[1 mark]

10 seconds

9 seconds

6 seconds

4 seconds

The rocket was in the air between 4 and 10 seconds. The difference of these is 6



- 19 (b) Using the graph, estimate the speed of the rocket after 6 seconds.  
State the units of your answer.

[3 marks]

$$\frac{16-0}{10-2}$$

A tangent is drawn at 6 seconds to estimate the gradient of the curve at that point.  
Gradient = (change in y)/(change in x). Change in y is found by 16 - 0 and change in x is found by 10 - 2 using the points indicated with crosses on the tangent

The unit is m/s as the change in y in metres was divided by the change in x in seconds

Answer 2 m/s

- 20 A square has an area of 0.25 square metres.  
Circle the length, in **centimetres**, of one side of the square.

[1 mark]

0.5 cm

5 cm

50 cm

500 cm

Let x be the side length of the the square.  $x^2 = \text{area} = 0.25$ . Square rooting both sides gives that x is 0.5 but this is in metres so is 50cm

Turn over for the next question

Turn over ►



21

 $x$  is an integer.Prove that  $35 + (3x + 1)^2 - 2x(4x - 3)$  is a square number.**[4 marks]**

$$35 + 9x^2 + 6x + 1 - 8x^2 + 6x$$

Expanding the square bracket using 'square the first term, double the product of the two terms, square the last term'. Expanding the other bracket

$$x^2 + 12x + 36$$

Collecting like terms to simplify

$$(x + 6)^2$$

Factorising by finding two numbers which multiply to 36 and add to 12 and putting these in brackets with  $x$ . 6 and 6 do this so it would be  $(x + 6)(x + 6)$ . This can be written as a square bracket and shows that it is a square number



22

Liam is trying to remember a 3-digit code.

He knows the rule that

the first digit is a cube number

the second digit is a factor of 16

the third digit is an odd number.

Liam tries at random a code that matches the rule.

Work out the probability that this is the correct code.

[4 marks]

1, 8

Listing out the single digit cube numbers

1, 2, 8, 4

Listing out the single digit factors of 16

1, 3, 5, 7, 9

Listing out the single digit odd numbers

$$\frac{1}{2 \times 4 \times 5}$$

Using the product rule for counting. There are 2 possibilities for the first digit, 4 for the second and 5 for the third and multiplying these together works out how many possible codes there are. 1 out of these codes is correct

Answer  $\frac{1}{40}$



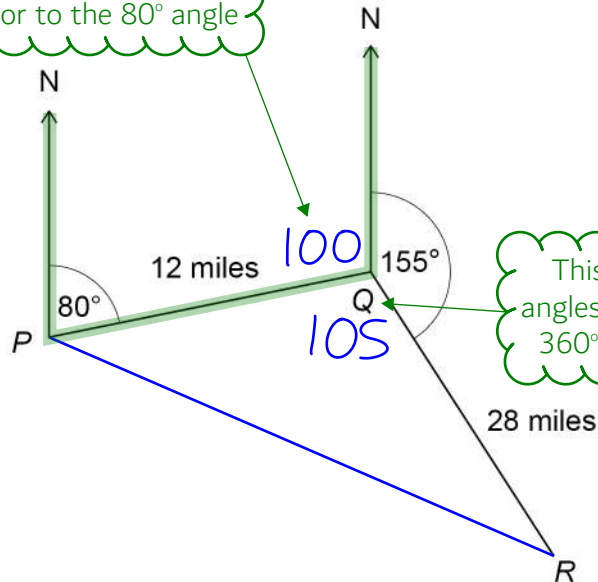
23

A ship sails from  $P$  to  $Q$  and then from  $Q$  to  $R$ .

$Q$  is 12 miles from  $P$ , on a bearing of  $080^\circ$

$R$  is 28 miles from  $Q$ , on a bearing of  $155^\circ$

This angle must be  $100^\circ$  as it is co-interior to the  $80^\circ$  angle



This angle must be  $105^\circ$  as angles around a point add up to  $360^\circ$ .  $360 - 100 - 155 = 105$

Work out the direct distance from  $P$  to  $R$ .

[4 marks]

$$a^2 = b^2 + c^2 - 2bc \cos A$$

It is not a right angled triangle so Pythagoras' Theorem can't be used. There aren't two pairs of opposite sides and angles so the sine rule can't be used. Therefore the cosine rule needs to be used

$$\sqrt{12^2 + 28^2 - 2 \times 12 \times 28 \times \cos 105}$$

Rearranged to make  $a$  the subject by square rooting both sides as  $PR$  is opposite the angle and the angle must be  $A$ .  $a$  is opposite  $A$ . Substituting 12 for  $b$ , 28 for  $c$  and 105 for  $A$

Answer 33.2 miles





24

The flight of a plane was in two stages.  
The table shows information about the flight.

	Distance (miles)	Speed (mph)	Time (hours)
1st stage	731	$x$	$\frac{731}{x}$
2nd stage	287	$x - 24$	$\frac{287}{x - 24}$

In total, the flight lasted 2 hours.

Work out the value of  $x$ .

[5 marks]

$$\frac{731}{x} + \frac{287}{x-24} = 2$$

The distances and speeds are irrelevant so can be ignored. Adding the times must give 2

$$731(x-24) + 287x = 2x(x-24)$$

Multiplying both sides by the denominators to cancel them out

$$731x - 17544 + 287x = 2x^2 - 48x$$

Expanding the brackets

$$2x^2 - 1066x + 17544 = 0$$

Bringing into the quadratic form  $ax^2 + bx + c = 0$

$$x = \frac{-(-1066) \pm \sqrt{(-1066)^2 - 4 \times 2 \times 17544}}{2 \times 2}$$

Solving using the quadratic formula

The other solution of  $x$  is 17 but is ignored as this would give a negative time for the 2nd stage

Answer

516

Turn over ►



25 The equation of a curve is  $y = x^2 + 14x + 52$

By completing the square, work out the coordinates of the turning point.

You **must** show your working.

[3 marks]

$$y = (x+7)^2 + 52 - 7^2$$

Completing the square by halving the coefficient of  $x$ , putting this in a bracket with  $x$  and squaring it. Subtracting  $7^2$  as when expanding the square bracket this will be in addition to the  $x^2 + 14x$  and if it wasn't subtracted this would change the value of the right side

The turning point occurs when the square bracket is equal to 0 as this is the smallest a squared number can be.  $x = -7$  for this to happen. When the bracket is 0,  $y = 52 - 7^2 = 3$

Answer ( -7 , 3 )

END OF QUESTIONS

