# AQA

Please write clearly in block capitals.

Centre number ☐☐☐☐☐　　Candidate number ☐☐☐☐

Surname _____

Forename(s) _____

Candidate signature _____

# GCSE
# MATHEMATICS

## Higher Tier　　　Paper 2　Calculator
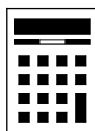
**H**

Thursday 8 November 2018　　Morning　　　Time allowed: 1 hour 30 minutes

## Materials

For this paper you must have:
- a calculator
- mathematical instruments.

## Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

## Advice

In all calculations, show clearly how you work out your answer.

| For Examiner's Use | |
|---|---|
| Pages | Mark |
| 2–3 | |
| 4–5 | |
| 6–7 | |
| 8–9 | |
| 10–11 | |
| 12–13 | |
| 14–15 | |
| 16–17 | |
| 18–19 | |
| 20–21 | |
| 22–23 | |
| **TOTAL** | |

**.CG Maths.**

Answer **all** questions in the spaces provided

**1** What does (A ∩ B) represent in     P(A ∩ B) ?

Circle your answer.

*The intersection of A and B*

**[1 mark]**

A or B or both                                        A but not B

not A and not B                                        A and B

**2** *P* is (4, 9) and *Q* is (–2, 1)

Circle the midpoint of *PQ*.

**[1 mark]**

(1, 5)                (3, 4)                (3, 5)                (6, 8)

*Working out the mean of the x-coordinates works the x-coordinate of the midpoint*

**3** Which of these is a geometric progression?

Circle your answer.

**[1 mark]**

1  3  5  7  9                                1  3  6  10  15

1  4  9  16  25                                1  3  9  27  81

*Geometric: each term is multiplied by the same number to get the next term*

**.CG Maths.**

**4**   The bearing of *A* from *B* is 310°

Circle the bearing of *B* from *A*.

**[1 mark]**

050°          110°          130°          220°

This is the angle we are trying to find.
Co-interior angles add up to 180

A

Find this angle by using the fact there
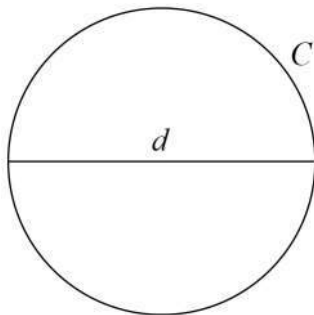are 360 degrees in total around a point

B

310°
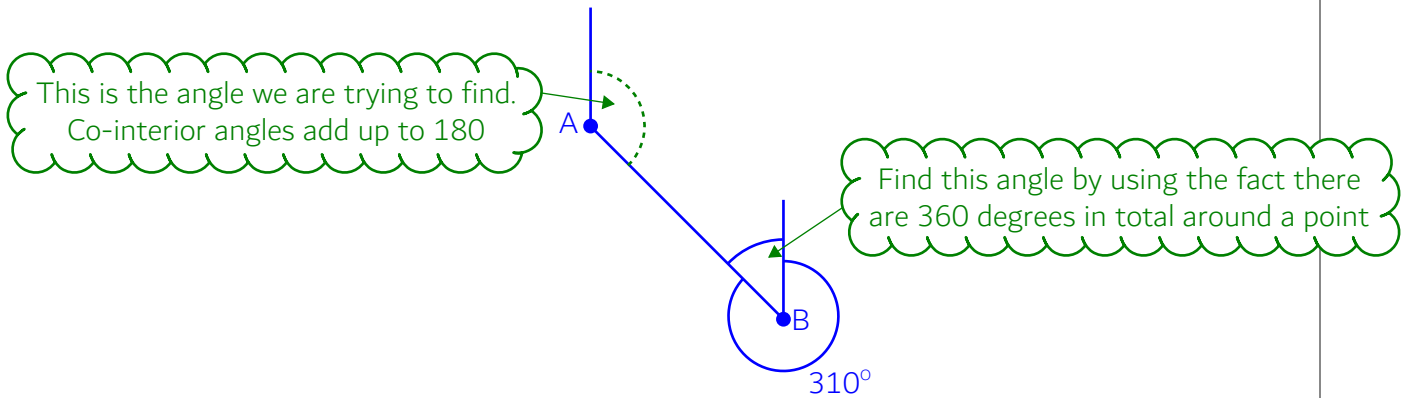
**5**   A circle has circumference $C$ and diameter $d$.

$C$

$d$

$C = kd$

What **value** does the constant $k$ represent?

**[1 mark]**

Answer _____

Circumference = π x diameter

5

**Turn over ▶**

.CG Maths.

**6** Here is some information about 20 trains leaving a station.

| Number of minutes late, $t$ | Number of trains | Midpoint | |
|---|---|---|---|
| $0 \leqslant t < 5$ | 12 | | |
| $5 \leqslant t < 10$ | 7 | | |
| $10 \leqslant t < 15$ | 1 | | |
| $t \geqslant 15$ | 0 | | |

*There are no trains in this category so it can be ignored*

*Each category has a range of 5. Dividing 5 by 2 then adding this on to each of the lowest number in each category works out the midpoints*

**6 (a)** Work out an estimate of the mean number of minutes late.

**[3 marks]**

*Mean = total/number. An estimate of the total is found by multiplying the midpoint by the frequency for each category then adding them all together. The number is the total number of trains*

Answer _____ minutes

**6 (b)** The station manager looks at the information in more detail.

| Number of minutes late, $t$ | Number of trains |
|---|---|
| $0 \leqslant t < 2$ | 12 |
| $2 \leqslant t < 4$ | 0 |
| $4 \leqslant t < 6$ | 7 |
| $6 \leqslant t < 8$ | 0 |
| $8 \leqslant t < 10$ | 0 |
| $10 \leqslant t < 12$ | 1 |

He works out an estimate of the mean using this information.

How does his estimate compare with the answer to part (a)?

Tick **one** box.

[1 mark]

☐ Higher than part (a)

☐ Same as part (a)

☐ Lower than part (a)

☐ Not possible to tell

The midpoints of the categories for the 12, 7 and 1 trains are lower

**Turn over for the next question**

**7** Work out the values of $a$ and $b$ in the identity
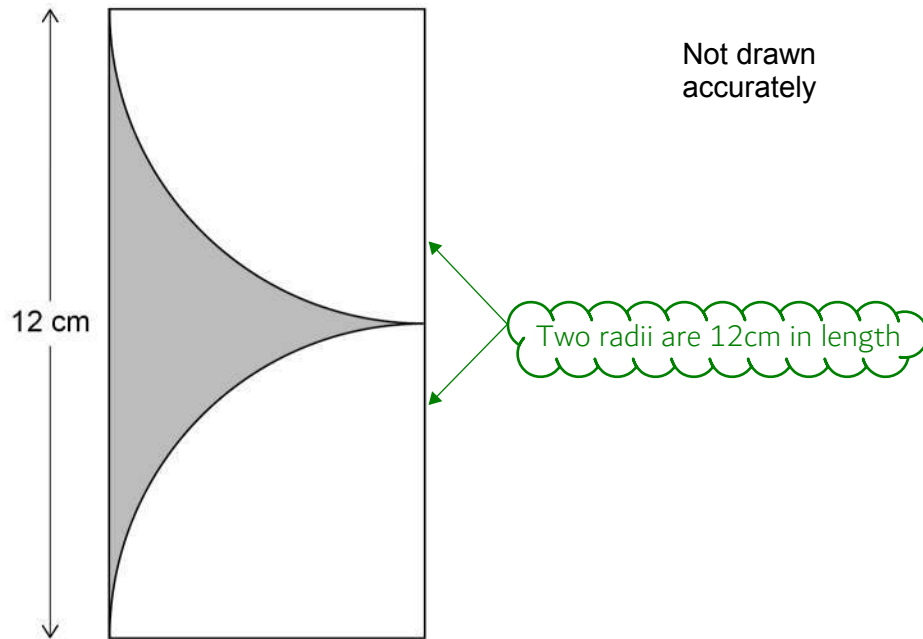
$$5(7x + 8) + 3(2x + b) \equiv ax + 13$$

**[4 marks]**

Expand the brackets then equate the coefficients. There must be the same amount of x and constants on the left as on the right

$a =$ _____ $b =$ _____

**.CG Maths.**

**8** Two identical quarter circles are cut from a rectangle as shown.

Not drawn accurately

12 cm

Two radii are 12cm in length

This length is a radius

Work out the shaded area.

**[4 marks]**

Subtracting the area of the two quarter circles from the area of the rectangle leaves the shaded area.
Area of rectangle = length x width.
Area of circle = $\pi r^2$, where r is the radius.
Find a quarter of the area of the full circle to find the area of the quarter circle

Answer _____ cm$^2$

.CG Maths.

07

8

**9**  The diagrams show the position of a tap when off and fully on.

The tap is fully on when the angle of turn is 180°

Off                                  Fully on

● 180°

When fully on, water flows out of the tap at 14 litres per minute.

The rate at which water flows out is in direct proportion to the angle of turn.

The tap is turned 135°

135°

The water flows into a tank with a capacity of 79.8 litres.

Will it take **less than** $7\frac{1}{2}$ minutes to fill the tank?

You **must** show your working.

**[4 marks]**

$S\frac{d}{t}$  ←  This is basically a speed distance time problem as the rate it is filled is the speed, the capacity is the distance and we are trying to calculate the time. From the formula triangle, time = distance/speed
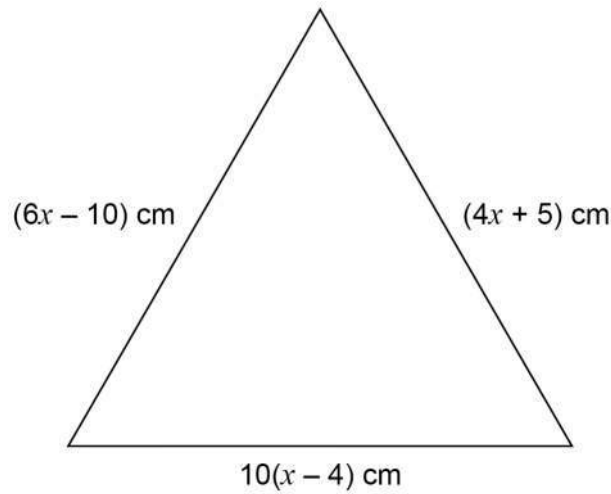
To work out the rate it is filled, express the number of degrees the tap is turned as a fraction of the number of degrees when it is fully on. Work out this fraction of the 14 litres per minute

.CG Maths.

**10**     This triangle is equilateral.

Not drawn accurately

(6x – 10) cm       (4x + 5) cm

10(x – 4) cm

Is the perimeter of the triangle greater than one metre?

You **must** show your working.

**[5 marks]**

The triangle is equilateral so all the sides are equal. Setting two of the sides equal to each other creates an equation in terms of x which can be solved.

Once x is found, substitute it into one of the expressions for the side lengths to work out the value of the length. Use this to calculate the perimeter in centimetres.

Convert the one metre into centimetres and compare it to the perimeter we have calculated

**Turn over ▶**

.CG Maths.

09

**11** An approximation for the value of $\pi$ is given by

$$4\left(1 - \frac{22}{57} + \frac{22}{85} - \frac{22}{105} + \frac{22}{117} - \frac{22}{242}\right)$$

Use your calculator to show that this approximation is within 0.1 of 3.14

**[2 marks]**

Type the approximation into the calculator exactly as it looks above

Subtracting the approximation from 3.14 works out the difference. As this is less than 0.1, it is within 0.1 of 3.14

**12** Work out $\dfrac{9.12 \times 10^{10}}{3.2 \times 10^{4}}$

Give your answer in standard form.

**[2 marks]**

Type it into the calculator exactly as it is above

To be in standard form, it needs to be in the form $a \times 10^{n}$, where $1 \le a < 10$ and n is an integer

Answer _____

.CG Maths.

**13** Ashraf is going to put boxes into a crate.

The crate is a cuboid measuring   2.5 m by 2 m by 1.2 m

Each box is a cube of length 50 cm

He does these calculations.

| | | |
|---|---|---|
| volume of crate | = | $2.5 \times 2 \times 1.2$ |
| | = | $6 \text{ m}^3$ |
| volume of one box | = | $0.5 \times 0.5 \times 0.5$ |
| | = | $0.125 \text{ m}^3$ |
| number of boxes | = | $6 \div 0.125$ |
| | = | $48$ |

He claims,

"I can put 48 boxes in the crate."

Evaluate Ashraf's method **and** claim.

**[2 marks]**

If a box is 10m long, it might have a smaller volume than the crate but it is too long to fit. Volume may not be an appropriate way of measuring how many boxes can fit

**14** The cross section of a prism has $n$ sides.

Circle the expression for the number of edges of the prism.

**[1 mark]**

$2n$          $3n$          $n + 2$          $2n + 3$

Consider a triangular prism: the cross section (highlighted in green) has 3 sides and the prism has 9 sides. Then consider a different prism (maybe a cuboid). Try each expression to see which one works for both

**7**

**Turn over ▶**

.CG Maths.

**15** The volume of a medal is 45 cm$^3$

The medal is made from copper and tin.

| volume of copper : volume of tin = 22 : 3 |

The density of copper is 8.96 g/cm$^3$

The density of tin is 7.31 g/cm$^3$
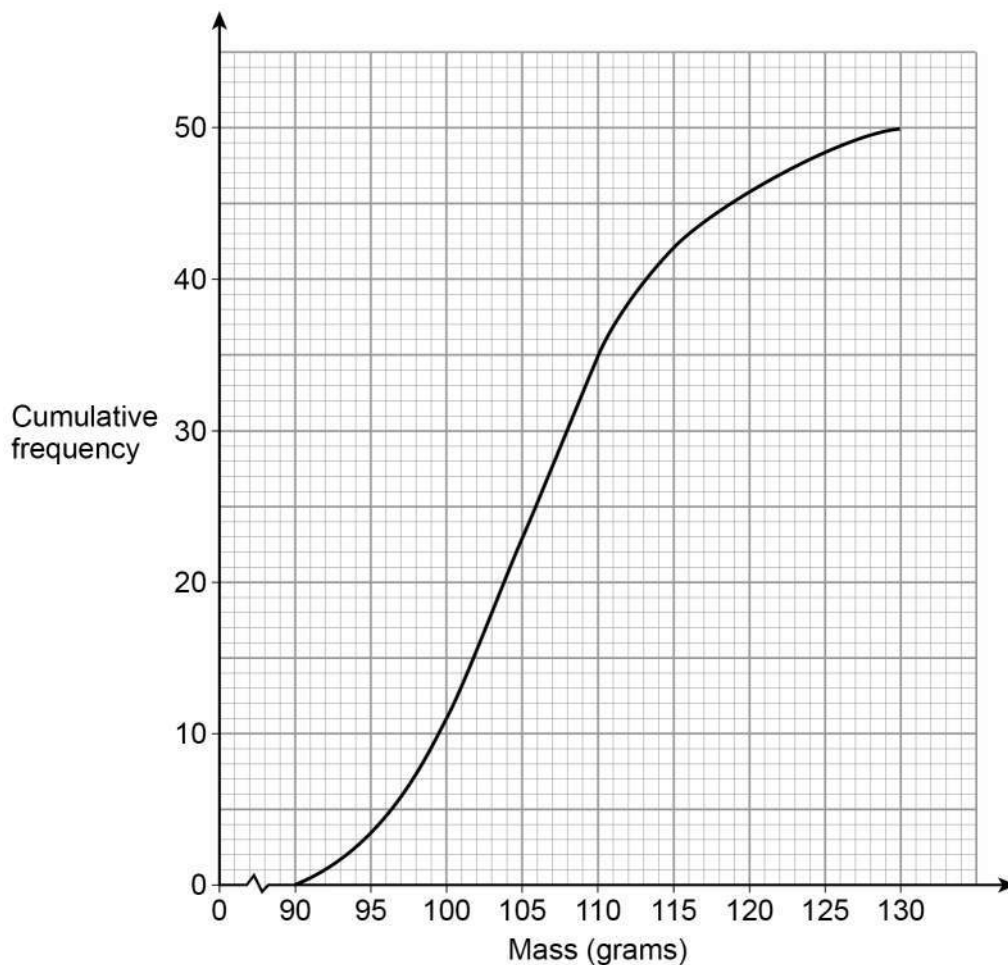
Work out the mass of the medal.

**[4 marks]**

*From the formula triangle, mass = density x volume*

D $\overset{m}{\phantom{.}}$ V

*Add the mass of the copper and the mass of the tin to get the mass of the medal. To work out the volume of each metal, divide 45cm³ into the ratio of the volumes*

Answer _____ grams

.CG Maths.

**16** The cumulative frequency graph shows information about the masses of 50 apples.



**16 (a)** Use the graph to estimate the median mass of the apples.

**[1 mark]**

*The median is roughly half way through the data. Drawing a line across from half of the 50 apples to the line then down gives an estimate for the median*

Answer _____ grams

**16 (b)** Estimate the proportion of the apples that have a mass greater than 115 grams.

**[2 marks]**

*Drawing a line up from 115 to the line then across estimates how many apples are 115g or less*

Answer _____

7

**Turn over ▶**

.CG Maths.

**17**   $a$ is a prime number.

$b$ is an even number.

$N = a^2 + ab$

Circle the correct statement about $N$.

**[1 mark]**

could be
even or odd                    always even

~~always prime~~                always odd

*It can't be always prime as a can be brought out as a factor*

*A prime number could be odd or even (as 2 is prime and even and all the other primes are odd). Consider whether a² would be even or odd or either and whether ab would be even or odd or either. Then consider what adding these together would mean*

**18**   A bag contains 20 discs.

10 are red, 7 are blue and 3 are green.

**18 (a)**   Marnie takes a disc at random before putting it back in the bag.

Nick then takes a disc at random before putting it back in the bag.

Olly then takes a disc at random.

Work out the probability that they all take a red disc.

**[2 marks]**

*Red AND red AND red; AND means to multiply the probabilities of each event. As the disk is replaced, the probability of getting red is the same for Marnie, Nick and Olly. There are 10 red disks out of a total of 20*

Answer _____

**18 (b)** All 20 discs are in the bag.

Reggie takes three discs at random, one after the other.

After he takes a disc he does **not** put it back in the bag.

Reggie's first disc is blue.

Work out the probability that all three discs are different colours.

**[3 marks]**

*There is space here for a tree diagram but it isn't needed*

*It is given the first is blue. The next two must be different and can't be blue. They could be red AND green OR green AND red. AND means to multiply the probabilities of each event and OR means to add the probabilities. As there is 1 fewer disk each time (he does not put it back in the bag), the denominator decreases below 20*

Answer _____

**6**

**Turn over ▶**

.CG Maths.

**19**

| Lunch |
| :---: |
| Choose one starter and one main course |

There are four starters and ten main courses to choose from.

Two of the starters and three of the main courses are suitable for vegans.

What percentage of the possible lunches have **both** courses suitable for vegans?

**[3 marks]**

*Use the product rule for counting to work out the number of possibilities which are suitable for vegans and the number of possibilities in total. Express the number suitable for vegans as a fraction of the total number then convert it into a percentage*

Answer _____ %

**20**     $n$ is a positive integer.

Prove algebraically that     $2n^2\left(\dfrac{3}{n} + n\right) + 6n\left(n^2 - 1\right)$     is a cube number.

**[3 marks]**

*Expand the brackets. Collect like terms and simplify. Express as a number (in terms of n) cubed*

**.CG Maths.**

**21** $y$ is inversely proportional to $\sqrt{x}$

$y = 4$ when $x = 9$

**21 (a)** Work out an equation connecting $y$ and $x$.

**[3 marks]**

Inversely proportional means 'proportional to 1 over'. So y $\quad$ 1/√x. 1/√x can be multiplied by anything but nothing can be added and it will still be proportional. So multiply it by k, which represents any number, and convert it into an equation. Rearrange to find k then substitute the value of k back into the equation

Answer _____

**21 (b)** Work out the value of $y$ when $x = 25$

**[2 marks]**

Substitute 25 for x in the equation found in part (a)

Answer _____

**Turn over for the next question**

11

**Turn over ▶**

IB/M/Nov18/8300/2H

**22**      Simplify fully     $\dfrac{x^5 - 4x^3}{3x - 6}$

**[3 marks]**

*To simplify a fraction, cancel out any common factors from the numerator and denominator. To find the common factors, factorise the numerator and denominator fully to express them as factors multiplied together*

Answer _____

**23**      *PQR* is a straight line.

$PQ : QR = 3 : 1$

$\overrightarrow{PQ} = \mathbf{a}$

Not drawn accurately

P ●————————————————————————●————————● R
                                                                Q

Circle the vector  $\overrightarrow{RQ}$

**[1 mark]**

$\dfrac{1}{3}\mathbf{a}$          $\dfrac{1}{4}\mathbf{a}$          $-\dfrac{1}{3}\mathbf{a}$          $-\dfrac{1}{4}\mathbf{a}$
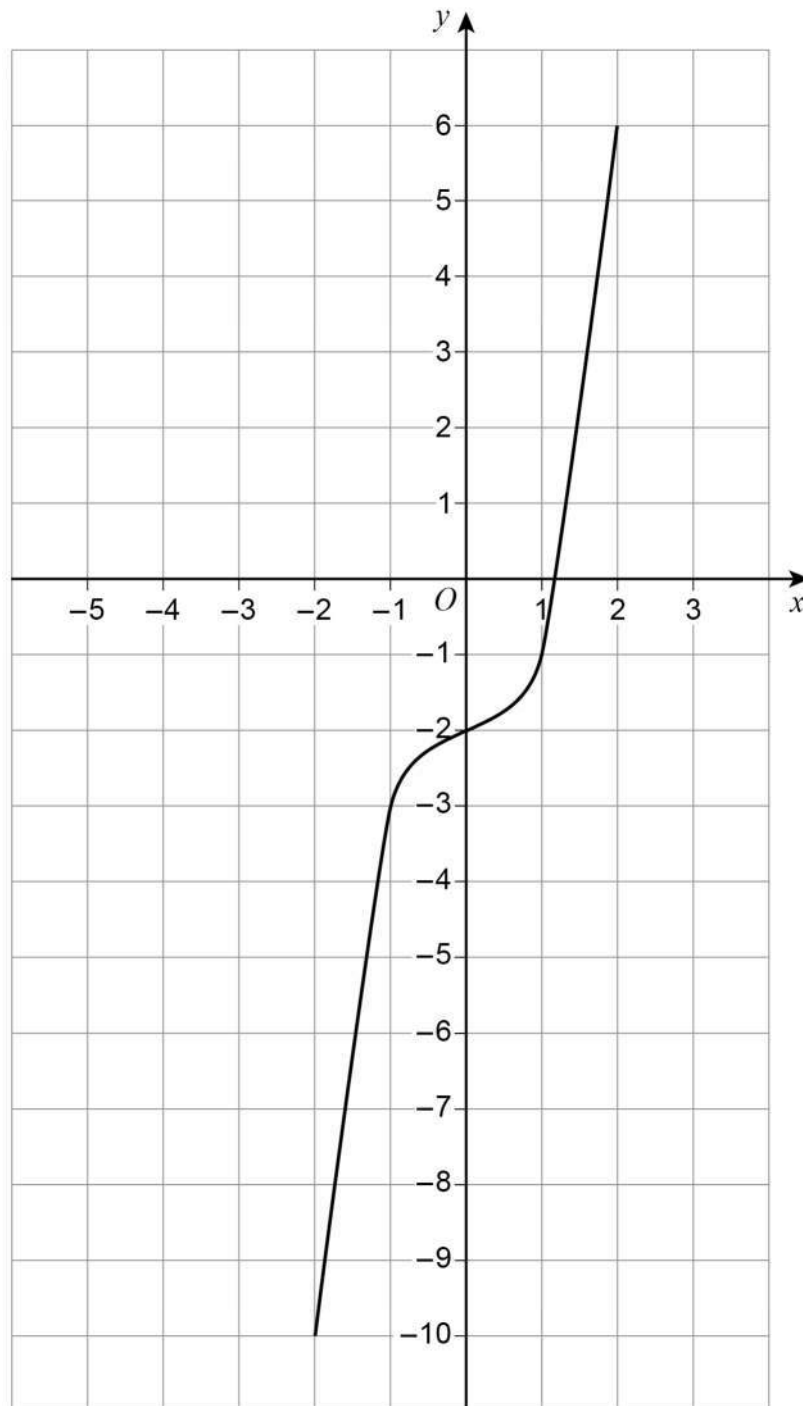
*$\overrightarrow{PQ}$ is a and is represented by 3 parts of the ratio. $\overrightarrow{QR}$ is represented by 1 part so is 1/3 a. $\overrightarrow{RQ}$ is the same size but opposite direction to $\overrightarrow{QR}$*

.CG Maths.

**25**     *ABC* and *ACD* are triangles.
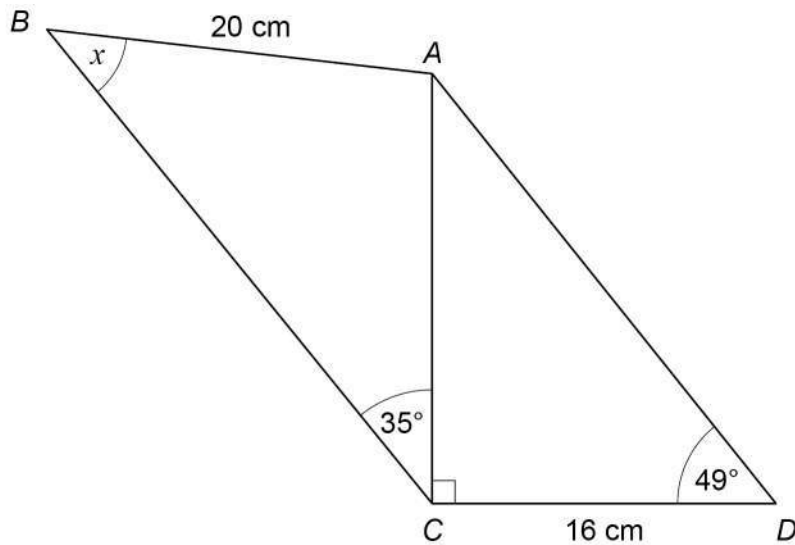


Not drawn accurately

Work out the size of angle $x$.

**[5 marks]**

$$S \overset{O}{\underset{H}{}} C \overset{A}{\underset{H}{}} T \overset{O}{\underset{A}{}}$$

AC can be found by using right-angled trigonometry in triangle ACD. Listing out SOH CAH TOA and tick what we have and what we are trying to find. If there are two ticks on one of the formula triangles, that one can be used. Cover over what we are trying to find and the formula triangle will tell us what to do

$$\frac{Sin A}{a} = \frac{Sin B}{b}$$

Once AC is found, the sine rule can be used to find angle x as there are opposite pairs of sides and angles. Writing the sine rule with the angles as numerators to make it easier to rearrange to find an angle

Rearrange the sine rule to make one of the angles the subject (either A or B) then substitute in the sides for a and b and substitute in the angles. Side a is opposite angle A

Answer _____ degrees

.CG Maths.

**26** $f(x) = \dfrac{x}{x+2}$ $\qquad g(x) = x^2 - 2$

Work out  fg($x$)

Give your answer in the form $\quad a + bx^n \quad$ where $a$, $b$ and $n$ are integers.

**[3 marks]**

Put g(x) into f(x) by substituting x² - 2 for x in f(x). Then divide the terms on the numerator by the denominator separately. b/xᵃ = bx⁻ᵃ

Answer _____

**27** The point $\left(3, \dfrac{1}{64}\right)$ lies on the curve $\quad y = k^x \quad$ where $k$ is a constant.

Show that the point $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ lies on the curve.

**[3 marks]**

The point (3 , 1/64) lies on the curve and therefore satisfies the equation. Substitute 1/64 for y (as this is the y-coordinate of the point) and 3 for x (as this is the x-coordinate of the point) in the equation then rearrange to find k. Substitute in the x-coordinate of the point (1/2 , 1/2) and the value of k to show that the y value is 1/2
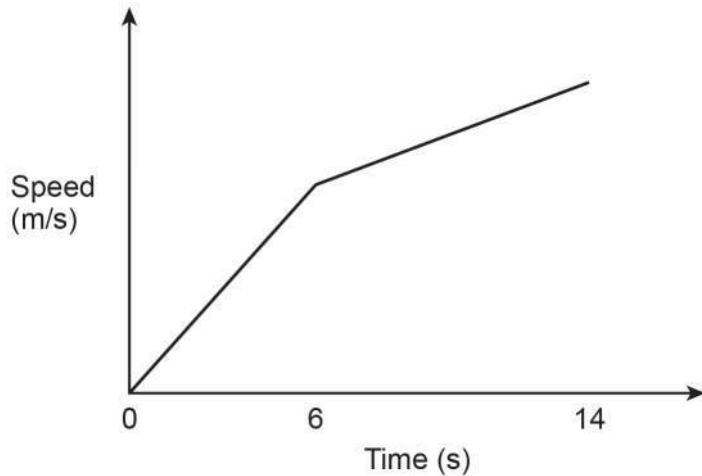
.CG Maths.

21

11

**28**   Izzy runs an 80-metre race in 14 seconds.

During the first 6 seconds her speed increases at a constant rate.

During the last 8 seconds her speed increases at a different constant rate.

Her speed at 14 seconds is 2 m/s more than her speed at 6 seconds.

Here is a sketch of her speed-time graph.

Not drawn accurately



**28 (a)**   Work out her acceleration during the last 8 seconds.

State the units of your answer.

**[2 marks]**

Acceleration = (change in speed)/(change in time). The last 8 seconds is from 6 seconds to 14 seconds. The units of speed and time can be divided to find the unit of acceleration

Answer _____

.CG Maths.

*Do not write outside the box*

**28 (b)**    When Izzy finishes the 80-metre race, her speed is $v$ m/s

Work out the value of $v$.

**[4 marks]**

The distance is equal to the area under the graph, which can be split into a triangle and trapezium. Area of triangle = 1/2 x base x height. Area of trapezium = 1/2 (a + b)h, where a and b are the parallel sides and h is the distance between them. v is the final speed at 14 seconds so the speed at 6 seconds is v - 2.

Express the distance in terms of v, set it equal to 80 then rearrange to make v the subject

Answer _____

**END OF QUESTIONS**

**6**

.CG Maths.