OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

H

Monday 8 November 2021 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space, use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** the questions.

1 This table shows the names and areas of five lakes.

Name of Lake	Area in km^2	
Ladoga	1.81×10^4	18100
Mweru	5.12×10^3	5120
Tana	3.20×10^3	3200
Topozero	9.86×10^2	986
Victoria	6.89×10^4	68900

Converting all of the areas into ordinary form to compare their areas. It is possible to compare them without doing this as they are all in standard form

(a) Write the area of Lake Mweru as an ordinary number.

Typing the standard form into the calculator converts it into ordinary form

(a) ....5120.......... km^2 **[1]**

(b) Write the lakes in the order of their area, starting with the **smallest**.

Topozero, Tana, Mweru, Ladoga, Victoria **[2]**

smallest ... *largest*

(c) Calculate the difference between the areas of Lake Ladoga and Lake Tana.
Give your answer in standard form, correct to **2** significant figures.

$1.81 \times 10^4 - 3.20 \times 10^3 = 14900$

Difference = largest - smallest. The answer of 14900 needs to be divided by 10 4 times to get a decimal between 1 and 10. So 1.49 x 10^4 is the difference in standard form. The second significant figure is the 4. The 9 after this causes the 4 to round up to a 5 then everything after it is set to 0 and ignored

(c) ....1.5×10^4.......... km^2 **[4]**

2 Azmi, Beth and Callum share a flat.

(a) The monthly rent is £760.
They share the rent in the ratio 2 : 3 : 3.

How much does Beth pay for rent each month?

$\frac{760}{2+3+3} \times 3$ ← 2 + 3 + 3 expresses how many parts there are in total in the ratio. This many parts represent the total monthly rent so dividing the £760 by this many parts works out the value of 1 part of the ratio. Multiplying this by the 3 parts representing the rent Beth pays works out how much Beth pays for rent each month

(a) £ ....285.......... **[2]**

(b) Azmi, Beth and Callum also share the fuel bill in the ratio 2 : 3 : 3.
Callum pays £36 for fuel each month.

How much does Azmi pay for fuel each month?

$\frac{36}{3} \times 2$ ← 3 parts of the ratio represent the amount Callum pays for fuel each month. Dividing the £36 by the 3 parts works out what 1 part of the ratio represents. Multiplying the value of 1 part by the 2 parts representing Azmi works out how much Azmi pays

(b) £ ....24.......... **[2]**

3 Multiply out and simplify.

$3(x+2)-(x-1)$

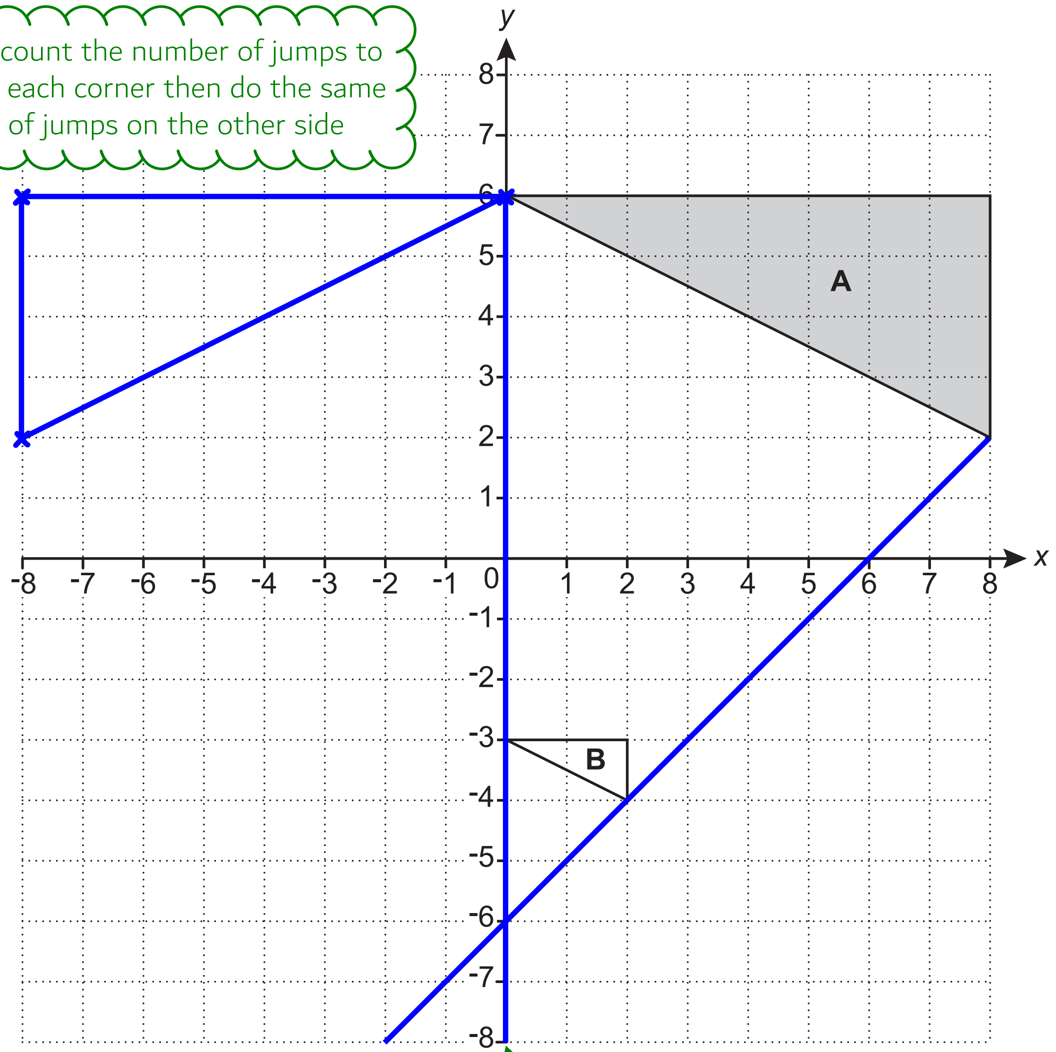
$3x+6-x+1$ ← Expanding the brackets

Collecting like terms
$3x - x = 2x$
$6 + 1 = 7$

$2x+7$ .... **[2]**

4 Triangle **A** and triangle **B** are drawn on the coordinate grid.

To reflect, count the number of jumps to the line for each corner then do the same number of jumps on the other side



(a) Reflect triangle **A** in the line $x = 0$. **[2]**

The y-axis is the line x = 0

(b) Describe fully the **single** transformation that maps triangle **A** onto triangle **B**.

Enlargement, scale factor 1/4, centre (0 , -6)

.... **[3]**

It is an enlargement as it has changed size. The scale factor is 1/4 as the sides on B are 1/4 of the size of the sides on A. The centre of enlargement is found by drawing straight lines through the corners of both shapes and seeing where they meet

The fraction (or proportion, which can be expressed as a decimal) of the times it lands on each number is the relative frequency

5 Ling throws a six-sided dice 300 times.
The table shows the frequencies of their results.

(a) Complete the table to show the relative frequencies.

Number on dice	1	2	3	4	5	6
Frequency	42	27	57	60	39	75
Relative frequency	$\frac{42}{300}$	$\frac{27}{300}$	0.19	$\frac{60}{300}$	$\frac{39}{300}$	$\frac{75}{300}$

[2]

(b) Ling thinks that the dice may be biased.

(i) Explain why evidence from the table could support their opinion.

It didn't land on each number the same number of times

We could expect the frequencies to all be similar if it was not biased

[1]

(ii) Explain why the dice may, in fact, **not** be biased.

Any frequencies are possible as long as there is a chance for landing on each number

For example we could toss a fair coin 5 times in a row and all could be heads. This does not mean it is biased as there is a chance of this happening. Probabilities are only estimates of the relative frequency we should expect and relative frequencies are only estimates of probability

[1]

6 A bag of sweets contains jellies, mints and toffees.

The ratio of jellies to mints is n : 2.
The ratio of mints to toffees is 5 : 3n.

Work out the ratio of jellies to toffees.
Give your answer in its simplest form.

J	M	T
n	2	
	5	$3n$
$5n$	10	$6n$

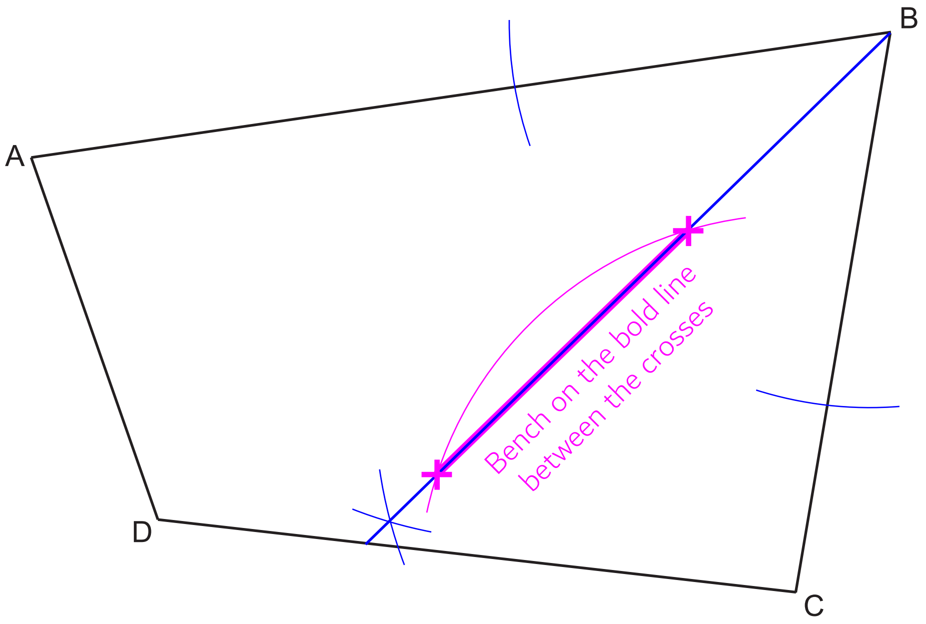
Writing the given ratios in a column. Mints is in common to both ratios. 10 is a common multiple of 2 and 5. Multiplying both sides of the first ratio by 5 gives 10 parts for mints and multiplying both halves of the second ratio by 2 gives 10 parts for mints. The combined ratio is 5n : 10 : 6n

Ignoring the mints leaves the ratio of jellies to toffees as 5n : 6n, which can be simplified by dividing both sides by n

......5...... : ......6...... **[4]**

7 The scale drawing represents a park, ABCD.

Scale: 1 cm represents 10 m



For (a): Construct an angle bisector on angle ABC. Use a compass to scribe two arcs on AB and BC from B which have the same radius. Then scribe an arc from each of the first two arcs with a radius which is greater than half of the distance between the first two arcs. Draw a straight line from B to the cross where the second two arcs meet.

For (b): 50m is represented by 5cm. Scribe an arc with a radius of 5cm which crosses the path twice. Put crosses where the arc and the path meet. The bench can be placed on the path between these two crosses

A straight path goes across the park from B.
The path is always the same distance from side AB and side BC.

(a) Construct the route followed by the path.
Show all your construction lines. Shown in blue **[2]**

(b) A bench is to be placed on the path.
The bench must be no more than 50 m from C. Shown in pink

Construct the locus of the possible positions of the bench.
Indicate clearly on the diagram where the bench can be placed. **[3]**

8 **(a)** Train A travels 120 km at a constant speed of 80 km/h.
Train B travels 120 km at a constant speed of 50 km/h.

How many more minutes does train B take to travel 120 km than train A?

$s\,{}^{d}\,t$

Writing the formula triangle for speed, distance, time

$\frac{120}{50}\times 60 - \frac{120}{80}\times 60$

From the formula triangle, covering over t gives that time = distance/speed. Dividing the 120km by each of the speeds in km/h gives the time in hours as the speed is in terms of kilometres and hours. There are 60 minutes in an hour so multiplying the times in hours by 60 converts them into minutes. Subtracting the time taken for train A in minutes from the time taken for train B in minutes gives how many more minutes train B takes than train A

(a) ....54........................ minutes **[4]**

(b) Train C has a speed of x km/h.

Write an algebraic expression for train C's speed in metres per second.

Multiplying x by 1000 converts it into metres per hour. Dividing by 60 converts it into metres per minute. Dividing by 60 again converts it into metres per second. The expression does not need to be simplified

(b) .... $\frac{x\times 1000}{60\times 60}$ .... m/s **[2]**

9 The width, w, of a kitchen cupboard is 60 cm, correct to the nearest centimetre.

(a) Complete the error interval for the width, w.

$60 \pm \frac{1}{2}$

The resolution is 1cm. Adding and subtracting half of this to the 60cm works out the upper and lower bound

(a) ....59.5........ $\leqslant w <$ ....60.5........ **[2]**

(b) Six of these kitchen cupboards are to be placed side by side along a kitchen wall.
The wall is 363 cm long, correct to the nearest centimetre.

(i) Show that the six cupboards may **not** fit along the wall. **[3]**

$\underline{363-\frac{1}{2}}-\underline{6\times 60.5}=-0.5$

Subtracting the upper bound of the length of 6 of the kitchen cupboards from the lower bound of the length of the wall gives a negative result, meaning that the cupboards could be longer than the wall and therefore may not fit

The resolution of the measurement of the wall is 1cm. Subtracting half of this from the 363cm works out the lower bound of the length of the wall

The upper bound of the length of 6 of the kitchen cupboards

(ii) Find the upper bound of the space remaining if six cupboards do fit along the wall.

$\underline{363+\frac{1}{2}}-\underline{6\times 59.5}$

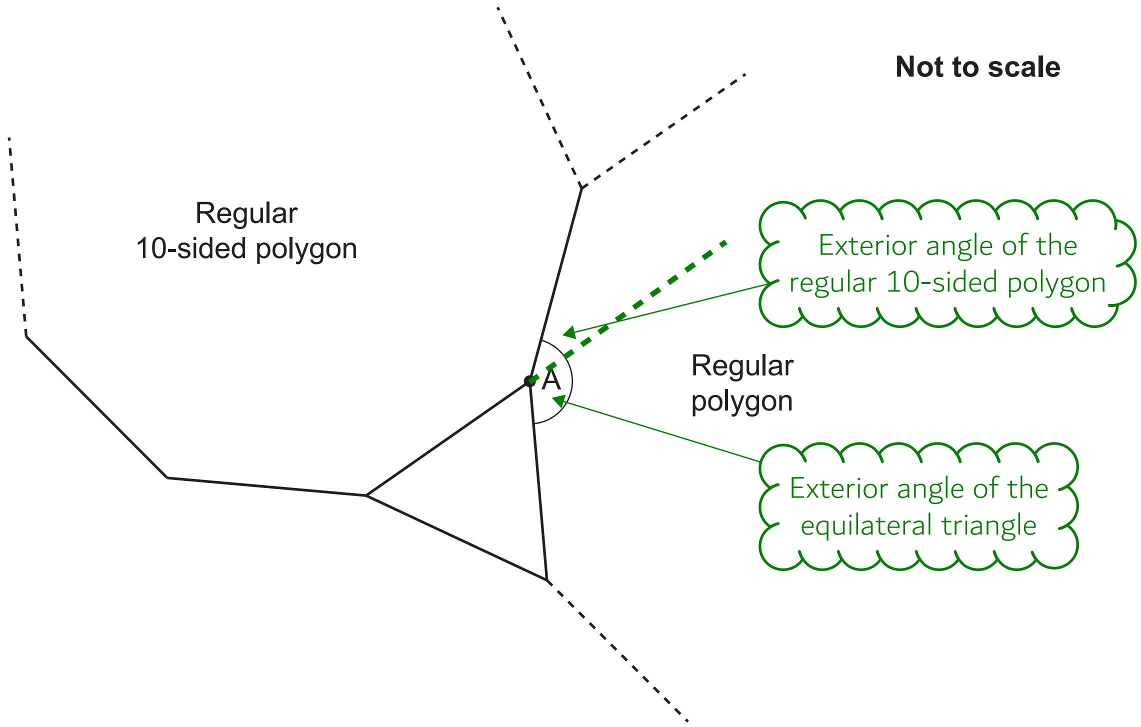
Subtracting the lower bound of the length of 6 of the kitchen cupboards from the upper bound of the length of the wall gives the upper bound of the space remaining

The resolution of the measurement of the wall is 1cm. Adding half of this to the 363cm works out the upper bound of the length of the wall

The lower bound of the length of 6 of the kitchen cupboards

(b)(ii) ....6.5........................ cm **[3]**

10 An equilateral triangle, a regular 10-sided polygon and another regular polygon meet at a point.



(a) Show that angle A is 156°. **[3]**

$\frac{360}{3} + \frac{360}{10} = 156$

The exterior angles of any polygon add up to 360°. So dividing 360° by the number of sides works out each exterior angle. Adding the exterior angle of the regular 10-sided polygon and the exterior angle of the equilateral triangle gives angle A

(b) Work out the number of sides of the other regular polygon.

$\frac{360}{180-156}$

The interior angle and exterior angle lie on a straight line and angles around a point on a straight line add up to 180°. So 180 - 156 expresses the exterior angle of the other regular polygon. The exterior angles of any polygon add up to 360°. So dividing 360 by the exterior angle works out how many exterior angles there are and therefore how many side it has

(b) ....15.................. **[2]**

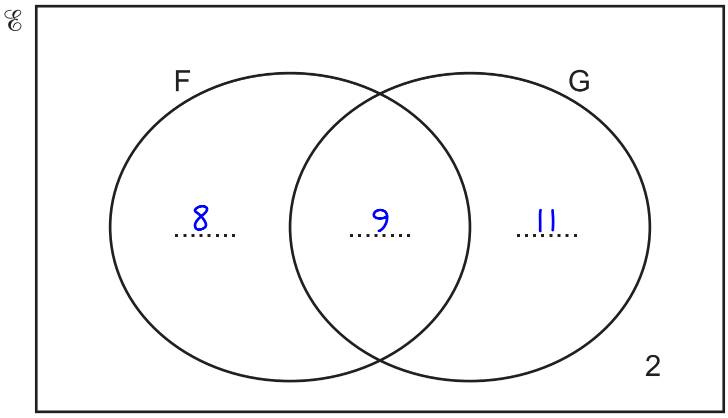
11 In a class of 30 students

- 17 study French (F)
- 20 study German (G)
- 2 do not study either subject.

$30-2=28$
$17+20=37$
$37-28=9$
$17-9=8$
$20-9=11$

11 Subtracting the 2 on the outside from the 30 students works out that there must be 28 in total inside the rings. 17 + 20 works out that there would be 37 in total in the rings if there was nothing in the centre. Working out the difference between the 37 and the 28 works out how that 9 must go in the centre, as this would take 9 off the total. Subtracting the 9 from the 17 and 20 works out how many goes in just the F and G rings

(a) Complete the Venn diagram.



[3]

(b) Two of the 30 students are chosen at random.

Calculate the probability that one of these two students studies French but not German and the other studies German but not French.
You must show your working.

$$\frac{8}{30}\times\frac{11}{29}+\frac{11}{30}\times\frac{8}{29}$$

French but not German AND German but not French OR German but not French AND French but not German. AND means to multiply the probabilities, OR means to add the probabilities. The number of students in total decreases by 1 for the second pick

(b) .... $\frac{88}{435}$ .... **[5]**

12 A solid metal sphere has mass 235 g.
The density of the metal is 7.78 g/cm^3.

Show that the surface area of this sphere is 46.9 cm^2, correct to **3** significant figures.
You must show your working.

[For a sphere with radius r: Volume $= \frac{4}{3}\pi r^3$ Surface area $= 4\pi r^2$.] **[6]**

$d \; M \; V$

Writing the formula triangle for density, mass, volume

$\frac{4}{3}\pi r^3 = \frac{235}{7.78}$

From the formula triangle, covering v gives volume = mass/density. So the volume of the sphere is 235/7.78. Setting the formula of the volume of the sphere equal to this

$r = \sqrt[3]{\dfrac{\left(\frac{235}{7.78}\right)}{\frac{4}{3}\pi}}$

Rearranged to find r, the radius, by dividing both sides by 4/3 and π then cube rooting both sides

$= 1.9...$

Storing the exact value of 1.931967807 as A on the calculator

$4\pi \times 1.9...^2 = 46.90$

$= 46.9$ (3sf)

Using the formula for the surface area of the sphere and substituting the exact value of the radius for r. Quoting the surface area truncated to 4 significant figures to show that it rounds to 46.9 to 3 significant figures

13 A straight line passes through the point (8, 1) and is perpendicular to the line $y = 4x - 2$.

Find the equation of the line, giving your answer in the form $y = mx + c$.

The equation is in the form y = mx + c, where m is the gradient and c is the y-intercept

$m = -\frac{1}{4}$ ← The gradient of the line y = 4x - 2 is 4. The gradient of the perpendicular line must be the negative reciprocal of this so is -1/4

$c = y - mx$ ← Rearranged y = mx + c to make c the subject by subtracting mx from both sides

$= 1 - \left(-\frac{1}{4}\right) \times 8$ ← Substituted 1 for y, -1/4 for m and 8 for x

$= 3$ ← c = 3

Substituted -1/4 for m and 3 for c in the general equation of the straight line

$y = -\frac{1}{4}x + 3$ .... **[4]**

14 y is inversely proportional to the square root of x.
$y = 5$ when $x = 36$.

(a) Find a formula linking x and y.

$y \propto \frac{1}{\sqrt{x}}$ ← Writing out the proportion. Inversely means '1 over'

$y = \frac{k}{\sqrt{x}}$ ← Converting the proportion into an equation by multiplying the right side by k, which represents an unknown constant

$k = y\sqrt{x}$ ← Rearranging to find k by multiplying both sides by $\sqrt{x}$

$= 5\sqrt{36}$ ← Substituting in the given values of x and y

$= 30$

Substituting 30 for k in the equation

(a) $y = \frac{30}{\sqrt{x}}$ **[3]**

(b) Find the value of x when $y = 20$.

$y\sqrt{x} = 30$ ← Rearranging to make x the subject. First multiplying both sides by $\sqrt{x}$ to eliminate it as a denominator

$\sqrt{x} = \frac{30}{y}$ ← Dividing both sides by y

$x = \left(\frac{30}{y}\right)^2$ ← Squaring both sides

$= \left(\frac{30}{20}\right)^2$ ← Substituting 20 for y

(b) $x =$ 2.25 **[3]**

15 (a) Show that the equation $x^3 - 5x - 1 = 0$ has a solution between $x = 2$ and $x = 3$. **[3]**

$2^3 - 5(2) - 1 = -3$

$3^3 - 5(3) - 1 = 11$

Substituting both 2 and 3 into the equation

There is a change in sign, therefore the solution is between 2 and 3

As it is a continuous function and one of the results is below 0 and the other is above 0, it must give 0 somewhere between 2 and 3

(b) Find this solution correct to **1** decimal place.
You must show your working.

$F(x) = x^3 - 5x - 1$

Using table mode, setting f(x) = x^3 - 5x - 1

$F(2.3) = -0.333$

$F(2.4) = 0.824$

Start: 2. End: 3. Step: 0.1

This gives all of the values of f(x) for x from 2 to 3 to 1 decimal place. There is a change in sign from x = 2.3 to x = 2.4 so the solution must be between these

$F(2.33) = -6.63 \times 10^{-4}$

$F(2.34) = 0.112904$

Start: 2.3. End: 2.4. Step: 0.01

This gives all of the values of f(x) for x from 2.3 to 2.4 to 2 decimal places. There is a change in sign from x = 2.33 to x = 2.34 so the solution must be between these

A number between 3.33 and 3.34 will round to 3.3 to 1 decimal place

(b) $x =$ ....2.3........................ **[4]**

.CG Maths.

16 The following kinematics formulas may be used in this question.

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

The initial velocity of a particle is 20 m/s.
The acceleration of the particle is -8 m/s^2.
After t seconds, the particle has travelled 25 m.

(a) Show that $4t^2 - 20t + 25 = 0$. **[3]**

The second formula can be used as the distance (s) is given, the initial velocity (u) is given and the acceleration (a) is given and we are looking for an equation in terms of t

$25 = 20t + \frac{1}{2}(-8)t^2$ ← Substituting 25 for s, 20 for u and -8 for a

$-4t^2 + 20t - 25 = 0$ ← Simplifying and subtracting 25 from both sides

$4t^2 - 20t + 25 = 0$ ← Dividing all terms on both sides by -1 to flip the signs

(b) Solve $4t^2 - 20t + 25 = 0$.

$t = \frac{--20 \pm \sqrt{(-20)^2 - 4 \times 4 \times 25}}{2 \times 4}$ ← Solving using the quadratic formula

Both solutions are 2.5

(b) $t =$ ......2.5...... **[3]**

(c) Show that the particle is stationary when it has travelled 25 m.

The third formula can be used as the initial velocity (u) is given, acceleration (a) is given, distance (s) is given and we are looking for the final velocity (v)

$V = \sqrt{u^2 + 2as}$ Rearranged to make v the subject by square rooting both sides

$= \sqrt{20^2 + 2 \times -8 \times 25}$ Substituting 20 for u, -8 for a and 25 for s

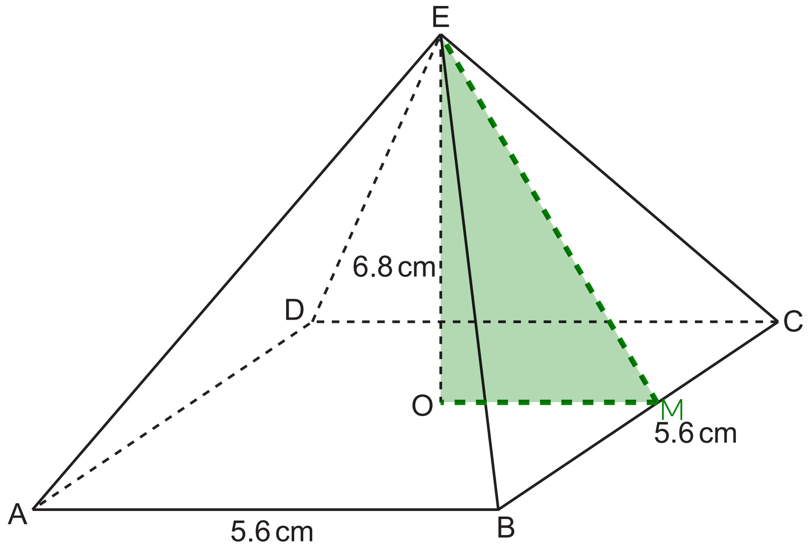
$= 0$

Therefore it is stationary as its velocity is 0m/s

....

.... **[3]**

17 The diagram shows a pyramid ABCDE.



Not to scale

The pyramid has a square horizontal base ABCD with side 5.6 cm.

The vertex E is vertically above the centre O of the base.
The height OE of the pyramid is 6.8 cm.

Calculate the surface area of the pyramid.
You must show your working.

$a^2+b^2=c^2$

Pythagoras' Theorem can be used to work out the height of each triangle, EM, as it is the missing side in the green right-angled triangle

$c=\sqrt{a^2+b^2}$

c is the longest side so represents EM. Rearranging to make c the subject by square rooting both sides

a and b are the two shorter sides. So substituting in 6.8 for a and side OM for b expresses the height of each triangle, EM. Side OM is half of AB, which is 5.6

$\frac{1}{2}\times5.6\times\sqrt{6.8^2+\left(\frac{5.6}{2}\right)^2}\times4+5.6\times5.6$

Adding the area of the 4 triangles and the square base gives the surface area

Area of triangle = 1/2 x base x height. The base of each triangle is 5.6 and the height is side EM. Multiplying the area of one of the triangles by 4 as there are 4 of them

Area of square = length x width. Both the length and width are 5.6

............113.7............ cm^2 **[5]**

18 Rearrange this formula to make y the subject.

$$\frac{5y+2}{y} = \frac{3t-7}{2}$$

$10y + 4 = 3ty - 7y$

Multiplying both sides by the denominators eliminates them

$17y - 3ty = -4$

Adding 7y, subtracting 3ty and subtracting 4 from both sides to get all the y terms on the same side and all the others on the other side

$y(17-3t) = -4$

Bringing y out as a factor on the left

Dividing both sides by (187 - 3t) makes y the subject

$y = \dfrac{-4}{17-3t}$ .... **[5]**

END OF QUESTION PAPER