OCR
Oxford Cambridge and RSA
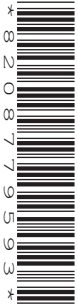
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Hints

H

Tuesday 2 November 2021 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number [ ][ ][ ][ ][ ]   Candidate number [ ][ ][ ][ ]

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start to write your answer.

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Turn over

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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Answer **all** the questions.

1 Calculate.

(a) $(6^2 + 5)^3$

(a) .... **[1]**

(b) $\sqrt{\dfrac{8.4^2 - 1.9^2}{2.5 + 5.7}}$

Write your answer correct to **3** significant figures.

Use the figure after the third significant figure to round the third significant figure. Then set everything after the third significant figure to 0 and ignore them. Significant figures are figures after the first non-zero

(b) .... **[3]**

2 Calculate the area of this trapezium.

8.7 cm

Not to scale

6.8 cm

12.3 cm
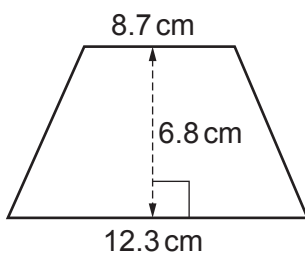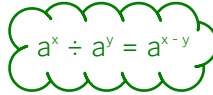
Area of trapezium = 1/2 x (a + b) x h, where a and b are the parallel sides and h is the perpendicular distance between them

.... cm^2 **[2]**

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3 Simplify.

$x^{12} \div x^4$

$a^x \div a^y = a^{x-y}$

.... **[1]**
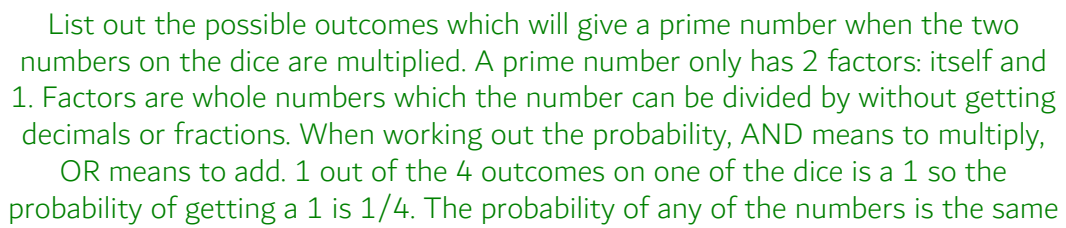
4 Li throws two fair four-sided dice, each numbered 1, 2, 3 and 4.
Li multiplies together the two numbers that the dice land on to produce a score.

Find the probability that Li's score is a prime number.

List out the possible outcomes which will give a prime number when the two numbers on the dice are multiplied. A prime number only has 2 factors: itself and 1. Factors are whole numbers which the number can be divided by without getting decimals or fractions. When working out the probability, AND means to multiply, OR means to add. 1 out of the 4 outcomes on one of the dice is a 1 so the probability of getting a 1 is 1/4. The probability of any of the numbers is the same

.... **[4]**

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5 **(a)** Fountain A squirts water every 24 minutes.
Fountain B squirts water every 42 minutes.
They squirt water together at 15:19.

Find the next time they squirt water together.

> Express 24 and 42 as a product of prime factors. Work out the lowest common multiple by multiplying the highest power of each prime factor of both numbers. This works out after how many minutes they squirt water together next. Adding the time taken for them both to squirt water together to the time they squirt water together works out the time they next squirt water together

> The calculator can be used to express numbers as a product of primes. Newer models of the Casio calculator can calculate the lowest common multiple of two numbers. Time can be put into the calculator in the form hh°mm°ss°, where hh is the hours, mm is the minutes and ss is the seconds

(a) .... **[4]**

(b) A school sends 60 students from Year 8 and 105 students from Year 9 to a museum.

The school divides these students into groups using the following rules.

- The groups must all be the same size.
- All students in any group must be from the same year.
- There should be as few groups as possible.

Find the size of each group and the total number of groups.

> Express 60 and 105 as a product of prime factors. Work out the highest common factor by multiplying the lowest power of each prime factor of both numbers. This works out the greatest number of students in a group (which leads to the fewest number of groups). Dividing the number of students by the number of students in each group works out the number of groups

> The calculator can be used to express numbers as a product of primes. Newer models of the Casio calculator can calculate the highest common factor of two numbers

Size of each group = ...

Total number of groups = ... **[4]**

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6 A shop sells the same milk in three different sized cartons.
The diagram shows the price of each carton.

300 ml for 31p

500 ml for 47p

2 litres for £1.96
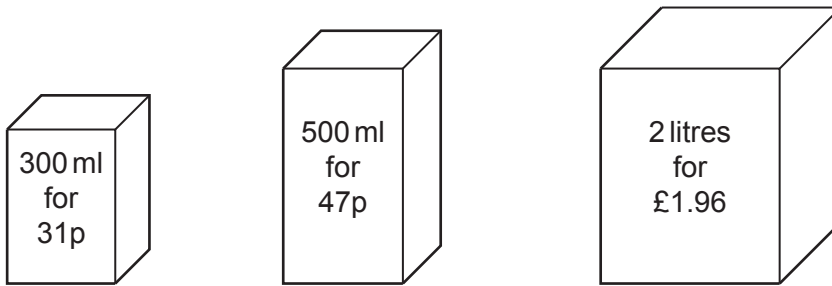
(a) Which carton is the best value for money?
Show how you decide.

> Dividing the number of millilitres by the cost in pence works out how many millilitres each carton is per penny. The carton which is the best value for money is the one with the most millilitres per penny

....

....

.... **[3]**

(b) A student only buys milk on a Saturday morning.
They use 120 ml of milk each day.
Any unused milk has to be thrown away at the end of the following Friday.
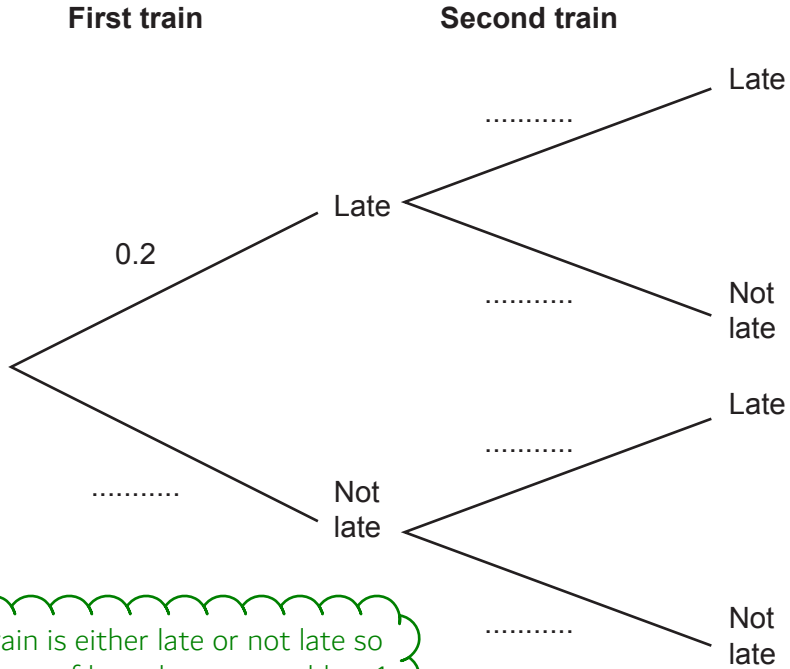
Show that it is cheaper for the student to buy the milk they need in 300 ml cartons than in 500 ml cartons. **[3]**

> Count how many days it is from Saturday morning to the end of the following Friday. Multiply this by the 120ml to work out how much milk is used. Dividing this by the 300ml cartons works out how many 300ml cartons are needed. Round the amount up to a whole number then multiply this number by the cost of each 300ml carton to work out the cost of the milk needed. Do a similar method for the 500ml cartons. The cost of the 500ml cartons should be more than the 300ml cartons

Turn over

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7 **(a)** Over a long period of time, it is found that the probability of a train from Bewford to London being late is 0.2.

(i) One morning there are two trains from Bewford to London.

Use the information to complete the tree diagram.

First train   **Second train**

```
                                              Late
                                  ...........
                          Late <
                  0.2             ...........
                                              Not
                                              late

                                              Late
                                  ...........
       ...........       Not <
                          late    ...........
                                              Not
                                              late
```

It is certain that each train is either late or not late so the probabilities of each set of branches must add to 1

[2]

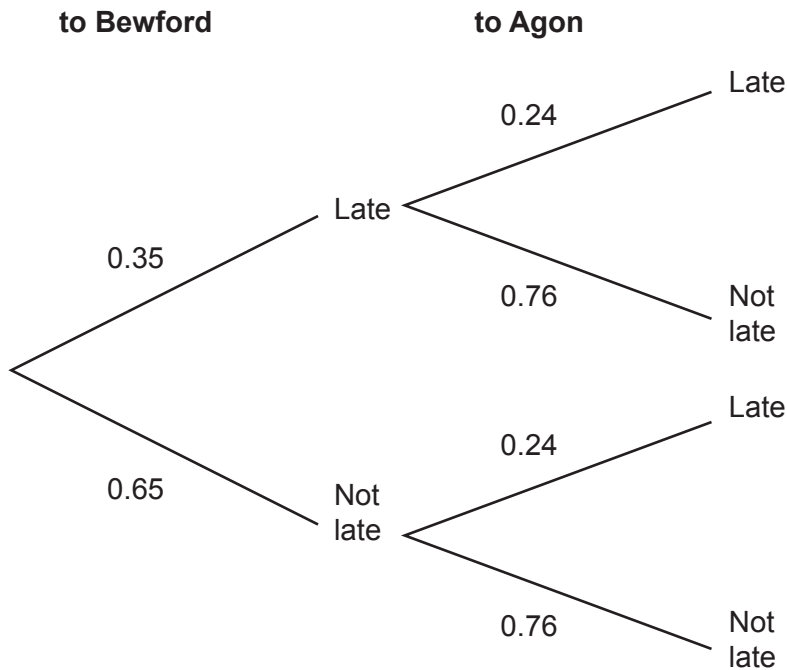**(ii)** Work out the probability that both trains are **not late**.

Not late AND not late. Assuming the two events are independent, AND means to multiply the probabilities

(a)(ii) .... **[2]**

(iii) Give a reason why the probabilities used in the tree diagram for the second train may **not** be reliable.

.... *Are the events definitely independent (meaning that the first train has no effect on the second train)?* ....

.... **[1]**

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(b) Morgan takes a train from London to Bewford and then another train to Agon.
The tree diagram shows the probabilities of Morgan's trains being late or not late.

to Bewford   **to Agon**

```
                                              Late
                                    0.24
                          Late
              0.35
                                    0.76     Not
                                             late

                                              Late
                                    0.24
              0.65        Not
                          late
                                    0.76     Not
                                             late
```

Morgan will **not catch** the train to Agon if the train to Bewford is late and the train to Agon is not late.

Work out the probability that Morgan will **catch** the train to Agon.

> It is certain that Morgan will either catch the train to Agon or not catch the train to Agon. Therefore the probabilities must add to 1 and subtracting the probability of not catching the train to Agon from 1 leaves the probability of catching the train to Agon. The probability of not catching the train to Agon is the probability of the train to Bewford being late AND the train to Agon not being late. AND means to multiply the probabilities

(b) .... **[3]**

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8 Jamie invests £6000 at a simple interest rate of $r\%$ each year.
After 6 years the value of their investment is £7170.

Find the value of r.

> £7170 - £6000 works out how much interest was gained. Putting this over the original £6000 expresses the interest as a fraction. Multiplying this fraction by 100 converts it into a percentage. As it is simple interest, the interest gained is the same each year so the percentage can be divided by 6 to give the interest rate each year

$r =$ .... **[4]**

9 The price of a plane ticket is increased by 15% to £1426.

Find the original price of the plane ticket.

> Reducing the £1426 by 15% does not work as the 15% is of the original price, not of the £1426. Let 100% be the original price. 100% + 15% expresses the percentage of the original price the ticket has increased to. Dividing the £1426 by this works out 1% of the original price. Multiplying this by 100 works out 100%, which is the original price

£ .... **[3]**

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10 Alex, Blake and Charlie play a computer game.

Alex goes first and scores *n* points.

- Blake scores 8 points less than 3 times the number of points scored by Alex.
- Charlie scores 25 more points than Blake.
- The three people score a total of 618 points.

Work out how many points they each score.
You must show your working.

> Express each person's score in terms of n. Adding the expressions of the number of points each person scores gives the total number of points scored. Simplify the expression of the total number of points scored by collecting like terms. This must be equal to the 618 points. Rearrange to find n, which is Alex's score. Then Blake's score can be worked out. Then Charlie's score can be worked out

Alex = ...

Blake = ...

Charlie = ... **[7]**

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11 The diagram shows triangles ABC and ADE.
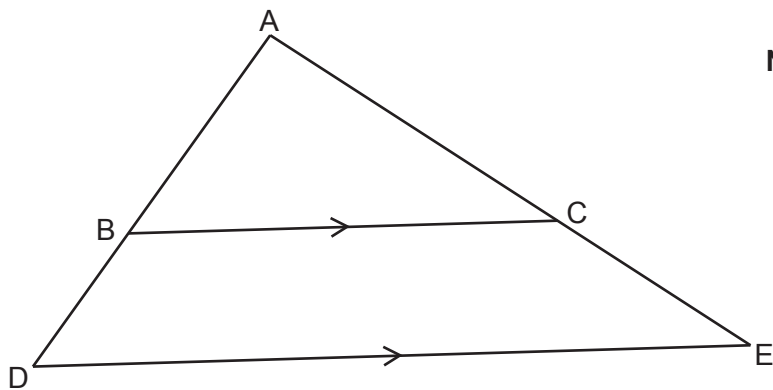
Not to scale

B lies on AD and C lies on AE.
BC is parallel to DE.

Complete these statements to show that triangles ABC and ADE are similar.

Angle ABC = angle ADE because they are corresponding angles.

Angle ACB = angle .... because .....they are corresponding angles...

Angle BAC is ....

Triangles ABC and ADE are similar because .....they have the same angles.......................................

....

[3]

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12 (a) The region **R** is shown on this grid.
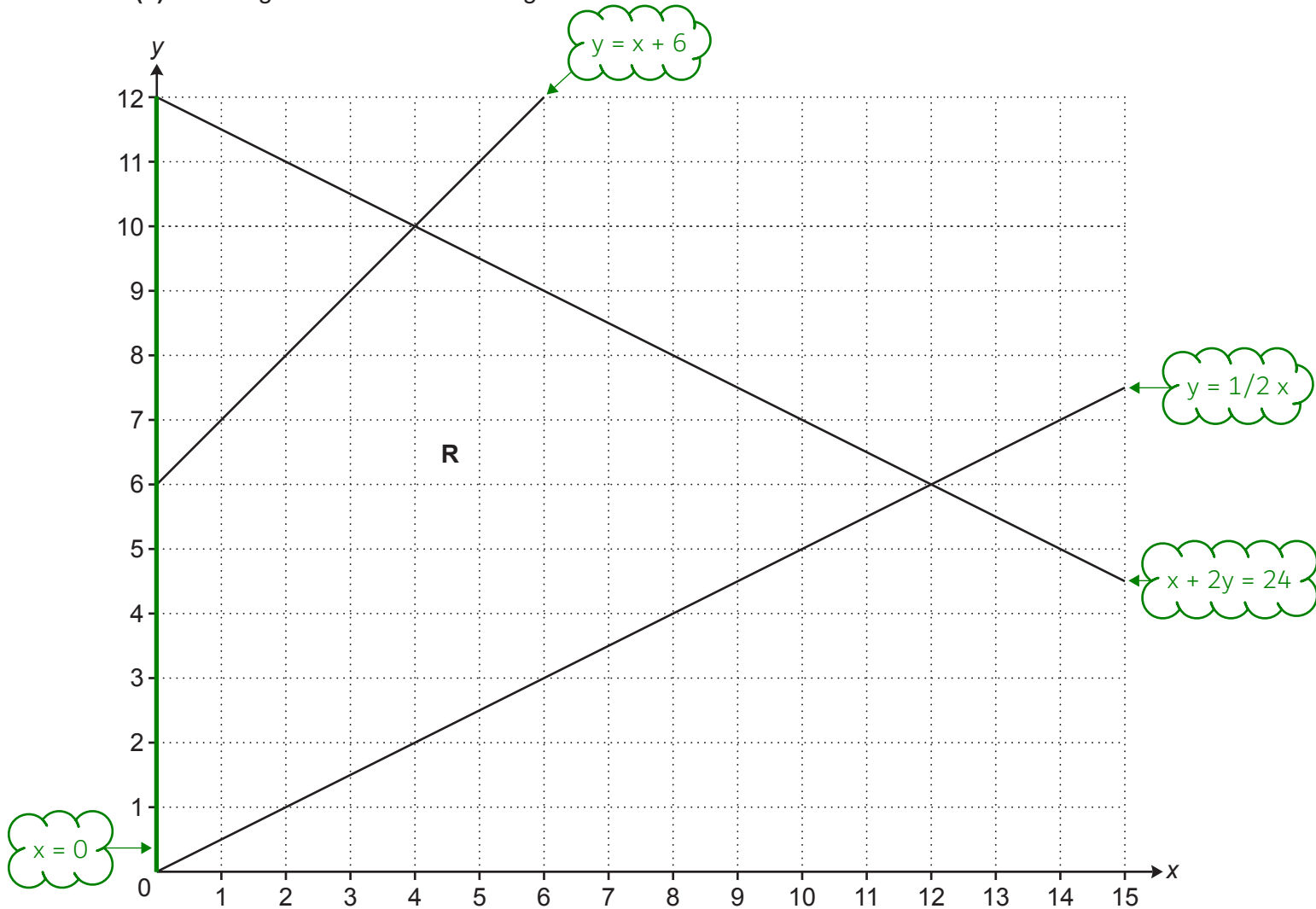
Region **R** is defined by four inequalities.
One of the inequalities is $x \geqslant 0$.

Use the symbols $\leqslant$ and $\geqslant$ to complete the other three inequalities.

$$x \geqslant 0$$

$$y \;\text{...............}\; \frac{1}{2}x$$

$$x + 2y \;\text{...............}\; 24$$

$$y \;\text{...............}\; x + 6$$ **[2]**

(b) The inequality $x \geqslant 0$ is replaced by a new inequality.
Region **R** is then a kite.

Write down the new inequality.

Kites are four-sided shapes and have a line of symmetry. Draw on a line which can create this shape. Work out the equation of the straight line. The general equation of a straight line is y = mx + c, where m is the gradient and c is the y-intercept. Gradient = (change in y)/(change in x). Then convert the equation into an inequality which describes the region

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13 The height, h cm, of each member of a tennis club is recorded.
The histogram shows some of the results.

Height (h cm)
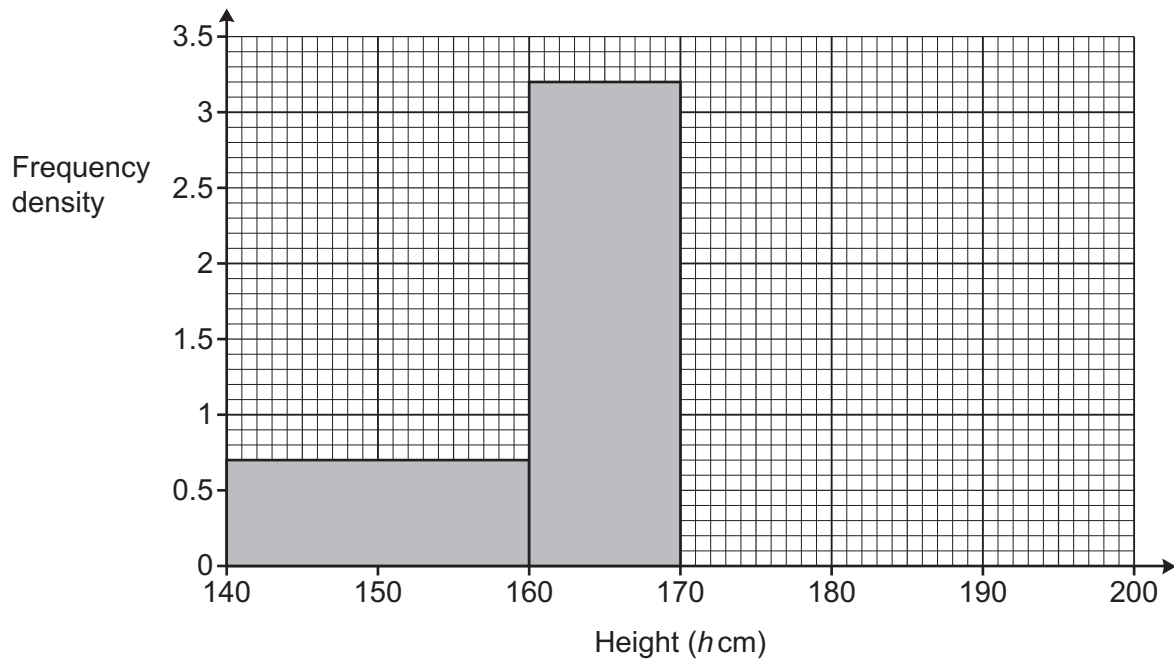
40% of the members have a height in the interval $160 \leqslant h < 170$.
30% of the members have a height in the interval $170 \leqslant h < 180$.
100% of the members have a height in the interval $140 \leqslant h < 200$.

Complete the histogram for the intervals $170 \leqslant h < 180$ and $180 \leqslant h < 200$. **[6]**

$c^F d$ ← Writing the formula triangle for class width, frequency, frequency density. C stands for class width, F stands for frequency and d stands for frequency density

Class width is how wide the bar is on the histogram. It is found by subtracting the lower bound from the upper bound of each bar. Work out the frequency of the second bar, which is 40% of the members. Then work out the total number of members by finding 1% and multiplying this by 100. Working out 30% gives the frequency for the third bar and this can be used to work out it's frequency density. Work out the frequency of the first bar. Subtracting all of the other frequencies from the total number of members gives the frequency for the fourth bar, which can be used to work out it's frequency density

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14 Find the coordinates of the turning point of the graph of $y = x^2 + 6x + 17$.

Complete the square. $ax^2 + bx + c = a(x + b/2a)^2 + c - a(b/2a)^2$. Once in the completed the square form, the turning point can be worked out by considering that it occurs when the square bracket is equal to 0 as the smallest a squared value can be is 0. Work out what x must be for this to happen and we then have the x-coordinate. The y-coordinate can also be worked out when the bracket is equal to 0

(........................ , ...) **[4]**

15 Here are the first four terms of a quadratic sequence.

-1   3   13   29

The nth term is $an^2 + bn + c$.

Find the values of a, b and c.

Work out the differences then the second difference. a is half of the second difference. List out the sequence of an^2 then work out what must be added to each term to get the original sequence. This will form a linear sequence which is in the form bn + c. b is the amount the sequence changes by between each term and c is the 0th term. Adding the an^2 sequence and the bn + c sequence gives the original sequence in the form an^2 + bn + c

$a = $ ....

$b = $ ....

$c = $ .... **[4]**

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16 The formula

$$P = 6800 \times 1.045^n$$

is used to predict the population, *P*, of an island *n* years after 2018.

(a) Write down the population of the island in 2018.

2018 is 0 years after 2018 so n is 0. Anything to the power of 0 is 1

(a) .... **[1]**

(b) Write down the percentage growth rate used in the formula.

The formula is basically a compound interest formula. To convert the decimal multiplier into a percentage it should be multiplied by 100

(b) .... % **[1]**

(c) (i) Work out the population predicted by the formula for the year 2030.

2030 - 2018 expresses the difference between the two years and therefore how many years after 2018 the 2030 is, which is the value of n. Substitute this into the formula. The population should be a whole number

(c)(i) .... **[2]**

(ii) Give **one** reason why the answer to **(c)(i)** may **not** be reliable.

.... Consider what would happen if the population ....

.... continued at the same rate over a long period of time .... **[1]**

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17 The diagram shows triangle ABC.

A

53°

48 mm

Not to scale

B

C

85 mm
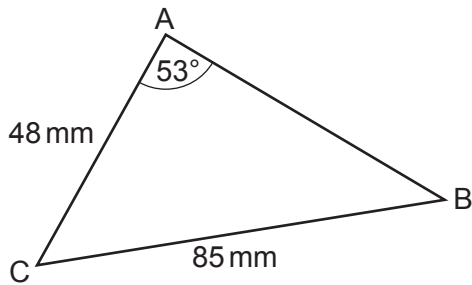
We do not know if it is a right angled triangle so non-right angled trigonometry will be needed. There is not enough information in the triangle to work out AB using either the sine rule or cosine rule straight away. So we need to work out another angle first

AC = 48 mm, BC = 85 mm and angle BAC = 53°.

Calculate length AB.
You must show your working.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

There are opposite pairs of sides and angles so the sine rule can be used to work out angle B. Quoting the sine rule with the angles as numerators

a is opposite A, b is opposite B and c is opposite C. To get rid of sin, do the inverse sin ($\sin^{-1}$) of both sides. There are 180° in total in a triangle so C can be found once B is found

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

Once C is found, there are opposite pairs of sides and angles so the sine rule can be used to work out side c. Quoting the sine rule with the sides as numerators

.... mm **[6]**

Turn over

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18 (a) For each graph below, select its possible equation from this list.

$y = x$ $y = x^2$ $y = \dfrac{1}{x}$

$y = \sin x$ $y = \cos x$ $y = \tan x$

$y = 3^x$ $y = \left(\dfrac{1}{3}\right)^x$

(i)

(a)(i) $y =$ .... **[1]**

(ii)
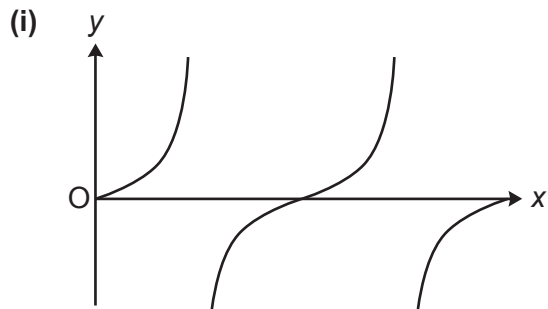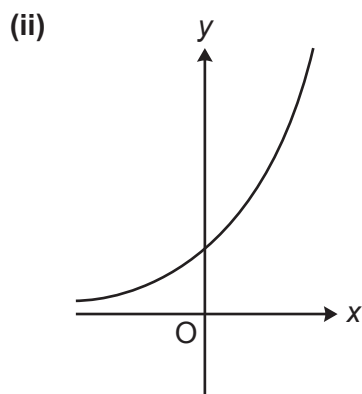
(ii) $y =$ .... **[1]**

These are both typical graphs so we could just memorise what they look like

Alternatively, table mode can be used on the calculator to give a table of values of each equation and we can roughly imagine if each one would look like these graphs

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(b) Here is a sketch of $y = x^3$.
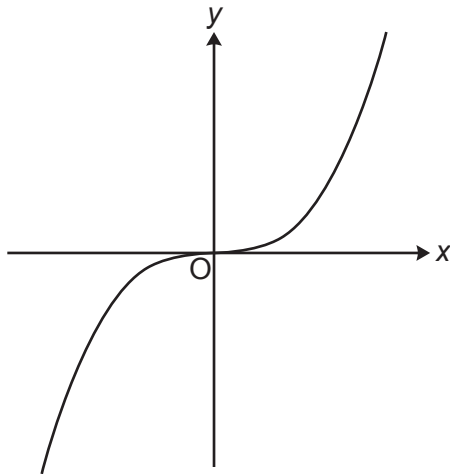
On the axes below, sketch the graphs of

(i) $y = -x^3$
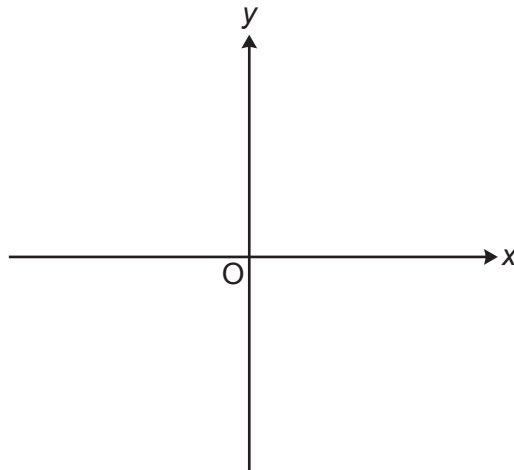
The graph reflects in the x-axis

[1]

(ii) $y = x^3 - 8$, showing the values of any intercepts with the axes.
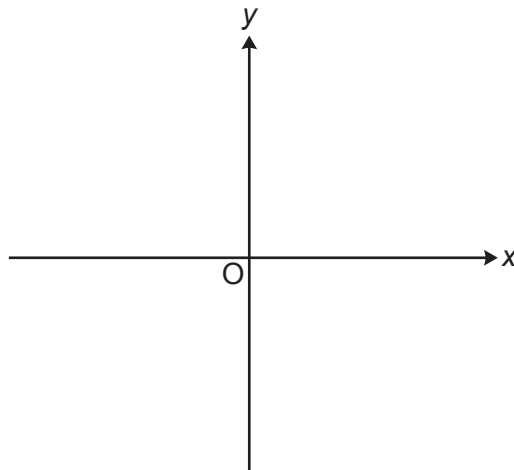
The x-intercept can be worked out by setting y equal to 0 then rearranging to find x. The y-intercept can be found by setting x equal to 0 and finding y

The graph translates 8 down

[3]

Turn over

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19 ABC is an isosceles triangle.
The sides of the triangle ABC are all tangents to a circle of radius 6 cm, centre O.

B

Drawing on length BO makes a right angled triangle

Not to scale

6 cm
O

70°
A   C
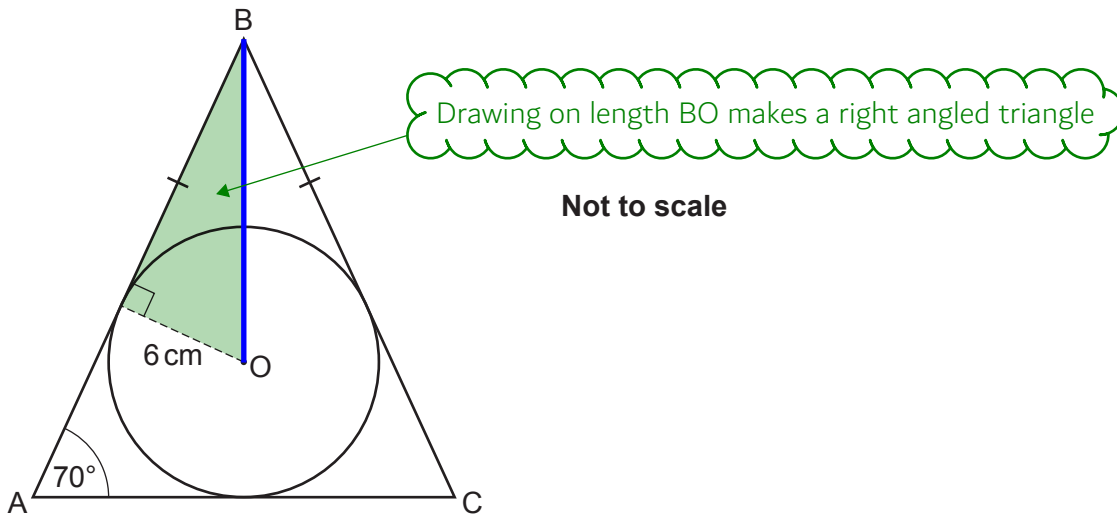
Angle BAC = 70° and BA = BC.

(a) Show that length BO is 17.54 cm, correct to **2** decimal places. **[4]**

The base angles of an isosceles triangle are the same. There are 180° in total in a triangle. Work out angle ABC. Isosceles triangles are symmetrical so both halves of the angle must be the same. Work out angle ABO.

S^OH C^AH T^OA

Right angled trigonometry can be used so quoting SOH CAH TOA as formula triangles. Tick what we have and what we are looking for. Two ticks on one of the formula triangle tells us that we can use that one

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(b) Find the area of triangle ABC.
You must show your working.
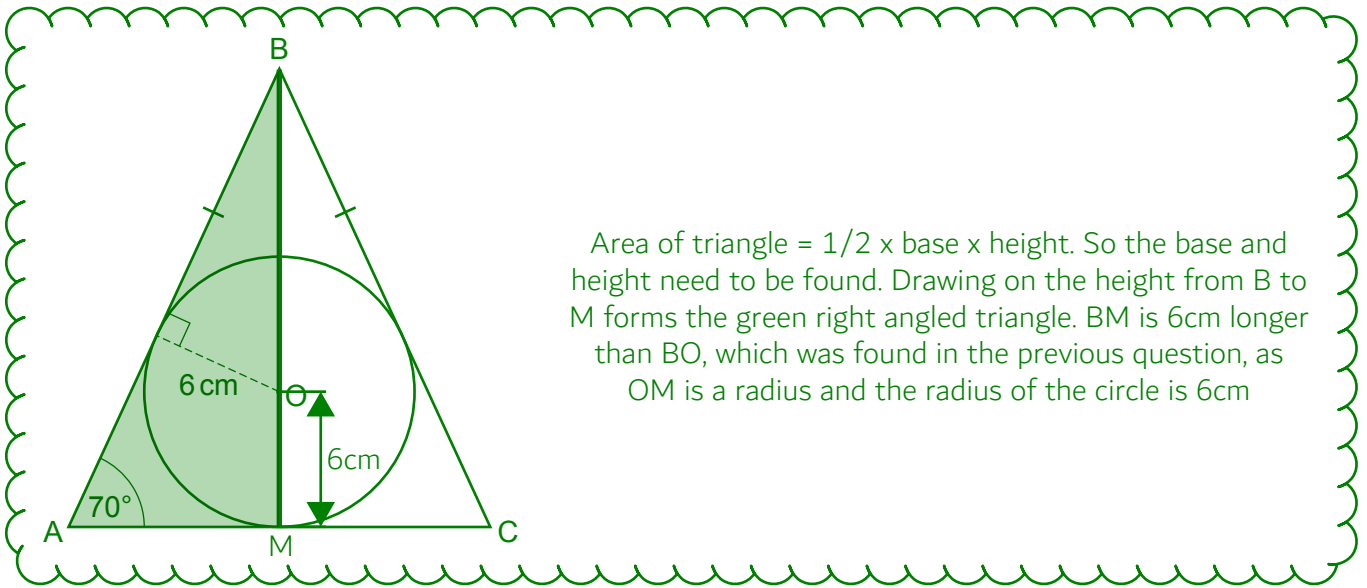
Area of triangle = 1/2 x base x height. So the base and height need to be found. Drawing on the height from B to M forms the green right angled triangle. BM is 6cm longer than BO, which was found in the previous question, as OM is a radius and the radius of the circle is 6cm

6 cm

6cm

70°

A   M   C

B

O

$S^O H C^A H T^O A$

Once side BM is worked out, right angled trigonometry can be used to work out side AM, which is half of the base

.... cm^2 **[5]**

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20 Solve algebraically.

$$y = x + 3$$
$$(x - 3)^2 + y^2 = 50$$

You must show your working.

> Substitute (x + 3) for y in the left side of the second equation to get an equation just in terms of x which can be solved. Expand each square bracket by squaring the first term, doubling the product of both terms, and squaring the last term. Simplify by collecting like terms. The x terms should cancel out to leave just x^2 terms and constants, so the equation should be able to be solved by rearranging. Remember that when square rooting there is also a negative value. Once both values of x have been found, they can be substituted into the first equation to find the y values

$x =$ .... $y =$ ....

$x =$ .... $y =$ .... **[5]**

END OF QUESTION PAPER

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