AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

GCSE
MATHEMATICS

H

Higher Tier Paper 2 Calculator

Thursday 7 November 2019 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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Answer **all** questions in the spaces provided

1 Expand $4x^2(3x + 5)$

Circle your answer.

[1 mark]

$32x^3$ [✓ $12x^3 + 20x^2$] $7x^3 + 9x^2$ $12x^2 + 5$

$4x^2 \times 3x = 4 \times 3 \times x^2 \times x = 12x^3$
$4x^2 \times 5 = 4 \times 5 \times x^2 = 20x^2$

2 How many millimetres are there in a kilometre?

Circle your answer.

[1 mark]

10^3 10^5 [✓ 10^6] 10^9

1 km = 1000 m = 100000 cm = 1000000 mm = 10^6 mm

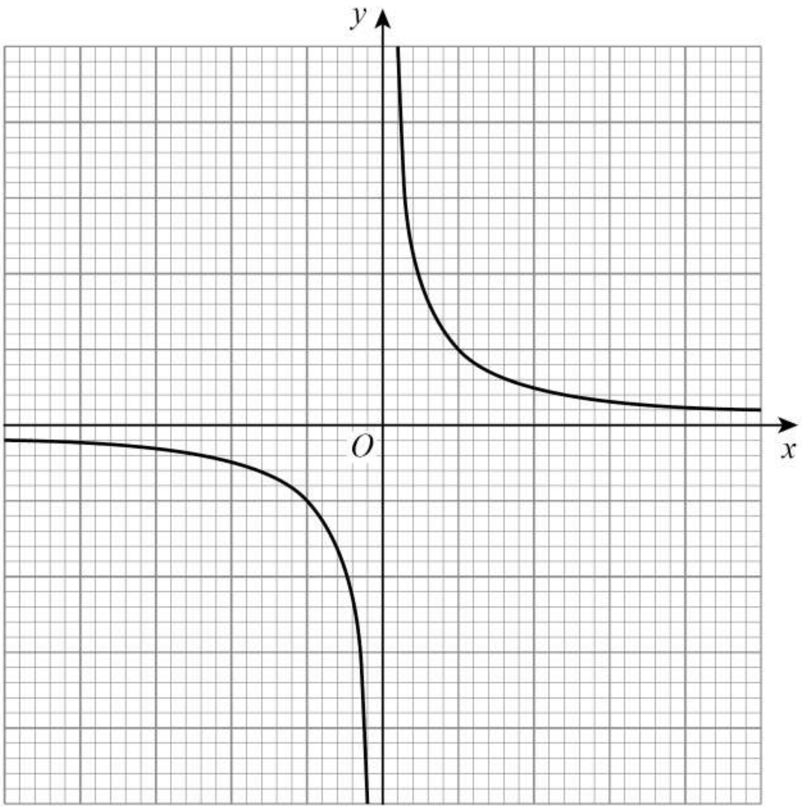
3 Circle the number half way between $\frac{7}{12}$ and $\frac{3}{4}$

[1 mark]

$\frac{7}{32}$ $\frac{5}{8}$ [✓ $\frac{2}{3}$] $\frac{1}{2}$

Doing the mean of the two fractions works out what is half way.
7/12 + 3/4 = 4/3
4/3 ÷ 2 = 2/3

4 Here is the sketch of a graph.



Circle the equation of the graph.

[1 mark]

$y = x$ $y = -x^2$ $y = -x^3$ [✓ $y = \frac{1}{x}$]

This is a typical graph so is one which might be known. Otherwise table mode could be used on the calculator to do a table of values for each equation. y = 1/x is the only one which would have this shape

5 Work out the lowest common multiple (LCM) of 120 and 144

[2 marks]

$120 = 2^3 \times 3 \times 5$

$144 = 2^4 \times 3^2$

Expressing both 120 and 144 as a product of prime factors. The calculator can be used to do this

$2^4 \times 3^2 \times 5$

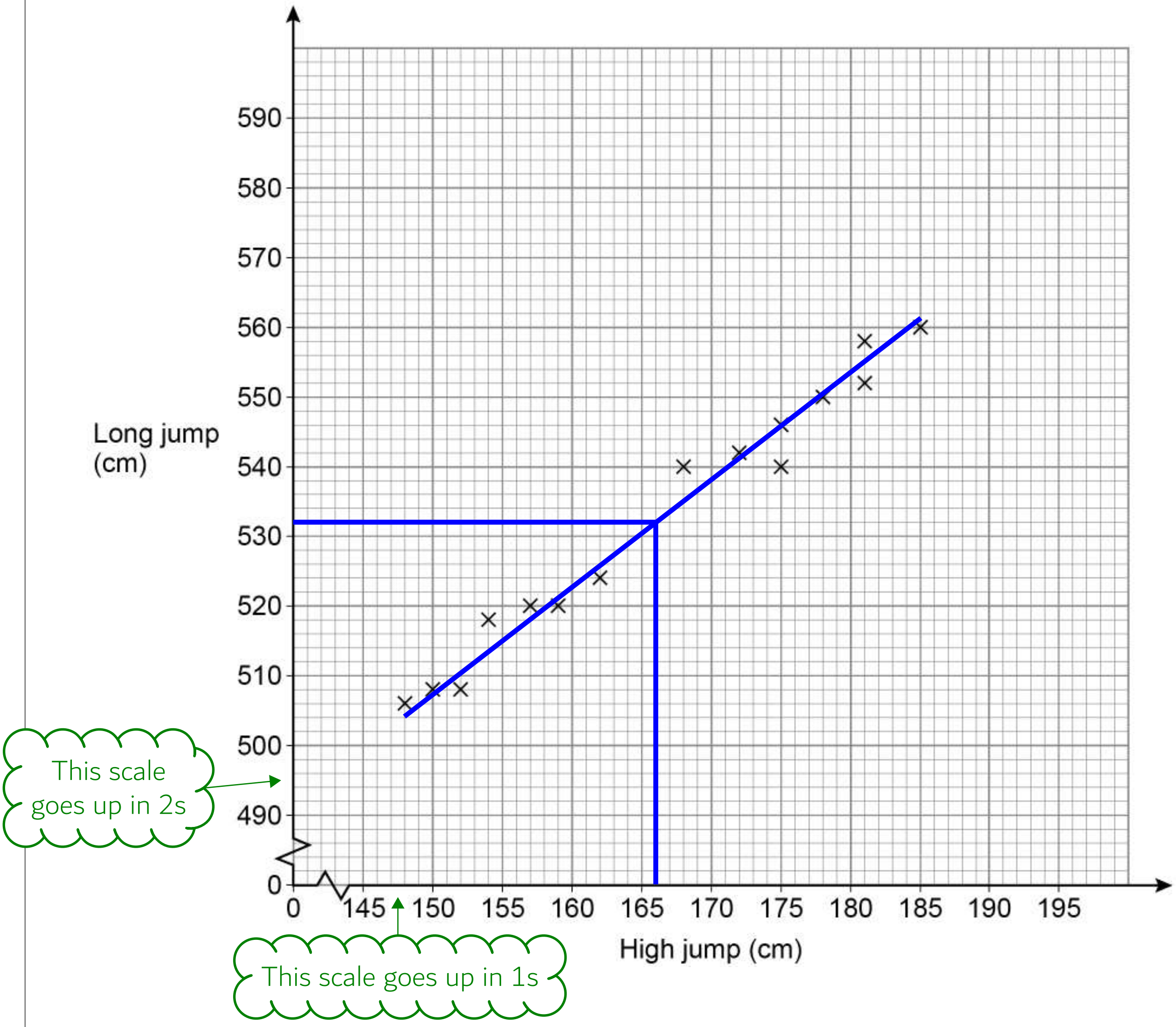
The lowest common multiple is the highest power of each prime in both lists multiplied together

Newer versions of Casio calculators can work out the lowest common multiple without having to do this method

Answer 720

6

6 The scatter graph shows the best high jump and the best long jump for 15 boys.



6 (a) Write down the type of correlation shown.

[1 mark]

Answer Positive

As one variable (the high jump) increases, so does the other variable (the long jump). So this is positive correlation

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6 (b) Liam has a best high jump of 166 cm

Use a line of best fit to estimate his best long jump.

[2 marks]

Answer 532 cm

Drawing a line of best fit using a clear ruler by trying to get the same amount of points above and below the line and have it going in the same direction as the points. Then reading up from 166 cm on the high jump to the line and across to the long jump

6 (c) Another boy has a best high jump of 195 cm

Give a reason why you should **not** use a line of best fit to estimate his best long jump.

[1 mark]

Outside the range of the data

The data goes up to 185 cm for the high jump. It is not certain that the positive correlation will continue in a straight line beyond this point

Turn over for the next question

4

7 A car journey is in two stages.

Stage 1 The car travels 110 miles in 2 hours.

Stage 2 The car travels 44 miles at the same average speed as Stage 1

Work out the time for Stage 2

Give your answer in minutes.

[3 marks]

$s^{d}t$

Writing the formula triangle for distance, speed, time

$110 \div 2$

Speed = distance ÷ time. The distance for Stage 1 is 110 miles and the time is 2 hours. Dividing these works out that the speed was 55 mph

$44 \div 55$

Time = distance ÷ speed. The distance for Stage 2 is 44 miles and the speed is 55 mph. Dividing these works out that the time was 0.8 hours

0.8×60

Answer 48 minutes

There are 60 minutes in an hour so multiplying the 0.8 hours by 60 converts it into 48 minutes

8 Here is an identity.

$$a(3x - 10) \equiv 21x + 2b$$

Work out the values of a and b.

[3 marks]

$3ax - 10a$

Expanding the brackets on the left to put it into the same form as the right

$3a = 21$

There must be the same number of x on both sides. There are 3a on the left and 21 on the right. So 3a must be equal to 21

$a = 7$

Dividing both sides by 3 eliminates the 3 on the left and gets a on its own

-10×7

Working out that the -10a is worth -70

$-70 = 2b$

There is -70 on the left. This must be equal to the 2b on the right

$a =$ 7 $b =$ -35

Dividing both sides by 2 eliminates the 2 on the right and gets b on its own

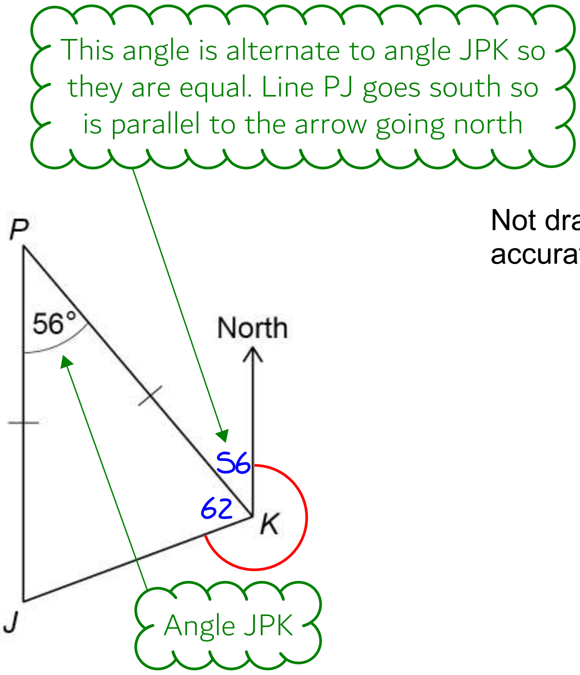
9 *J* and *K* are ships.

P is a port.

J is due South of *P*.

Angle *JPK* = 56°

JP = *KP*



Not drawn accurately

Work out the bearing of *J* from *K*.

[3 marks]

180 − 56 ← There are 180° in total in a triangle. So subtracting angle JPK from 180° works out that there are 124° left in the triangle

124 ÷ 2 ← The remaining two angles in the triangle are equal as the triangle must be isosceles (it has two equal sides) and these are the base angles. So the 124° can be divided by 2 to work out that each of the remaining angles are 62°

360 − 56 − 62

Answer 242 °

The bearing is the angle drawn in red as this is the number of degrees turned clockwise from north from K. There are 360° around a point so subtracting the two other angles around the point from 360° works out the bearing

Turn over for the next question

9

10 The 5th term of a linear sequence is 17

The 6th term of the sequence is 21

Work out the 100th term of the sequence.

[3 marks]

$21-17=4$

Linear sequences change by the same amount between each term. So subtracting the 5th term from the 6th term works out that the sequence increases by 4 between each term

$100-6$

Subtracting the 6 from 100 works out that the 100th term is 94 terms after the 6th term

$21+94\times4$

It must increase 94 lots of 4 between the 6th term and the 100th term. Adding 94 x 4 to the 6th term works out the 100th term

Answer 397

11 The value of a house is £120 000

The value is expected to increase by 5% each year.

Work out the expected value after 4 years.

Give your answer to 2 significant figures.

You **must** show your working.

[4 marks]

$120000\times\left(\frac{100+5}{100}\right)^4$

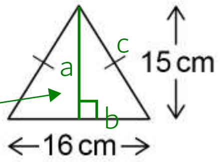
Adding 5% to 100% expresses the percentage it increases to each year. Putting this over 100 converts it into a fraction, which increases by 5% when multiplied by. Raising this to the power of 4 as it needs to be increased by 4 times

Answer £ 150000

The answer of 145860.75 is rounded to 2 significant figures

12 An isosceles triangle has base 16 cm and perpendicular height 15 cm

Drawing a line down the centre of the triangle would form a right-angled triangle with base 8 cm (as this is half of the 16 cm) and height 15 cm



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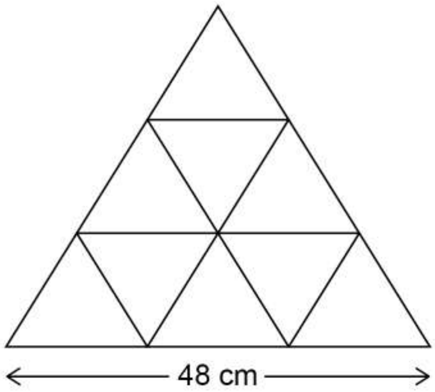
Some of these triangles are used to make a large triangle.



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Work out the perimeter of the large triangle.

[4 marks]

$a^2+b^2=c^2$ ← Pythagoras' Theorem can be used to find the missing length of the right-angled triangle formed as shown above

$c=\sqrt{15^2+8^2}$ ← Square rooting both sides finds c, which is the longest side and is the one we are trying to find. Substituting 15 for a and 8 for b. So the slanted side of each small isosceles triangle is 17 cm

$17 \times 6+48$ ← There are 6 of the slanted side on the permeter of the large triangle so doing 17 x 6 works out the total length of these. Then adding the 48 cm which is the base. Perimeter is all of the outside sides added together

Answer 150 cm

11

Turn over ▶

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13 200 people recorded the time they spent on social media one day.

The table shows the results.

Time, t (mins)	Frequency	Midpoint	
$0 \leqslant t < 30$	24	15	360
$30 \leqslant t < 50$	76	40	3040
$50 \leqslant t < 60$	52	55	2860
$60 \leqslant t < 90$	48	75	3600
	Total = 200		9860

÷200

13 (a) Work out an estimate of the mean time.

[3 marks]

Mean = total ÷ number, where total is the total time in minutes and number is the number of people. Using the midpoint of each category to estimate the time for all of the people in each category. The midpoints can be worked out by doing the mean of the upper and lower bound of each category.

0 + 30 = 30. Then 30 ÷ 2 = 15
30 + 50 = 80. Then 80 ÷ 2 = 40
50 + 60 = 110. Then 110 ÷ 2 = 55
60 + 90 = 150. Then 150 ÷ 2 = 75

Multiplying the midpoints by the frequencies for each category works out an estimate of the total time for each category.

24 x 15 = 360
76 x 40 = 3040
52 x 55 = 2860
48 x 75 = 3600

Then adding all of these totals works out that the estimated total time is 9860 minutes. Dividing this by the 200 people works out an estimate of the mean time

Answer 49.3 mins

13 (b) Draw a histogram to represent the results.

[4 marks]

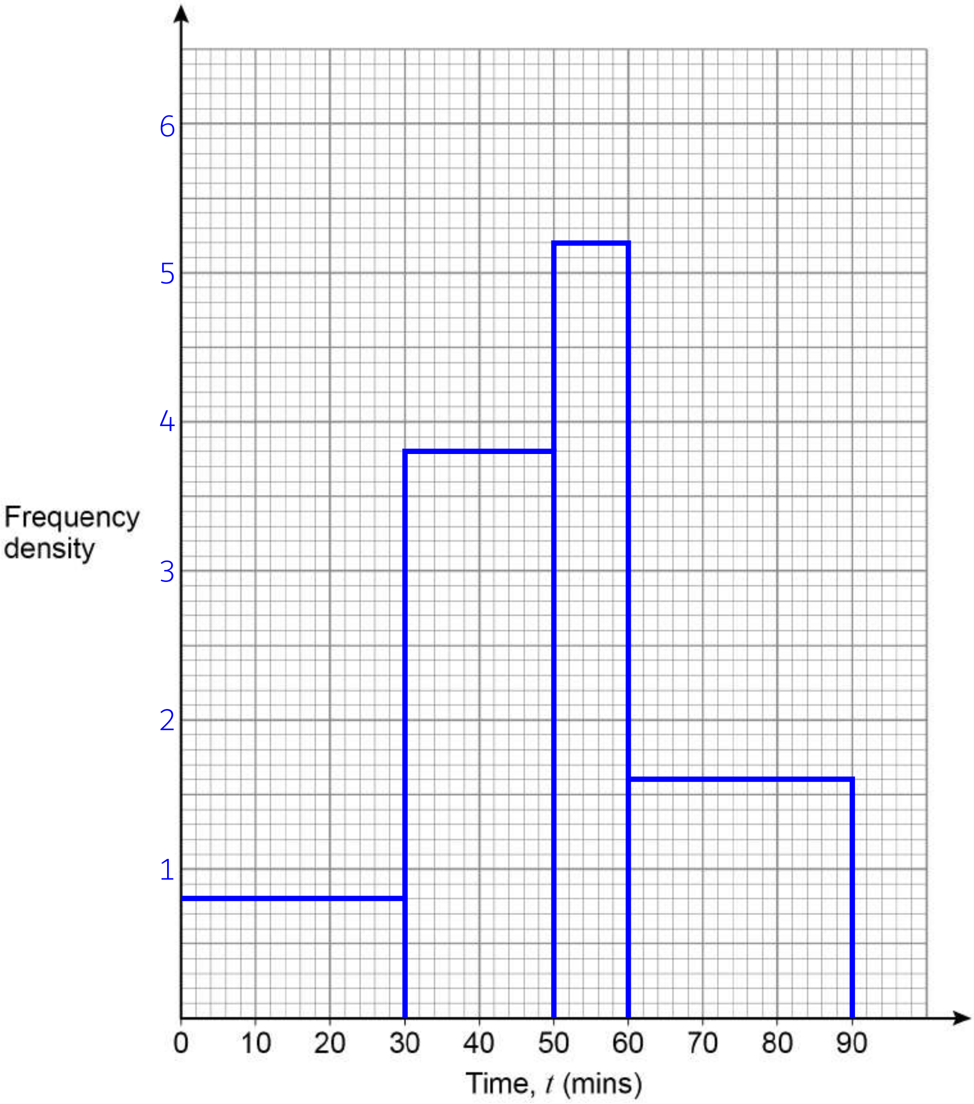
Time, t (mins)	Frequency	Class width	Frequency density
$0 \leqslant t < 30$	24	30	0.8
$30 \leqslant t < 50$	76	20	3.8
$50 \leqslant t < 60$	52	10	5.2
$60 \leqslant t < 90$	48	30	1.6

24 ÷ 30 = 0.8

76 ÷ 20 = 3.8

52 ÷ 10 = 5.2

48 ÷ 30 = 1.6



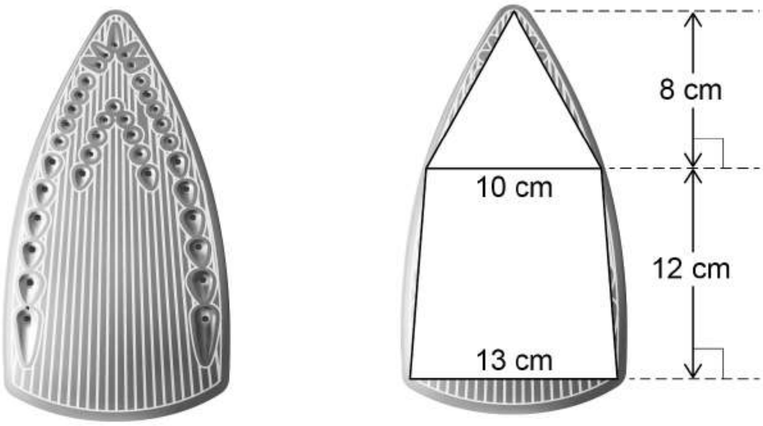
$c \, F \, d$

Writing a formula triangle for class width, frequency, frequency density. Class width is how wide each category on the graph. From the formula triangle, frequency density = frequency/class width

14 Ralf has an iron.

He models the base as a triangle joined to a trapezium.

Not drawn accurately



14 (a) The iron applies a force of 25 newtons (N)

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the pressure using Ralf's model.

[4 marks]

$\frac{1}{2} \times 10 \times 8 = 40$ ← Area of triangle = 1/2 x base x height. The base of the triangle is 10 cm and the height is 8 cm. So the area of the triangle is 40 cm^2

$\frac{1}{2}(10+13) \times 12 = 138$ ← Area of trapezium = 1/2 (a + b) x height, where a and b are the parallel sides and the height is the distance between them. a is 10 cm, b is 13 cm and the height is 12 cm. So the area of the trapezium is 138 cm^2

$40+138$ ← Adding the area of the triangle and the trapezium works out that the area of the iron in the model is 178 cm^2

$\frac{25}{178}$ ← From the formula, pressure = force/area

Answer 0.14 N/cm^2

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14 (b) Is the actual pressure greater than, equal to or less than your answer to part (a)?

Tick **one** box.

☐ greater than ☐ equal to ☑ less than

Give a reason for your answer.

[2 marks]

Area was greater

From the diagrams, it can be seen that the area of the iron is actually greater than what it is in the model. So the force should be divided by more, which will make the pressure less

15 Rearrange $y = \sqrt{w^3}$ to make w the subject.

Circle your answer.

[1 mark]

$w = y^6$ [✓ $w = \sqrt[3]{y^2}$] $w = \sqrt{y^3}$ $w = y^5$

$y^2 = w^3$ ← Squaring both sides eliminates the square root on the right

Then cube rooting both sides eliminates the power of 3 and gets w on its own

Turn over for the next question

7

16 (a) Show that $a\%$ of $b = b\%$ of a

[1 mark]

$\frac{a}{100} \times b = \frac{ab}{100}$ ← Percentage is out of 100 so putting a over 100 converts the percentage into a fraction, which finds a% of b when multiplied by b. Simplifying the expression by multiplying the a and b

$\frac{b}{100} \times a = \frac{ab}{100}$ ← Percentage is out of 100 so putting b over 100 converts the percentage into a fraction, which finds b% of a when multiplied by a. Simplifying the expression by multiplying the b and a

Both simplified expressions are the same, showing that the statement is correct

16 (b) Rosie says,

"160% of 40 = 140% of 60 because $a\%$ of $b = b\%$ of a"

Is she correct?

Tick a box.

☐ Yes ☑ No

Give a reason for your answer.

[1 mark]

$160 \neq 60$

a is not the same in the 160% and 60

17 A packet contains 80 sweets.

The flavour of each sweet is lemon, orange or apple.

A sweet is taken at random.

17 (a) P(lemon or orange) $\leqslant 0.85$

Work out the minimum possible number of **apple** sweets in the packet.

[2 marks]

$1-0.85$ ← The greatest probability is wanted for lemon or orange so that the probability of apple is as small as possible. The greatest this can be is 0.85. It is certain to be apple or not apple. So subtracting the 0.85 from 1 (which is the probability of something which is certain) leaves the minimum probability of apple

0.15×80 ← Multiplying the minimum probability of apple by the 80 sweets works out the minimum number of apple

Answer 12

17 (b) P(lemon or apple) < 0.71

There are 31 lemon sweets.

Work out the maximum possible number of **apple** sweets in the packet.

[2 marks]

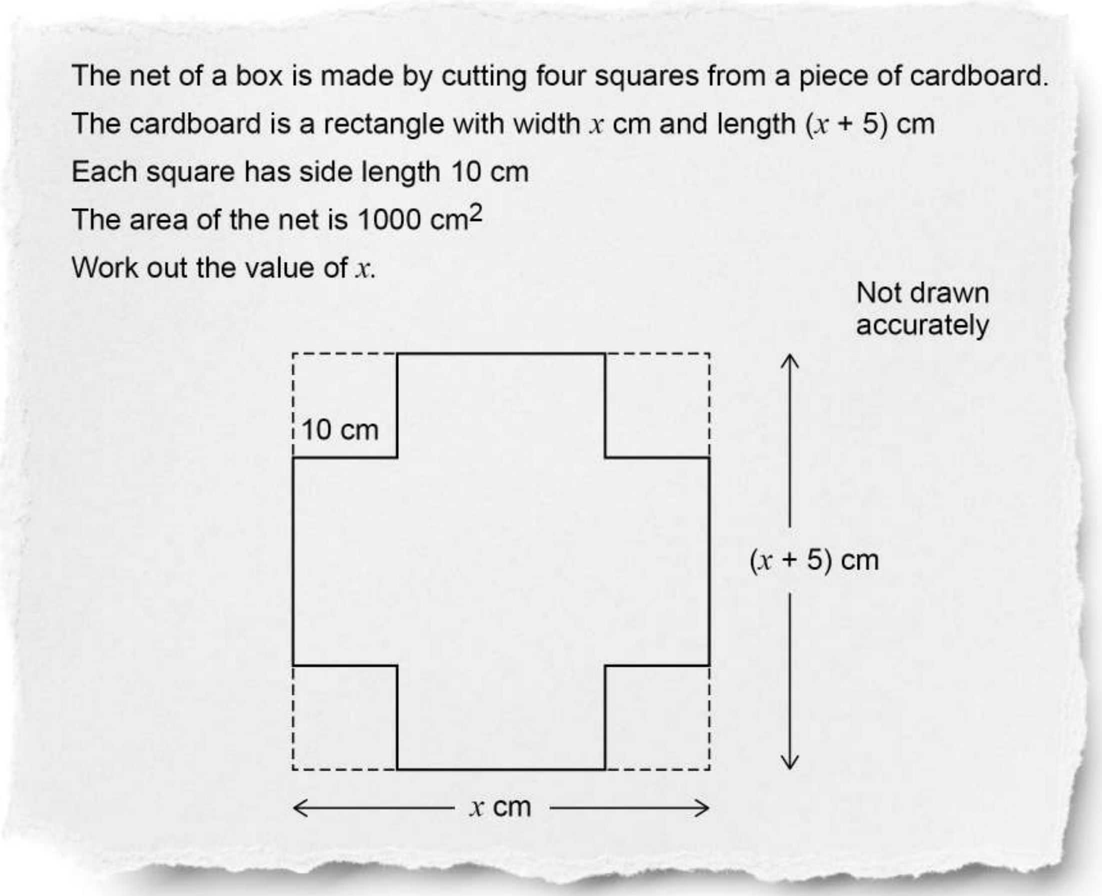
0.71×80 ← Multiplying the upper bound of the probability of lemon or apple works out that the upper bound of the number of lemon or apple is 56.8. There needs to be a whole number so rounding down to 56 means that the probability would be less than 0.71

$56-31$ ← Subtracting the 31 lemon from the 56 lemon or apple leaves 25 apple

Answer 25

6

18 Kate has the following question for homework.



The net of a box is made by cutting four squares from a piece of cardboard.

The cardboard is a rectangle with width x cm and length $(x + 5)$ cm

Each square has side length 10 cm

The area of the net is 1000 cm^2

Work out the value of x.

18 (a) Show that Kate can form the equation $x^2 + 5x - 1400 = 0$

[3 marks]

$x(x+5)-4\times10^2$

Area of rectangle = length x width. The length of the cardboard is x + 5 and the width is x. So multiplying these together expresses the area of the piece of cardboard. Area of square = length^2. The length of each square is 10 so squaring this expresses the area of one of the squares. Multiplying this by 4 as there are 4 of these squares. Subtracting this as the squares are cut from the piece of cardboard

$x^2+5x-400=1000$

Expanding the bracket. 4 x 10^2 = 400. Setting the expression of the area of the net equal to the 1000

$x^2+5x-1400=0$

Subtracting 1000 from both sides gives the desired equation

18 (b) Kate correctly factorises the equation to get $(x + 40)(x - 35) = 0$

Her answer to the homework question is $x = -40$ or $x = 35$

Is her answer correct?

Tick a box.

☐ Yes ☑ No

Give a reason for your answer.

[1 mark]

Length cannot be negative

x is a length

19 Circle the word that describes the graph $y = \sin x$

[1 mark]

[✓ periodic] exponential cubic quadratic

As it repeats every 360°

20 (7, 28) is a point on the graph $y = f(x)$

Circle the point which **must** be on the graph $y = f(x) + 2$

[1 mark]

(7, 26) [✓ (7, 30)] (5, 28) (9, 28)

Adding 2 to the right increases y by 2. 28 + 2 = 30. The x-coordinate is not changed

6

Turn over ▶

21 n is the middle integer of three consecutive positive integers.

The three integers are multiplied to give a product.

n is then added to the product.

Prove that the result is a cube number.

[4 marks]

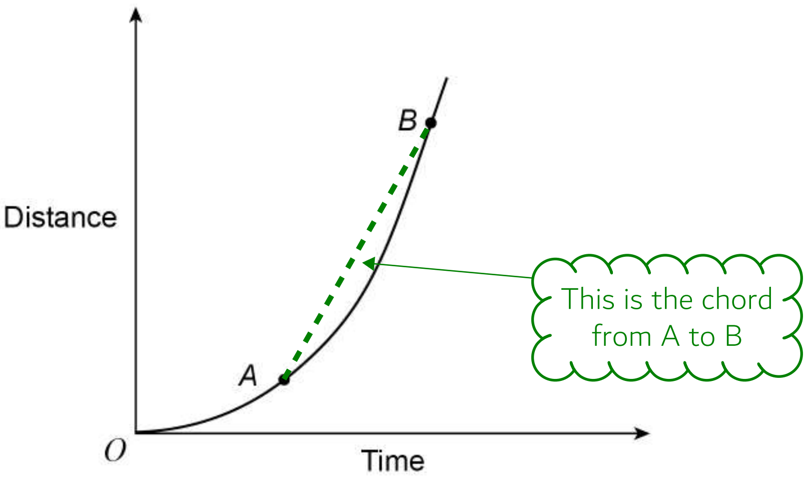
$n(n-1)$ ← n - 1 must be the integer before n. Multiplying n by this to express the product of the first two integers

$(n^2-n)(n+1)$ ← Expanding the bracket. n + 1 must be the integer after n. Multiplying the result by this to express the product of the three integers

$n^3+n^2-n^2-n+n$ ← Expanding the bracket. Adding the n to the result

n^3 ← Simplifying by collecting like terms. $n^2 - n^2 = 0$. $-n + n = 0$

22 Here is a sketch of a distance-time graph.



Which of these represents the average speed between *A* and *B*?

Tick **one** box.

[1 mark]

☐ The gradient of the tangent at *A* ← This gives the speed at A

☐ The gradient of the tangent at *B* ← This gives the speed at B

☑ The gradient of the chord from *A* to *B*

☐ The gradient of the chord from *O* to *B* ← This gives the average speed from the start to B

Gradient = (change in y)/(change in x). The change in y is change in distance and the change in x is change in time. So the gradient divides the distance between A and B by the time, which gives the average speed between A and B

Turn over for the next question

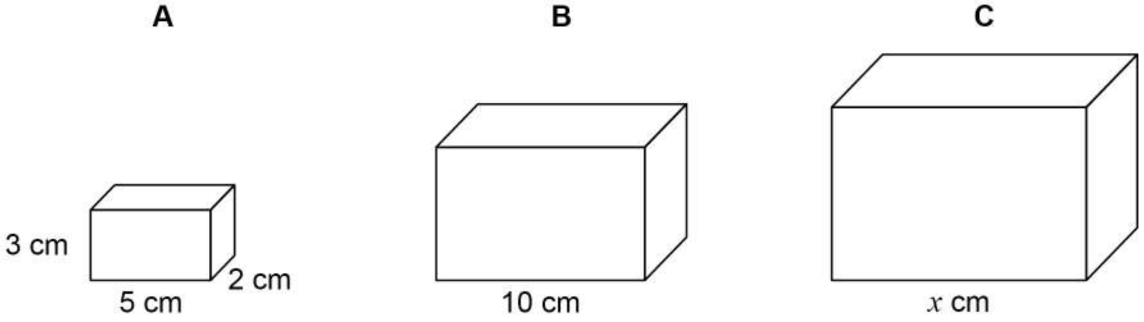
5

23 Here are three similar cuboids, A, B and C.

A has length 5 cm, width 2 cm and height 3 cm

B has length 10 cm

C has length x cm



23 (a) The total surface area of A is 62 cm^2

Tim wants to work out the total surface area of B.

Here is his working.

$$10 \div 5 = 2$$
$$62 \times 2 = 124$$
$$\text{Total surface area of B} = 124 \text{ cm}^2$$

Make **one** criticism of Tim's method.

[1 mark]

Should be 62 x 2^2

10 ÷ 5 works out that all the lengths of A have been multiplied by 2 to get the lengths of B. The unit of area is cm^2 so this scale factor should be squared when going from the area of A to the area of B

23 (b) Volume of A $\times \frac{125}{8}$ = Volume of C

Work out the value of x.

[3 marks]

$\sqrt[3]{\frac{125}{8}}$ ← 125/8 is the volume scale factor from A to C. The unit of volume is cm^3 so cube rooting this volume scale factor gives the length scale factor

5×2.5 ← Multiplying the length of A by the length scale factor works out x

Answer 12.5

Turn over for the next question

$\frac{}{4}$

24 Here are two inequalities.

$$-2 \leqslant x \leqslant 3$$

$$9 \leqslant x + y \leqslant 11$$

x and y are integers.

Work out the **greatest** possible value of $y - x$

[3 marks]

y - x will be greatest using the upper bound of y and the lower bound of x. So x should be -2. Subtracting x from both sides of the second inequality solves for y. Just dealing with the right side and ignoring the left side of the inequality as we are looking for the upper bound of y. So y should be 13

$y \leq 11 - -2$

Doing the chosen value of y subtract the chosen value of x

$13 - -2$

Answer 15

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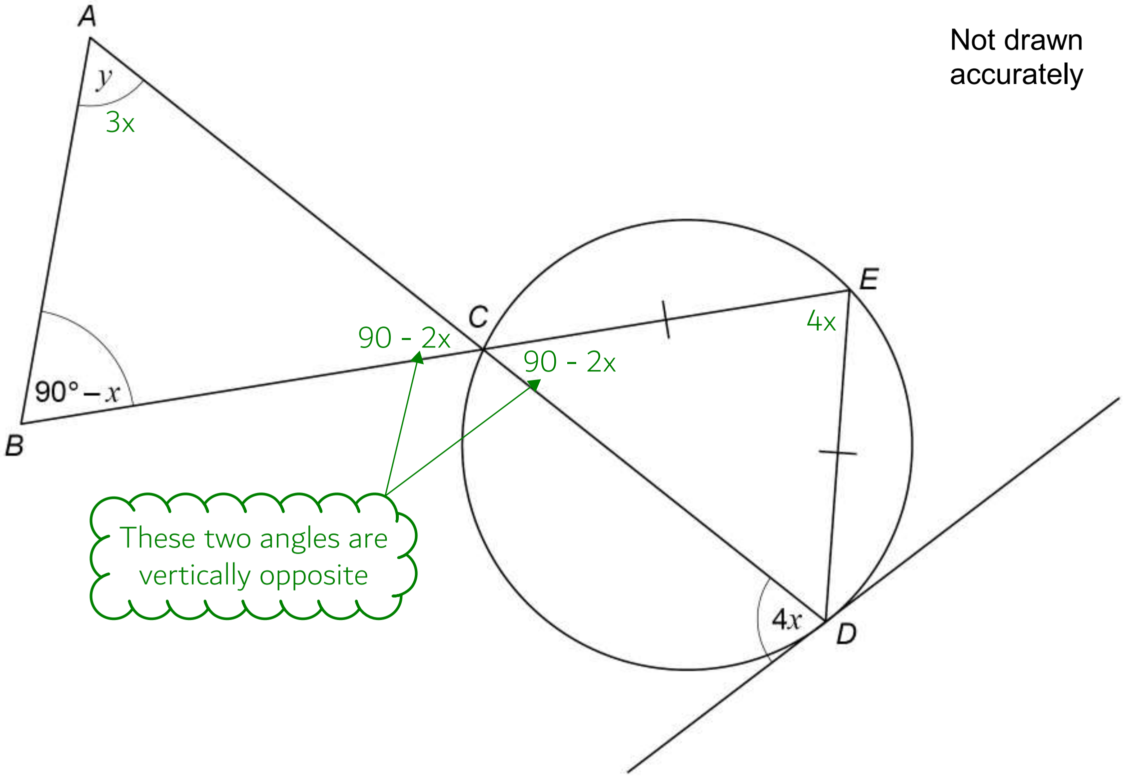
25 *C*, *D* and *E* are points on a circle.

$CE = DE$

The tangent at *D* is shown.

ACD and *BCE* are straight lines.



These two angles are vertically opposite

Not drawn accurately

Prove that $y = 3x$

[4 marks]

Angle CED = 4x because of the alternate segment theorem.

The angle between a tangent and a chord (4x) is equal to the interior opposite angle (CED)

(180 - 4x)/2
Angle ECD = 90 - 2x as there are 180° in a triangle
and the base angles of an isosceles triangle are equal.

Subtracting angle CED from 180 leaves the total of the other two angles in the triangle CED. The other two angles are equal as these are the base angles of an isosceles triangle (as it has two equal sides). So dividing 180 - 4x by 2 works out each base angle, one of these being angle ECD

Angle ACB = 90 - 2x as vertically opposite angles are equal.

y = 180 - (90 - x) - (90 - 2x) = 180 - 90 + x - 90 + 2x = 3x
as angles in a triangle add up to 180°

Subtracting angles ABC and ACB from 180 leaves angle BAC (which is y) in the triangle ABC.

7

26 P, Q and R have positive values.

P is directly proportional to the square of Q.

When $P = 1.25$, $Q = 0.5$

Q is inversely proportional to R.

When $Q = 0.5$, $R = 6$

Work out the value of R when $P = 0.8$

[5 marks]

$P \propto Q^2$

$Q \propto \frac{1}{R}$

Writing out the proportions to make it easier to visualise

$\sqrt{\frac{0.8}{1.25}} \times 0.5$

Using the first proportion, whatever happens to P, the square root of this needs to happen to Q. P is multiplied by 0.8/1.15 so Q must be multiplied by the square root of this. So when P = 0.8, Q = 0.4

$\left(1 \div \frac{0.4}{0.5}\right) \times 6$

Using the second proportion, whatever happens to Q, the reciprocal of this needs to happen to R. Q is multiplied by 0.4/0.5 so R must be multiplied by the reciprocal of this

Answer 7.5

Do not write outside the box

27 $x_{n+1} = \sqrt[3]{3x_n + 7}$

Use a starting value of $x_1 = 2$ to work out a solution to $x = \sqrt[3]{3x + 7}$

Give your answer to 3 decimal places.

[3 marks]

Entering 2 into the calculator and pressing = (or EXE). Entering $\sqrt[3]{3\text{Ans} + 7}$ and keep pressing = (or EXE) until all the digits in the answer stop changing. This gives 2.425988757, which can be given to 3 decimal places

Answer 2.426

END OF QUESTIONS

8