OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

Monday 7 November 2022 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number						Candidate number				

First name(s) ____________________

Last name ____________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Turn over

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

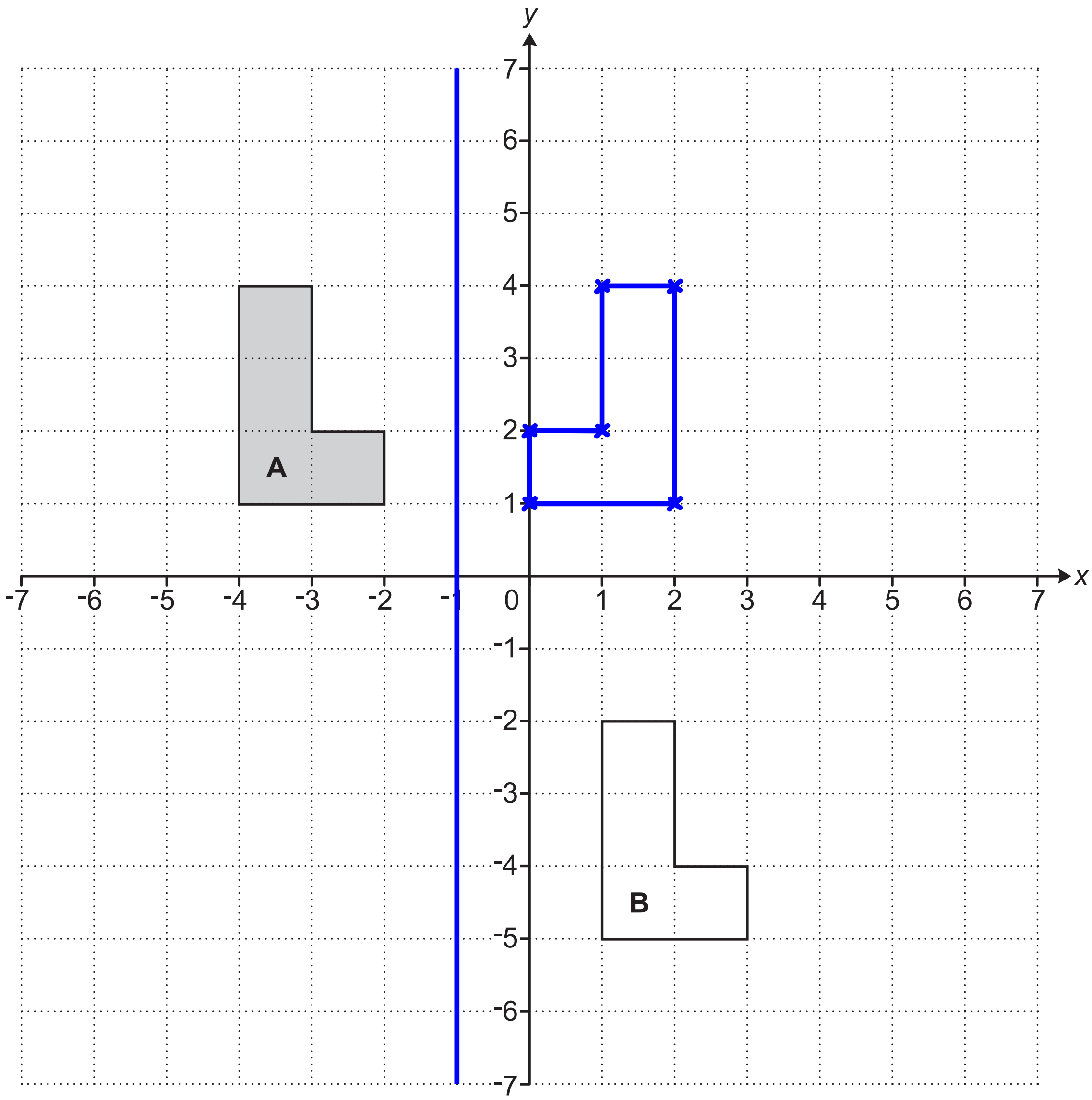
Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** the questions.

1 Shape **A** and shape **B** are drawn on the coordinate grid.



(a) Describe fully the single transformation that maps shape **A** onto shape **B**.

Translation by $\binom{5}{-6}$

It has moved 5 to the left and 6 down

[2]

(b) Reflect shape **A** in the line $x = -1$. **[2]**

Drawing the line x = -1 by considering that the x-coordinate is always -1. Reflecting the corners by counting the number of jumps to the line then doing the same number on the other side. Then connecting the corners

2 A recipe for a batch of jam needs 3 oranges, 5 lemons and 1.5 kg of sugar.
A cook uses the recipe to make lots of batches of jam.
They use 16 **more** lemons than oranges in total.

Find how much sugar the cook should use.

3:5:1.5 ← Writing the amount of oranges, lemons and sugar as a ratio

2p = 16 ← There are 2 more parts for lemons than oranges so this must represent the 16 more lemons than oranges

P = 8 ← Dividing both sides by 2 finds that 1 part of the ratio is worth 8

8 × 1.5 ← Multiplying the value of 1 part of the ratio by the 1.5 parts for sugar works out how much sugar should be used

........12........ kg **[3]**

3 In 1980, Ling's flat was worth £23 000.
Today, Ling's flat is worth 1200% of its value in 1980.

Calculate the value of Ling's flat today.

$23000 \times \frac{1200}{100}$ ← Putting the 1200% over 100 converts it into a fraction, which when multiplied by finds 1200% of the £23000

£ ....276000........ **[2]**

4 Sam and Taylor are playing a game against a computer.
They can win, draw or lose the game.

Sam says

I think the probability of us winning the game is 0.3.

Taylor says

I think the probability of us losing the game is 0.75.

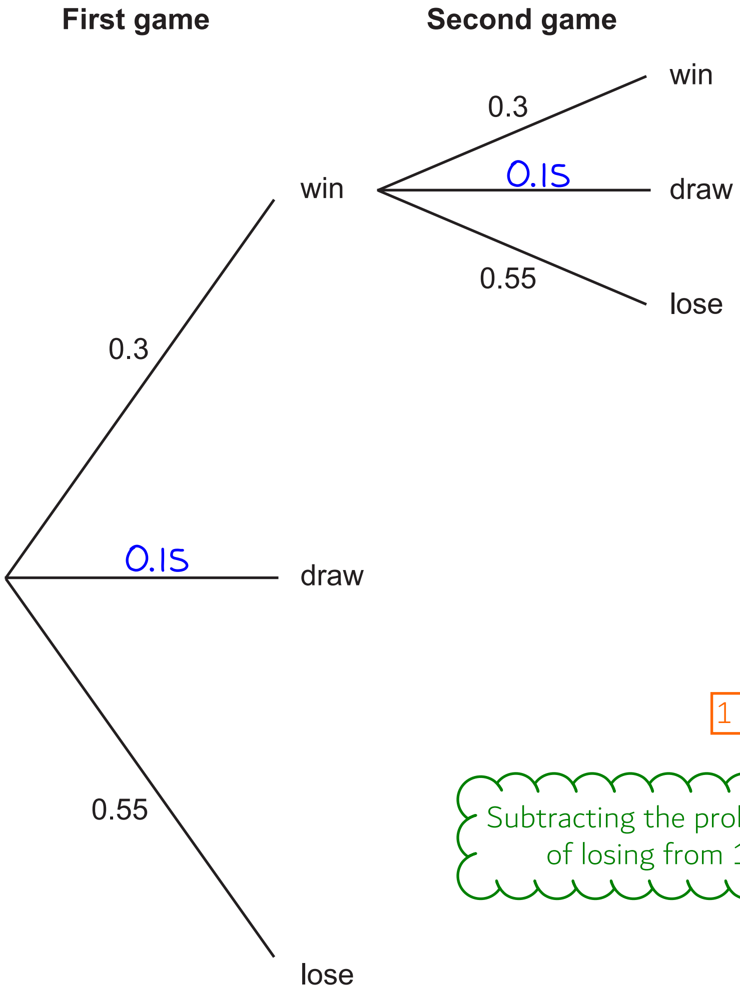
(a) Explain why Sam and Taylor cannot both be correct.

0.3 + 0.75 > 1

The probabilities cannot add up to more than 1 as this would mean that it would be more than certain for them to win or to lose **[1]**

(b) Sam is correct. The probability of them winning the game is 0.3.
Taylor is not correct. The probability of them losing the game is actually 0.55.

Complete this **partly drawn** tree diagram to show **all** the possible outcomes of playing the game twice.



1 - 0.3 - 0.55 = 0.15

Subtracting the probability of winning and the probability of losing from 1 gives the probability of drawing

[3]

(c) Find the probability of them winning the first game and losing the second game.

0.3×0.55 ← Winning AND losing. AND means to multiply the probabilities

(c) ....0.165.................... **[2]**

5 In space, distances can be measured in Astronomical Units.
In this question, use the conversion 1 Astronomical Unit $= 1.5 \times 10^8$ km.

(a) On a particular day the distance from Earth to Neptune is 29.09 Astronomical Units.

Calculate the distance from Earth to Neptune in kilometres on that day.
Give your answer in standard form.

$29.09 \times 1.5 \times 10^8$ ← Multiplying the number of Astronomical Units by the value of each one in kilometres converts it into kilometres

Using the calculator to put the answer of 4363500000 into ENG notation puts it into standard form in this case

(a) ......4.3635×10^9...... km **[3]**

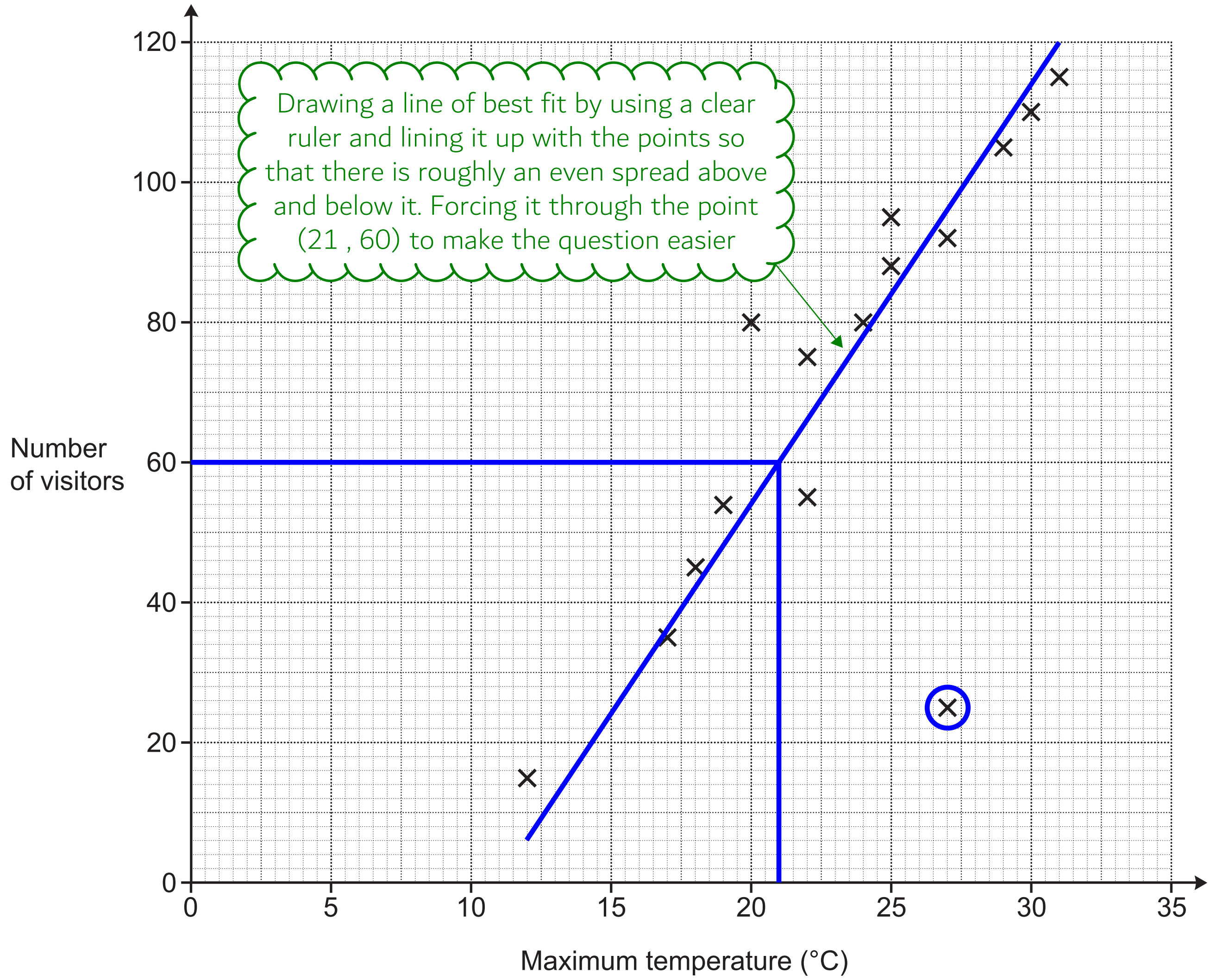
(b) On a particular day the distance from Earth to Mars is 78 340 000 km.

Calculate the distance from Earth to Mars in Astronomical Units on that day.

$\frac{78340000}{1.5 \times 10^8}$ ← Dividing the distance in kilometres by the value of each Astronomical Unit in kilometres works out how many lots of the Astronomical Units the distance is

(b) ......0.52...... Astronomical Units **[2]**

6 The scatter diagram shows the number of visitors to a children's playground and the maximum temperature on fifteen Saturdays in summer.



(a) Describe the type of correlation shown in the scatter diagram.

As both variables increase together

(a) ....Positive........ **[1]**

(b) One Saturday was a hot but stormy day.

(i) Circle the most likely point on the scatter diagram for this Saturday. **[1]**

(ii) Explain why you chose this point.

It was a hot day with fewer visitors

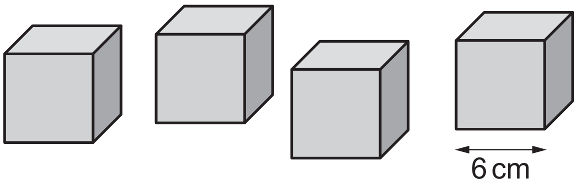
.... **[1]**

(c) Use a line of best fit to predict the number of visitors on a Saturday that has a maximum temperature of 21 °C.

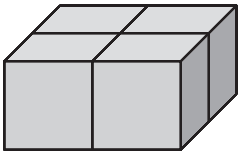
Reading up from 21 to the line and across to estimate the number of visitors when the maximum temperature was 21°C

(c) ....60........ visitors **[2]**

7 A child has four identical wooden cubes of side length 6 cm.



(a) They arrange the cubes in a 2 by 2 by 1 arrangement to form a cuboid.

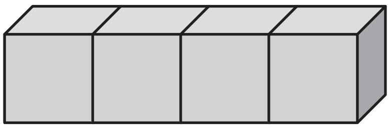


Show that the surface area of the cuboid is 576 cm^2. **[2]**

6×6 ← This works out that the area of each square face on each cube is 36 cm^2. Area of square = $length^2$

$36 \times 16 = 576$ ← There are 16 of the square faces on the surface of the cuboid. So multiplying the area of each one by 16 works out that the total surface area is 576 cm^2

(b) The child rearranges the cubes in a 4 by 1 by 1 arrangement to form a different cuboid.

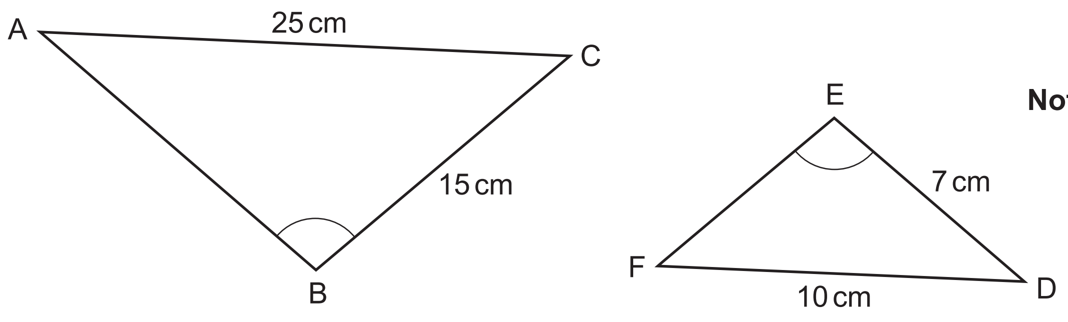


Calculate the percentage increase in surface area for this cuboid compared with the 2 by 2 by 1 cuboid.

$\frac{18-16}{16} \times 100$ ← There are 18 square faces on the surface of the different cuboid. 18 - 16 expresses the increase in the number of square faces on the surface. Putting this over the original 16 expresses it as a fraction increase. Multiplying this by 100 converts it into a percentage increase

(b) ....12.5.......... % **[4]**

8 Triangles ABC and DEF are mathematically similar.
Angle ABC = Angle DEF.



Not to scale

Calculate the perimeter of triangle ABC.

$25 \div 10$ ← FD is the smaller version of side AC. Dividing AC by FD works out that the scale factor is 2.5, which is the amount all of the sides on the smaller triangle have been multiplied by to get the sides on the larger triangle

7×2.5 ← Multiplying side ED by the scale factor works out side AB

$17.5 + 25 + 15$ ← Adding sides AB, AC and BC works out the perimeter of the triangle ABC

................57.5................ cm **[4]**

9 Given that $(2^k)^6 \times 8 = 2^{45}$, find the value of k.

$2^{6k} \times 2^3 = 2^{45}$ ← $(a^x)^y = a^{xy}$. $8 = 2^3$. Expressing everything as powers of 2

$6k + 3 = 45$ ← $a^x \times a^y = a^{x+y}$. The powers on the left must be equal to the powers on the right

$6k = 42$ ← Subtracting 3 from both sides gets the k term on its own

Dividing both sides by 6 gets k on its own

k = ...7................ **[3]**

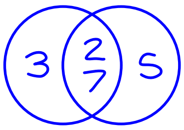
Turn over

10 The highest common factor (HCF) of two numbers is 14.
The lowest common multiple (LCM) of the same two numbers is 210.
The two numbers are **not** 14 and 210.

Find the two numbers.

$14 = 2 \times 7$
$210 = 2 \times 3 \times 5 \times 7$

Expressing both 14 and 210 as a product of prime factors using the calculator



Using a Venn diagram to arrange the prime factors. 2 and 7 must go in the middle and these make the HCF. 3 and 5 must go in the other parts but not in the same circle otherwise this would make 14 and 210

$2 \times 7 \times 3$
$2 \times 7 \times 5$

In one ring there is 2, 7 and 3. Multiplying these finds the number it represents. In the other ring there is 2, 7 and 5. Multiplying these finds the number it represents

.............42............ and ....70............ **[3]**

11 Factorise fully $30x^2 + 2x - 4$.

$2(15x^2 + x - 2)$

2 is a common factor of all terms so bringing this out as a factor and leaving the rest in a bracket

15×-2

In the bracket it is in the form $ax^2 + bx + c$. Multiplying a by c gives -30

$15x^2 - 5x + 6x - 2$

Using table mode on the calculator, enter f(x) = 30/x. Start: 1. End: 30. Step: 1. This lists out the factor pairs of 30 and helps to find the two numbers which multiply to -30 and add to 1 (which is b). -5 and 6 do this so splitting the middle x term into these numbers of x

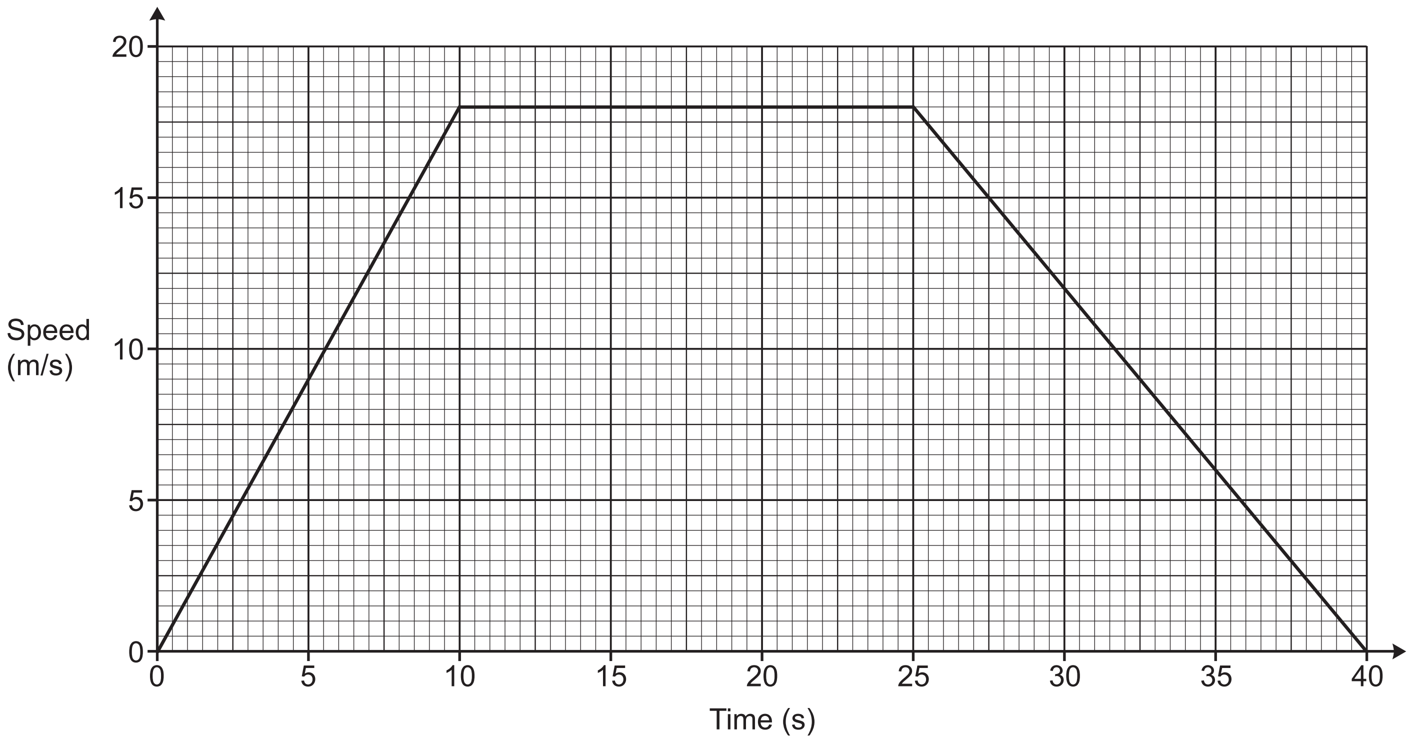
$5x(3x - 1) + 2(3x - 1)$

Factorising the first two terms and the last two terms separately

Bringing the two halves together and putting back with the 2 which was first brought out as a factor

.............$2(5x+2)(3x-1)$............ **[3]**

12 The graph shows the speed of a car during the first 40 seconds of a journey.



(a) Write down the acceleration of the car between 10 seconds and 25 seconds.

The speed is not changing therefore there is no acceleration

(a) ....0.................... m/s^2 **[1]**

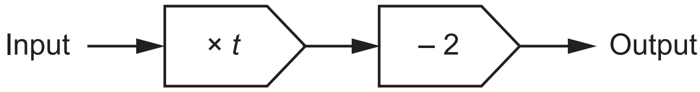
(b) Work out the average speed of the car, in m/s, during the 40 seconds.
You must show your working.

$\frac{1}{2}(15+40)\times 18$ ← The area under the line is the distance travelled. The whole shape is a trapezium. Area of trapezium = 1/2 (a + b)h, where a and b are the parallel sides and h is the distance between them

$495 \div 40$ ← The unit of m/s means to divide the distance in metres by the time taken in seconds. The total distance is 495 m and the time taken is 40 seconds

(b) ....12.4.................... m/s **[5]**

13 **(a)** Here is a function.



When the input is 6, the output is 18.

Find the value of t.

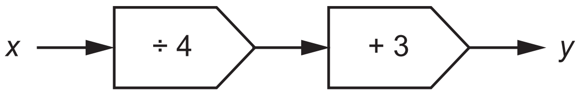
$6t-2=18$ ← Multiplying the input by t then subtracting 2 must give the output

$6t=20$ ← Adding 2 to both sides eliminates the -2 on the left and gets the t term on its own

Dividing both sides by 6 finds t. It can be left as an unsimplified fraction as it will not simplify to an integer

(a) $t =$ $\frac{20}{6}$ **[3]**

(b) Here is another function.
When the input is x, the output is y.



Write an algebraic expression for ***x* in terms of *y***.

Going backward in the function takes it from y to x. Doing the opposite operations in the opposite order. 3 must be subtracted by y then it must all be multiplied by 4 to get x

(b) $x=(y-3)\times4$ **[2]**

14 **(a)** The time taken to paint a wall is inversely proportional to the number of people painting. It takes 40 minutes for 3 people to paint the wall if nobody stops painting.

Layla, Mia and Nina start painting the wall.
After 10 minutes Layla stops painting.
She leaves Mia and Nina to finish painting the wall.

Assume that Layla, Mia and Nina paint at the same rate.

Work out the **total** time taken to paint the wall.

$40 \times 3 = 120$ ← Multiplying the 3 people by the 40 minutes works out that there is 120 minutes worth of work to be done

$10 \times 3 = 30$ ← Multiplying the 3 people by the 10 minutes they all work at the same time works out that 30 minutes worth of work is done

$120 - 30$ ← Subtracting the 30 minutes worth of work done from the 120 minutes worth of work which need to be done works out that 90 minutes worth of work still needs to be done

$90 \div 2$ ← Dividing the 90 minutes worth of work by the 2 people doing it works out that it will take 45 minutes

$45 + 10$ ← Adding the additional 45 minutes to the 10 minutes which has already been done

(a) ....55............ minutes **[3]**

(b) y is inversely proportional to x^3.
$y = 16$ when $x = 2$.

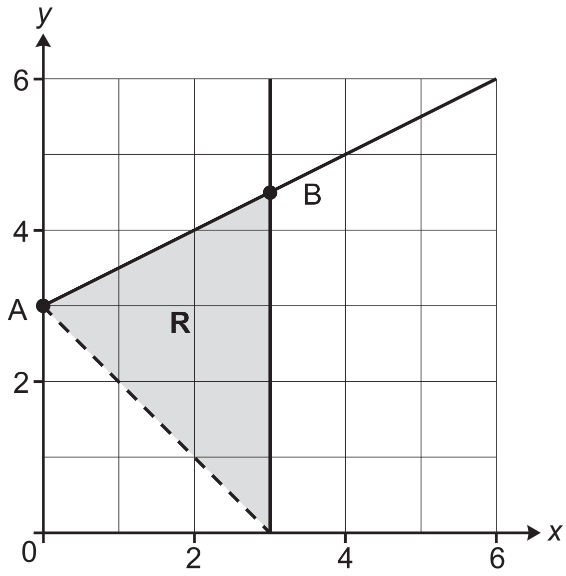
Find the value of y when $x = 8$.

$y \propto \frac{1}{x^3}$ ← Writing the proportion

$16 \div 4^3$ ← x has been multiplied by 4 from 2 to 8. So y must be divided by 4^3

(b) ....$\frac{1}{4}$............ **[3]**

15 The region **R** is shown on this grid.
A is the point (0, 3) and B is the point (3, 4.5).



(a) Show that an equation of the straight line through A and B is $2y = x + 6$. **[3]**

$\frac{6-3}{6-0}$ ← Working out the gradient using the two end points of the line. Gradient = (change in y)/(change in x). y changes from 3 to 6 and x changes from 0 to 6

$y=\frac{1}{2}x+3$ ← y = mx + c, where m is the gradient (which is 1/2) and c is the y-intercept (which is 3)

$2y=x+6$ ← Multiplying all terms on both sides of the equation by 2 gives the desired equation

(b) Write down the three inequalities that define region **R**.

$\frac{0-3}{3-0}$ ← Working out the gradient of the dashed line using the two end points of the line. Gradient = (change in y)/(change in x). y changes from 3 to 0 and x changes from 0 to 3

The equation of the vertical line is x = 3. x is less than or equal to 3 as the region is on the left and the line is solid. y is less than or equal to for the line going through A and B as the region is below and the line is solid. The equation of the dashed line is y = -x +3 and y is greater but not equal to as the region is above and the line is dashed →

(b) $x \le 3$

$y \le \frac{1}{2}x+3$

$y > -x+3$ **[5]**

16 A plane flies from London to Tokyo.
The distance is 9600 km, correct to the nearest 100 km.
The plane travels at an average speed of 820 km/h, correct to the nearest 10 km/h.

Calculate the shortest possible flight time of the plane.
Give your answer in hours and minutes, correct to the nearest minute.
You must show your working.

$s\,^{d}\,t$ ← Writing the formula triangle for distance, speed, time

$\left(9600-\frac{100}{2}\right)\div\left(820+\frac{10}{2}\right)$

From the formula triangle, time = distance/speed. The lower bound for the time is found by dividing the lower bound of the distance by the upper bound of the speed. The lower bound of the distance is found by subtracting half of the resolution of the measurement (which is 100 km as this is what it is to the nearest). The upper bound of the speed is found by adding half of of the resolution of the measurement (which is 10 km/h as this is what it is to the nearest)

The time in hours is converted into hours and minutes by using the calculator to convert it into a sexagesimal. The calculator shows 11°34'32.73'', which can be read as 11 hours, 34 minutes and 32.73 seconds. The 32.73 seconds is more than half a minute so the minutes must be rounded up to 35

..........11.......... hours ....35.......... minutes **[5]**

17 Charlie weighs many apples.

The weights of the apples are summarised below.

- heaviest apple = 75 g
- range = 50 g
- median = 60 g
- lower quartile = 45 g
- 50% of the apples weigh between 45 g and 65 g
- mean = 63 g

(a) **(i)** Write down the interquartile range for the weights of the apples.

50% of the apples are between the upper and lower quartile and the interquartile range is the difference between the upper and lower quartile. 65 - 45 = 20

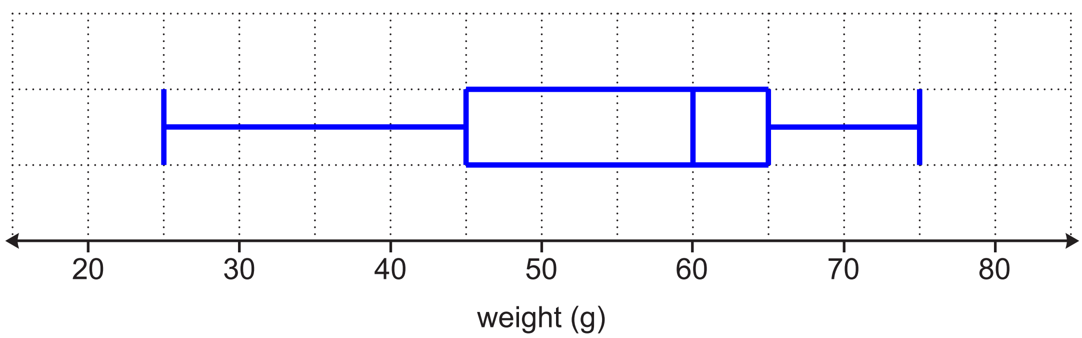
(a)(i) ....20........ g **[1]**

(ii) Write down the percentage of the apples that weigh between 45 g and 60 g.

1/4 of the data is between the lower quartile and the median

(ii) ....25........ % **[1]**

(b) Draw a box plot to show the distribution of the weights of the apples.



[3]

The lowest value must be 25 as the range is 50 and this is the difference between the largest and smallest. 75 - 50 = 25. The upper quartile must be 65 as 50% of the apples are between the upper and lower quartile

(c) Charlie eats two of the apples.
The apples that they eat weigh 58 g and 66 g.

Charlie says

The mean weight of all the apples was 63 g.
I ate one apple that weighed less than the mean and another apple that weighed more than the mean.
Therefore, the mean of the remaining apples will still be 63 g.

Is Charlie correct?
Explain your reasoning.

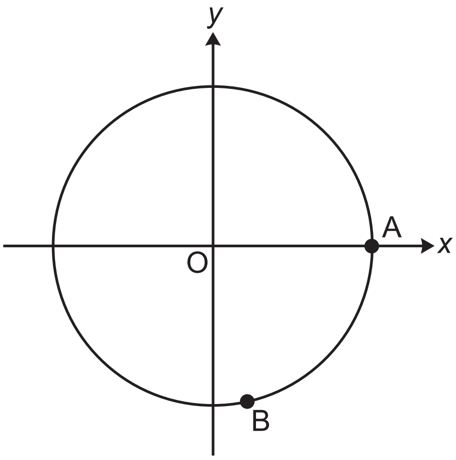
No, as 58 is 7 below the mean and 66 is 3 above the mean so the mean will now be more ....

....

.... **[2]**

The mean is a central value to all of the apples. Removing one above the mean lowers the mean and removing one below the mean increases the mean however the one below is further away from the mean so has more of an effect

18 A circle has equation $x^2 + y^2 = 100$.
The sketch shows the circle and two points, A and B, which lie on the circumference of the circle.



(a) Write down the coordinates of point A.

The general equation of a circle with its centre at (0 , 0) is $x^2 + y^2 = r^2$, where r is the radius. $r^2 = 100$ so r = 10. The radius is 10 and A is on the x-axis so must be 10 to the right of the centre

(a) (10 , 0) **[1]**

(b) Point B has *x*-coordinate 3.

Find the exact value of the *y*-coordinate of point B.

$3^2 + y^2 = 100$ ← Substituting the x-coordinate into the equation

$y^2 = 100 - 3^2$ ← Subtracting 3^2 from both sides

Square rooting both sides finds the exact value of y, which must be negative as it is below the x-axis

(b) $-\sqrt{91}$ **[3]**

(c) Another point, C, lies on the circle and has a *y*-coordinate that is seven times its *x*-coordinate.

Find the two possible pairs of coordinates for point C.
Give your answers in exact form.
You must show your working.

$(7x)^2$ ← Substituting 7x for y in y^2 as the y-coordinate is 7 times the x-coordinate

$x^2 + 49x^2$ ← Expanding the bracket and putting back into the left side of the equation

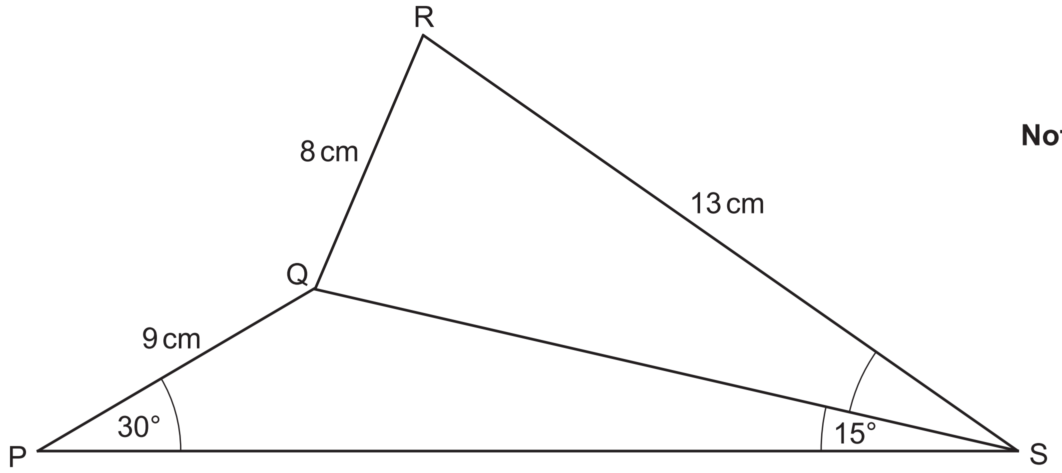
$50x^2 = 100$ ← Simplifying the left side and putting back into the equation

$x^2 = 2$ ← Dividing both sides by 50

Square rooting both sides finds the x-coordinate. Multiplying the x-coordinate by 7 gives the y-coordinate

(c) ($\sqrt{2}$, $7\sqrt{2}$) and ($-\sqrt{2}$, $-7\sqrt{2}$) **[5]**

19 PQS and QRS are triangles.



PQ = 9 cm, QR = 8 cm and RS = 13 cm.
Angle QPS = 30° and angle PSQ = 15°.

Calculate angle QSR.
Give your answer correct to **1** decimal place.
You must show your working.

$\frac{a}{\sin A} = \frac{b}{\sin B}$ ← There are opposite pairs of sides and angles in triangle PQS so the sine rule can be used to work out side QS

$QS = \frac{9\sin 30}{\sin 15}$ ← Multiplied both sides by sinA to make a the subject then substituted QS for a, 9 for b, 30 for A and 15 for B. Side a is opposite angle A and side b is opposite angle B. Storing the exact value of 17.38666487 as A on the calculator

$a^2 = b^2 + c^2 - 2bc\cos A$ ← There are not opposite pairs of sides and angles in triangle QRS so using the cosine rule to work out angle QSR

$a^2 - b^2 - c^2 = -2bc\cos A$ ← Subtracting b^2 and c^2 from both sides

$\frac{a^2 - b^2 - c^2}{-2bc} = \cos A$ ← Dividing both sides by -2bc

$\cos^{-1}\left(\frac{8^2 - 13^2 - QS^2}{-2 \times 13 \times QS}\right)$ ← Doing the inverse cos of both sides makes angle A the subject. Substituting 8 for a (as it is opposite angle A), 13 for b and the exact value of QS for c

Rounding the value of 25.71131253 to 1 decimal place →

25.7 ° **[6]**

Turn over for Question 20

Turn over

20 Write as a single fraction in its simplest form.

$$10 - \frac{6x+45}{3x+5}$$

$10(3x+5)$ ← 10 can be expressed as 10/1 as a fraction. Multiplying both the numerator and denominator by (3x + 5) makes it so that the denominators of both fractions are the same and they can be subtracted. Writing the numerator of the first fraction

$30x+50-6x-45$ ← Expanding the bracket and subtracting the numerator of the second fraction

Collecting like terms to simplify and putting over the common denominator. The fraction cannot be simplified any further as there are no common factors between the numerator and denominator

$\frac{24x+5}{3x+5}$ .... **[4]**

END OF QUESTION PAPER