OCR
Oxford Cambridge and RSA

Wednesday 14 June 2023 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

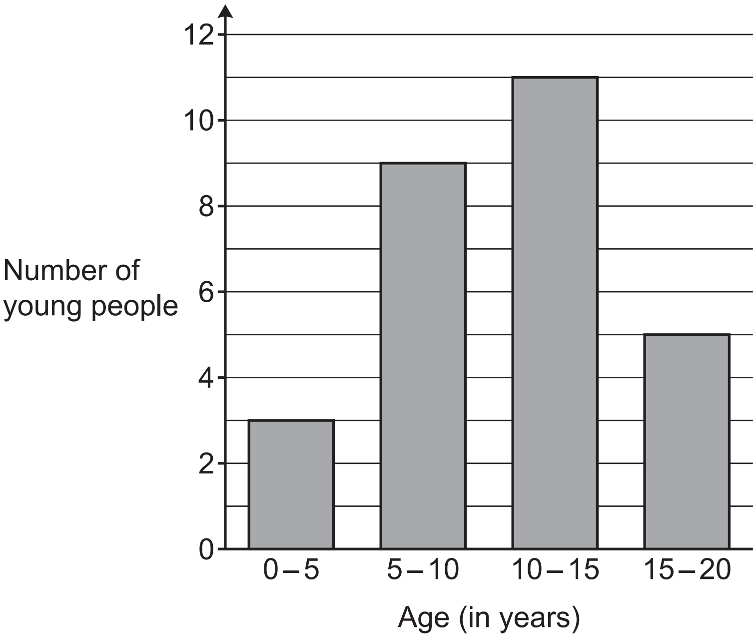
Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

1 Alex draws a bar chart to show the age of the young people attending a youth club.



Make **one** criticism of Alex's bar chart.

The groups overlap

For example, the age of 5 years could go in 0 – 5 or 5 – 10

[1]

2 **(a)** Rearrange this formula to make u the subject.

$$v^2 = u^2 + 2as$$

$u^2 = v^2 - 2as$ ← Subtracting 2as from both sides gets the u^2 on its own

Square rooting both sides gets u on its own

(a) .... $u = \sqrt{v^2 - 2as}$ .... **[2]**

(b) A rocket accelerates at 90 m/s^2 and travels 270 km.
The rocket's final velocity is 8000 m/s.

Using part **(a)**, or otherwise, calculate the rocket's initial velocity in m/s.

270×1000 ← The distance needs to be in metres to make it compatible with the other units. There are 1000 metres in a kilometre so multiplying the 270 by 1000 converts it into metres

$\sqrt{8000 - 2 \times 90 \times 270000}$ ← u is the initial velocity. v is the final velocity. a is the acceleration. s is the distance. Substituting in the known values into the right side of the formula with u as the subject finds the initial velocity

(b) .... 3924.3 .... m/s **[3]**

3 A bag contains 150 counters.
The counters are either red or yellow.

(a) Riley picks a counter from the bag, records its colour, and replaces it.
They do this nine times.

Here are Riley's results.

Red	𝍸
Yellow	||||

Use Riley's results to work out how many red counters are likely to be in the bag.

$\frac{5}{9} \times 150$ ← The tally chart shows that 5 out of the 9 counters were red. Doing this fraction of the 150 counters estimates how many red counters there are

The number of red counters needs to be a whole number so rounding $83.\dot{3}$ to the nearest whole number

(a) ....83.......... red counters **[3]**

(b) Ling uses the same bag of counters and picks the counters in the same way.

Here are Ling's results.

Red	𝍸 𝍸 ||
Yellow	𝍸 |||

Use Ling's results to estimate the probability of choosing a red counter from the bag.
Give your answer as a fraction in its simplest form.

$12+8$ ← The tally chart shows that there were 12 red and 8 yellow. Adding these works out that there Ling picks 20 counters

$\frac{12}{20}$ ← 12 out the 20 counters were red. Expressing this as a fraction

Entering 12/20 into the calculator simplifies it

(b) ....$\frac{3}{5}$.......... **[2]**

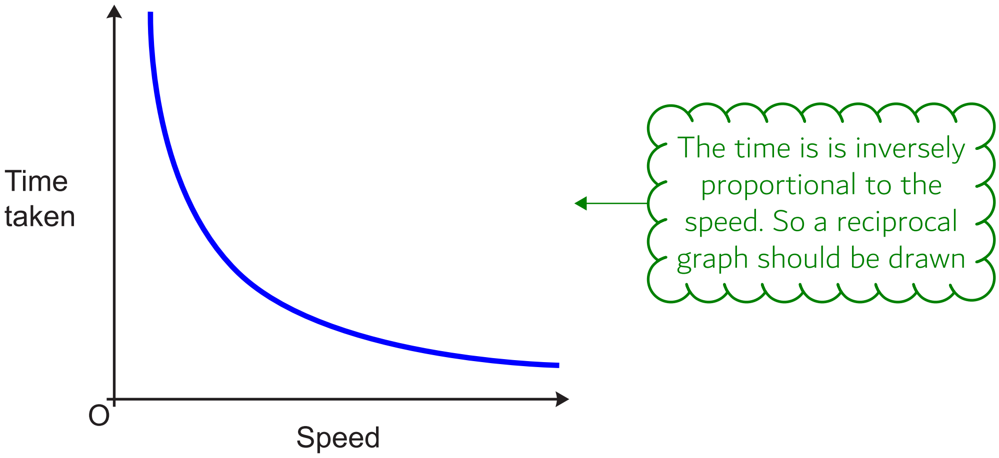
(c) Explain why Ling's results are likely to give a better estimate of the probability of choosing a red counter from the bag than Riley's results.

Ling picked more counters

The more counters picked, the more reliable the estimate of the probability **[1]**

4 (a) The time taken to complete a journey halves as the speed doubles.

On the axes below, sketch a graph to show this relationship.



[2]

(b) It takes 40 minutes to fill a garden pond using water from 5 identical hose pipes.

Assuming the rate of flow of water from each hose pipe is the same, work out how many minutes it would take to fill the same garden pond using 2 of these hose pipes.

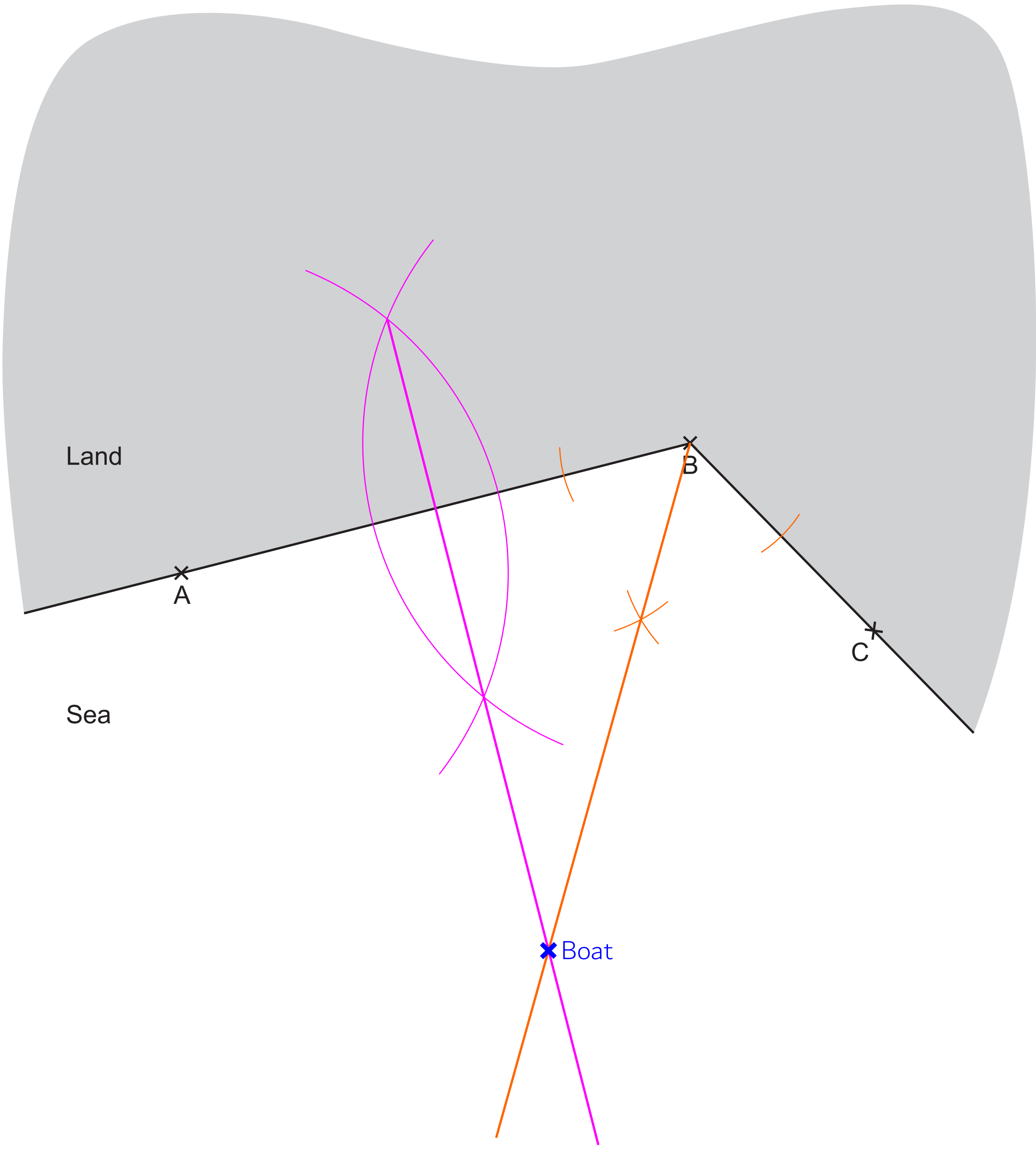
40×5 ← Multiplying the 40 minutes by the 5 hose pipes works out that 200 minutes worth of work is done

$200 \div 2$ ← Dividing the 200 minutes worth of work by the 2 hose pipes works out that it will take 100 minutes

(b) ....100.......... minutes **[2]**

5 The diagram represents a coastline.

A, B and C are lighthouses.



A boat is
- the same distance from A and B
- the same distance from AB and BC.

Using a ruler and compasses only, construct the position of the boat.
Label the position of the boat clearly. **[5]**

A perpendicular bisector of AB is shown in pink. An angle bisector of angle ABC is shown in orange. The boat is where both lines meet

6 At the end of each year, a driver records how many kilometres they have driven.

In 2021, they drove 18% more kilometres than in **2020**.
In 2022, they drove 25% more kilometres than in **2020**.

In 2022, they drove 3500 km.

(a) Kai says

I can work out how many kilometres were driven in 2020 by reducing 3500 by 25%.
3500 × 0.75 = 2625 km.

Explain why 2625 is **not** the correct number of kilometres driven in 2020.

Increasing 2625 by 25% does not give 3500

If 2625 km were driven in 2020, increasing this by 25% must give the distance driven in 2022. 2625 x 1.25 = 3281.25, not 3500

[1]

(b) Calculate the number of kilometres driven in **2021**.

$\frac{3500}{125} \times 100$

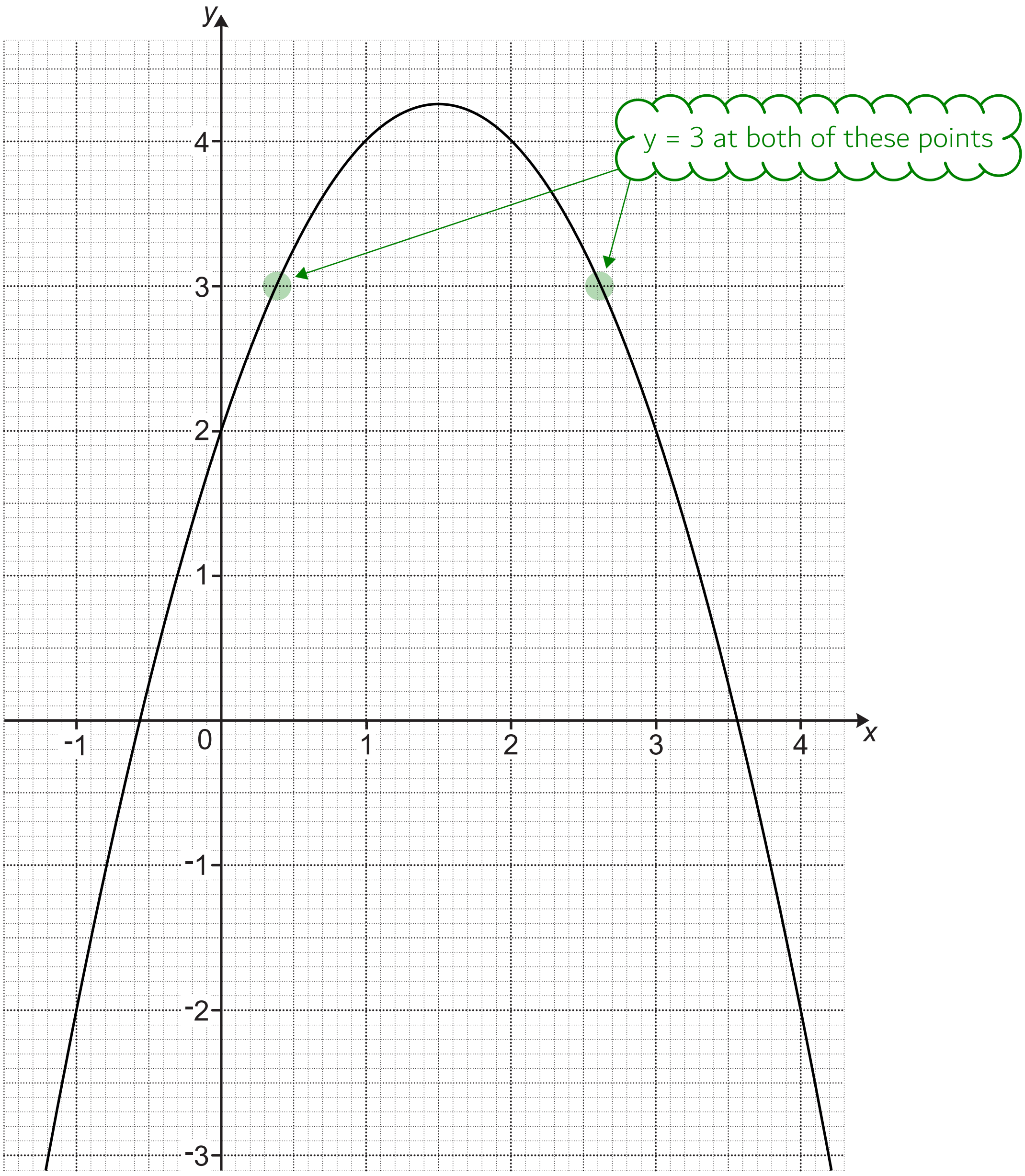
The distance in 2022 is 125% of the distance in 2020. Dividing the distance in 2022 by 125 expresses 1% of the distance in 2020. Multiplying this by 100 works out that 100% of the distance in 2020 is 2800 km

$2800 \times \frac{100+18}{100}$

Increasing the distance in 2020 by 18% gives the distance in 2021. Adding the 18% to 100% expresses the percentage it needs to increase to. Putting this over 100 converts it into a fraction. Multiplying the 2800 by this fraction increases it by 18%

(b) 3304 km **[4]**

7 The diagram shows the graph of $y = kx - x^2 + 2$, where k is an integer.



(a) Show that $k = 3$. **[2]**

The graph goes through the point (1 , 4). Substituting in the x and y-coordinates of this point into the equation. Not picking the point (0 , 2) as this would lead to multiplying the k by 0 and eliminating it making it impossible to work it out

$4 = k - 1^2 + 2$

Adding 1^2 and subtracting 2 from both sides finds that k = 3

$k = 3$

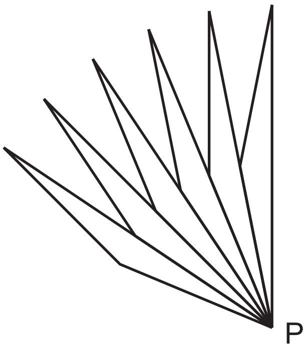
(b) Use the graph to solve $3x - x^2 + 2 = 3$.
Give your answers to **1** decimal place.

y has been replaced with 3 in the equation. So it is basically asking what x is when y = 3

(b) $x =$ ....0.4............ or $x =$ ....2.6............ **[2]**

8 Taylor designs a logo using isosceles triangles joined at a central point, P.

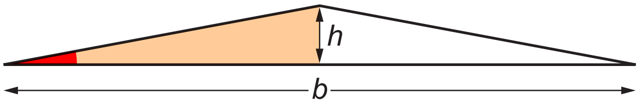
This is the start of Taylor's design.



Not to scale

The completed design will have rotational symmetry, order 60 about point P.

Each triangle has base, *b*, and height, *h*, measured in mm.



Not to scale

Calculate *h* when $b = 40$ mm.
Give your answer correct to **1** decimal place.

$360 \div 60 = 6$ — There are 360° around a point. The rotational symmetry is 60 so this means that 60 of the triangle will be around point P. Dividing the 360° by the 60 triangles works out that the angle coloured red must be 6°

$40 \div 2 = 20$ — Dividing b by 2 works out that the base length in the orange right-angled triangle is 20 mm

SOH CAH TOA — Right-angled trigonometry can be used to work out h in the orange right-angled triangle. Ticking O as we are looking for the opposite and ticking A as we have the adjacent. There are two ticks on the TOA formula triangle so this one can be used

$\tan 6 \times 20$ — From the TOA formula triangle, opposite = tan of the angle x adjacent

2.10... is rounded to 1 decimal place

2.1 .... mm **[4]**

9 On Heidi's bookcase, the ratio of fiction to non-fiction books is 2 : 3.
Heidi removes 2 fiction books from the bookcase.
The ratio of fiction to non-fiction books is then 5 : 8.

How many books are left on the bookcase in total?

16:24
15:24

The number of non-fiction books is the same before and after the 2 fiction books are removed. So making the same number of parts in both ratios for the non-fiction books makes them compatible as 1 part in the first ratio will be worth the same as 1 part in the second ratio. Multiplying both sides of the first ratio by 8 and multiplying both sides of the second ratio by 3 makes the number of parts for non-fiction 24 in both ratios

15+24

Adding the 15 parts and 24 parts in the second ratio works out that 39 parts of the ratio represents the number of books left on the bookcase

39x2

The 16 parts for fiction in the first ratio was reduced by 1 part to 15 parts in the second ratio. So 1 part of the ratio is worth 2 books. Multiplying the 39 total parts in the second ratio by the 2 books each part represents works out how many books are left on the bookcase

..........78.......... books **[4]**

10 (a) Show that 95 is **not** a prime number.

$95 = 5 \times 19$

The calculator can be used to express 95 as a product of prime factors. As other primes can be multiplied to give 95, it is not prime. Prime numbers only have two factors (themselves and 1) and 95 has both 5 and 19 as factors as well as 1 and 95

[1]

(b) (i) 2000 and 8750 are written below as the product of their prime factors.

$$2000 = 2^4 \times 5^3$$
$$8750 = 2 \times 5^4 \times 7$$

Find the highest common factor (HCF) of 2000 and 8750.

2×5^3

The highest common factor can be found by multiplying the lowest power of each prime in both lists

(b)(i) .... 250 .... **[2]**

(ii) Write 2×10^{12} as a product of its prime factors.

$2 \times (2 \times 5)^{12}$

Expressing 10 as a product of prime factors using the calculator gives 2 x 5

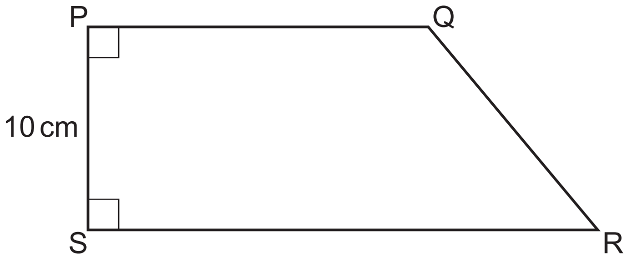
$2 \times 2^{12} \times 5^{12}$

Raising everything in the bracket to the power of 12

(ii) .... $2^{13} \times 5^{12}$ .... **[2]**

$2^1 \times 2^{12} = 2^{13}$

11 The diagram shows a quadrilateral, PQRS.



Not to scale

PS = 10 cm.
Angle QPS = Angle PSR = 90°.

SR is 6 cm longer than PQ.
The area of quadrilateral PQRS is A cm^2.

Write a simplified expression for the length PQ in terms of A.
You must show your working.

$\frac{1}{2}(x+x+6)\times 10$ ← Making an expression for the area of the trapezium.Area of trapezium = 1/2 (a + b) x h, where a and b are the parallel sides and h is the perpendicular distance between them. Letting x be side PQ. Side SR must be x + 6. PQ and SR are the parallel sides. h is 10

$5(2x+6)$ ← Simplifying the expression by doing the 1/2 multiplied by the 10 to give 5 and collecting like terms in the bracket

$10x+30=A$ ← Expanding the bracket then setting the expression equal to the actual area A

$10x=A-30$ ← Rearranging to make x the subject. First subtracting 30 from both sides to get the term involving x on its own

Dividing both sides by 10 makes x the subject. The right side of the this equation is the expression of PQ in term of A

$\frac{A-30}{10}$ .... **[5]**

12 A box contains 200 matches, correct to the nearest ten matches.

(a) Complete the error interval for n, the number of matches in the box.

$200 \pm \frac{10}{2}$ ← Adding and subtracting half of the resolution (which is 10) works out the upper and lower bound

(a) ......195...... $\leqslant n \leqslant$ ......204...... **[2]**

There needs to be 1 less than 205 as this rounds to 210 and the interval is stating that it can be equal to the number on the right

(b) The box is a cuboid with

- length 7 cm, correct to the nearest cm
- width 5 cm, correct to the nearest cm
- volume 248 cm^3, correct to the nearest cm^3.

Show that the smallest possible height of the box is 6 cm. **[3]**

$$\left(7+\frac{1}{2}\right)\times\left(5+\frac{1}{2}\right)\times H = 248-\frac{1}{2}$$

← Volume of cuboid = length x width x height. Using the upper bounds of both of the length and width and the lower bound of the volume as this will mean that the height must be the smallest it can be. Using H to represent the height

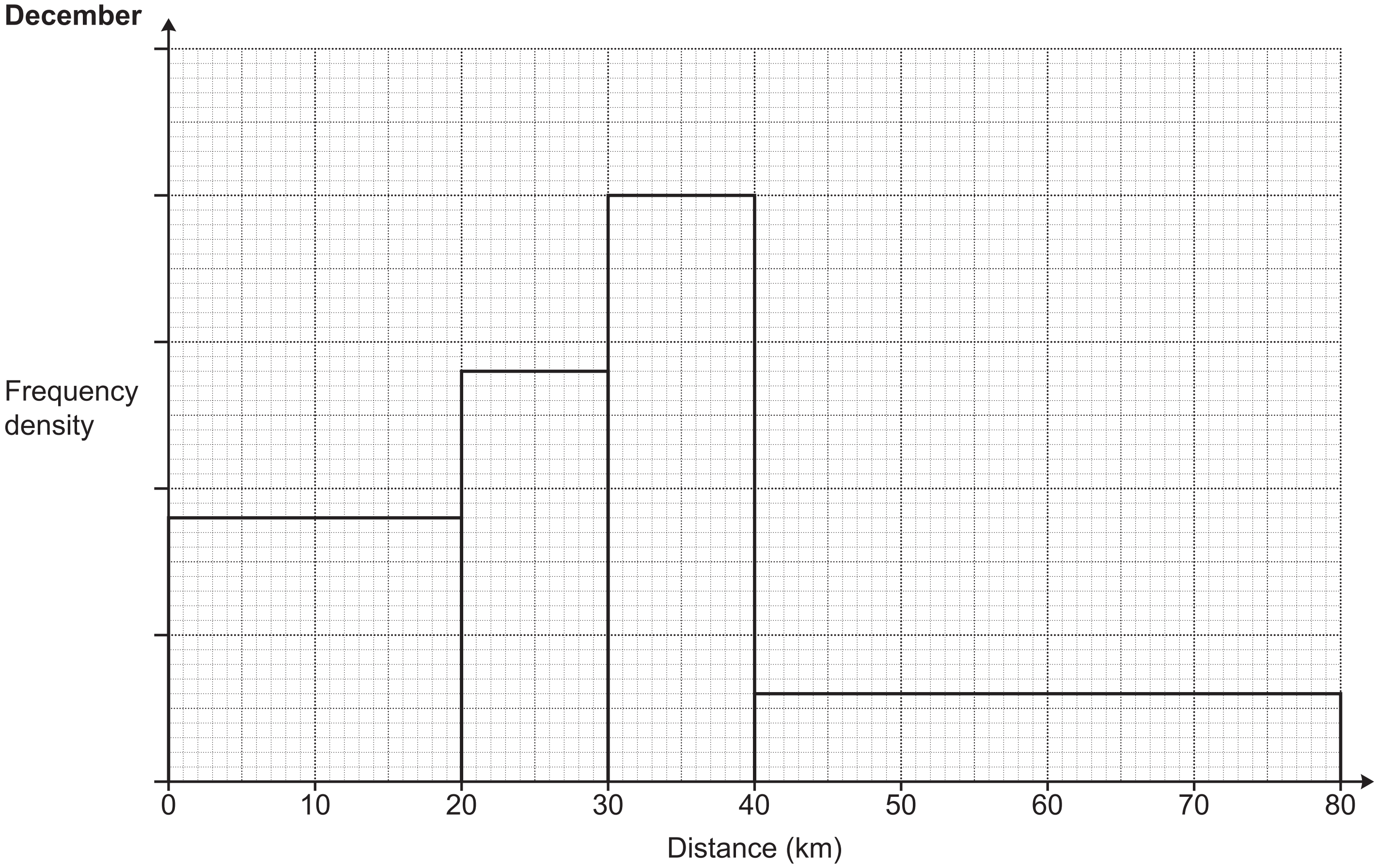
$$H = \frac{248-\frac{1}{2}}{\left(7+\frac{1}{2}\right)\times\left(5+\frac{1}{2}\right)}$$

← Rearranging to make H the subject by dividing both sides by everything H was multiplied by

$$= 6$$

← Putting the right side of the equation into the calculator shows than the lower bound of H is 6 cm. This is the smallest possible height of the box

13 A running club records the distances run by each member during December.
The results are shown in this histogram.



(a) 18 members run less than 20 km.

(i) Work out the number of members who run more than 30 km.

The method is shown on the next page

(a)(i) ....32............ **[3]**

(ii) Finley says

To estimate the range, I subtracted the smallest possible value from the largest possible value. So, 80 – 0 = 80 km.

Explain why Finley's method is likely to overestimate the true value of the range.

The data is grouped .... So the largest value is probably less than 80 and the smallest value is possibly more than 0 ....

.... **[1]**

$20 \times 18x = 18$

Frequency on a histogram is the area of each bar. So multiplying the class width by the frequency density gives the frequency. The first bar represents the members who run less than 20 km. 20 is the class width of the first bar as it goes from 0 to 20. The frequency density is 18x, where x is the value of one small box

$x = \frac{18}{20 \times 18}$

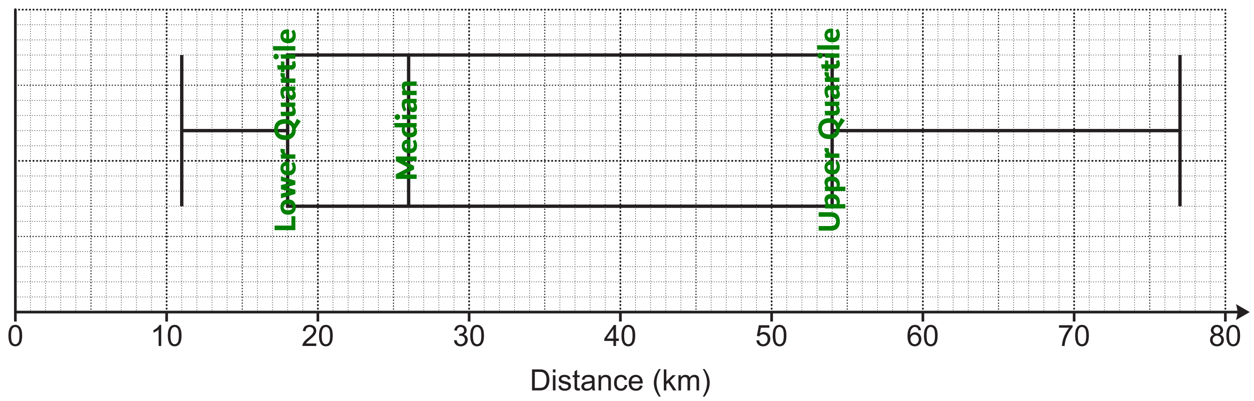
Rearranging to find x by dividing both sides by everything x was multiplied by. This finds that each small box is worth 0.05 on the y-axis

$10 \times 40 \times 0.05 + 40 \times 6 \times 0.05$

Frequency is the area of each box and the third and fourth bars represent the members who run more than 30 km. The class width of the third bar is 10 as it goes from 30 to 40. The frequency density of the third bar is 40 x 0.05 as it is 40 boxes tall and each box is worth 0.05. The class width of the fourth bar is 40 as it goes from 40 to 80. The frequency density is 6 x 0.05 as it is 6 boxes tall and each box is worth 0.05. Adding the frequencies of the third and fourth bar works out how many members run more than 30 km

(b) This box plot shows the distribution of the distance run by each member of the running club during July.

July



During **December**,

- the median distance run was 30 km
- the interquartile range of the distance run was 20 km.

Make **two** comparisons between the distances run during December and the distances run during July.
Include values to support your comparisons.

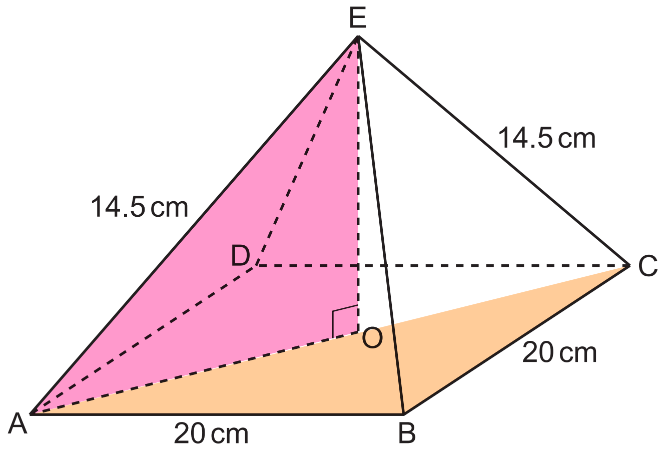
54 − 18 ← Subtracting the lower quartile from the upper quartile works out that the interquartile range for July was 36

1. July's median of 26 was less than December's median

2. July's interquartile range of 36 was greater than December's interquartile range

[4]

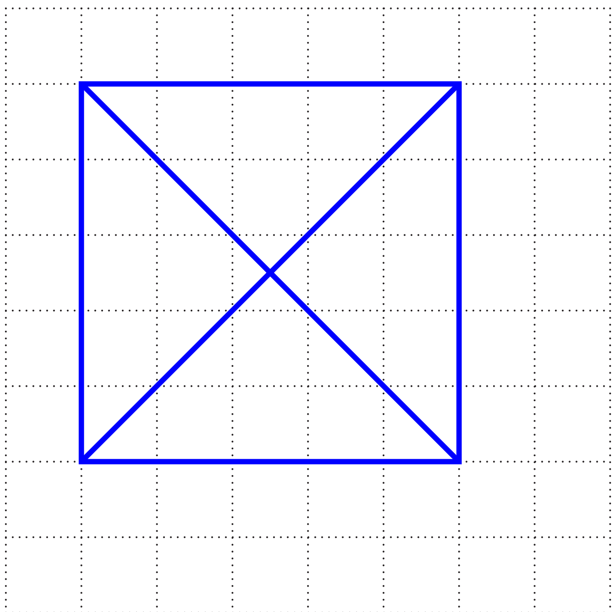
14 The diagram shows a square-based pyramid ABCDE.
O is the centre of the base.



The pyramid has base length 20 cm and each sloping edge has length 14.5 cm.

(a) Draw the plan view of the pyramid on the one-centimetre grid below.

Scale: 1 cm represents 4 cm.



[2]

The plan is the view from above. Drawing the square base then connecting the opposite corners. The side length of the square must be 5 cm as this multiplied by the 4 cm each 1 cm represents is 20 cm, which is the actual length of the square

(b) Calculate the volume of the pyramid.
You must show your working.

[The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ perpendicular height]

$20^2 + 20^2 = AC^2$ ← Using Pythagoras' Theorem in the orange right-angled triangle. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side

$AC = \sqrt{20^2 + 20^2}$ ← Rearranging to make side AC the subject by square rooting both sides

$20\sqrt{2} \div 2$ ← Halving side AC finds AO, which is a side in the pink right-angled triangle

$(10\sqrt{2})^2 + OE^2 = 14.5^2$ ← Using Pythagoras' Theorem in the pink right-angled triangle. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side

$OE = \sqrt{14.5^2 - (10\sqrt{2})^2}$ ← Rearranging to make OE the subject by subtracting $(10\sqrt{2})^2$ from both sides then square rooting both sides

$\frac{1}{3} \times 20^2 \times \frac{\sqrt{41}}{2}$ ← Working out the volume of the pyramid. The area of the base is 20^2 as it is a square and area of square = length^2. The perpendicular height is length OE

Rounded 426.87... to 1 decimal place

(b) ....426.9.......... cm^3 **[5]**

15 Two bottles are mathematically similar.

The small bottle holds 0.5 litres and has a height of 35 cm.
The large bottle holds 2 litres.

Calculate the height of the large bottle.

$2 \div 0.5$ ← Dividing the volume of the large bottle by the volume of the small bottle works out that the volume scale factor is 4

$35 \times \sqrt[3]{4}$ ← Cube rooting the volume scale factor gives the length scale factor. Multiplying the height of the small bottle by this works out the height of the large bottle

Rounding the answer of 55.55... to 1 decimal place →

.......................55.6....................... cm **[4]**

16 The price of a seat on a flight, £P, is given by

$P = 49 \times 1.009^n$

where n is the number of seats already sold on this flight.

(a) Write down the percentage increase in price of the second seat sold compared to the first seat sold.

The formula is basically saying that the price is multiplied by 1.009 for each seat already sold. Multiplying this by 100 converts it into 100.9%, which is an increase of 0.9%

(a) ....0.9........ % **[1]**

(b) Show that the price of the 40th seat sold is less than £70. **[2]**

49×1.009^{39} ← For the 40th seat, 39 seats are already sold. So substituting 39 for n in the formula

69.49 ← Rounding the result to the nearest penny gives £69.49, which is less than £70

17 The kth term of a sequence is r^k, where $r \neq 0$.
The sixth term is equal to three times the second term.

Find the value of r, giving your answer correct to **3** decimal places.

$r^6 = 3r^2$ ← On the 6th term k is 6. On the 2nd term k is 2. Multiplying the 2nd term by 3 must be equal to the 6th term

$r^4 = 3$ ← Dividing both sides by r^2. $r^6 \div r^2 = r^{6-2} = r^4$

$r = \sqrt[4]{3}$ ← Fourth rooting both sides gets rid of the power of 4 on the left and finds r

1.3160... is rounded to 3 decimal places

$r =$ ....1.316........ **[4]**

18 (a) Describe fully the graph of $x^2 + y^2 = 20$.

The general equation of a circle with centre (0 , 0) is $x^2 + y^2 = r^2$, where r is the radius. Square rooting the 20 finds the radius

Circle, centre (0 , 0), radius $\sqrt{20}$

.... **[3]**

(b) The graph of $y = 3x + 10$ intersects the graph of $x^2 + y^2 = 20$ at two points.

Use an algebraic method to work out the coordinates of the two points.
You must show your working.

Doing simultaneous equations works out the points of intersection of two equations

$(3x+10)^2$ ← Substituting 3x + 10 for y in y^2

$x^2+9x^2+60x+100=20$ ← Expanding out $(3x + 10)^2$ by squaring the first term, doubling the product of the two terms and squaring the last term. Substituting this for y^2 in the second equation

$10x^2+60x+80=0$ ← Collecting like terms and subtracting 20 from both sides to put it into the quadratic form: $ax^2 + bx + c = 0$

$x=\frac{-60\pm\sqrt{60^2-4\times10\times80}}{2\times10}$ ← $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← Solving the equation using the quadratic formula

$x=-2$, $y=4$ or $x=-4$, $y=-2$ ← Finding the values of x and substituting these into the equation y = 3x + 10 to find y for each value

(b) (-2 , 4) and (-4 , -2) **[6]**

19 **(a)** Show that $\sqrt{11} \times \sqrt{22} = 11\sqrt{2}$. **[1]**

$\sqrt{11} \times \sqrt{11} \times \sqrt{2}$ ← Using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ to split $\sqrt{22}$ into $\sqrt{11} \times \sqrt{2}$

$11\sqrt{2}$ ← $\sqrt{11} \times \sqrt{11} = \sqrt{11^2} = 11$

(b) Show that $\frac{\sqrt{11}}{13+\sqrt{22}}$ can be written in the form $\frac{a\sqrt{11} - 11\sqrt{2}}{b}$ where a and b are integers. **[4]**

$\frac{\sqrt{11}(13-\sqrt{22})}{(13+\sqrt{22})(13-\sqrt{22})}$ ← Rationalising the denominator by multiplying both the numerator and denominator $13 - \sqrt{22}$ (the same as the denominator but the plus becomes a subtraction)

$\frac{13\sqrt{11} - 11\sqrt{2}}{169 - 13\sqrt{22} + 13\sqrt{22} - 22}$ ← Expanding the brackets on the numerator and denominator

$\frac{13\sqrt{11} - 11\sqrt{2}}{147}$ ← Writing the final answer in the desired form

20 **(a)** Write $(2x-5)(x+4)$ in the form $2(x+a)^2-b$.

You must show working.

$2x^2+8x-5x-20$ ← Expanding the brackets

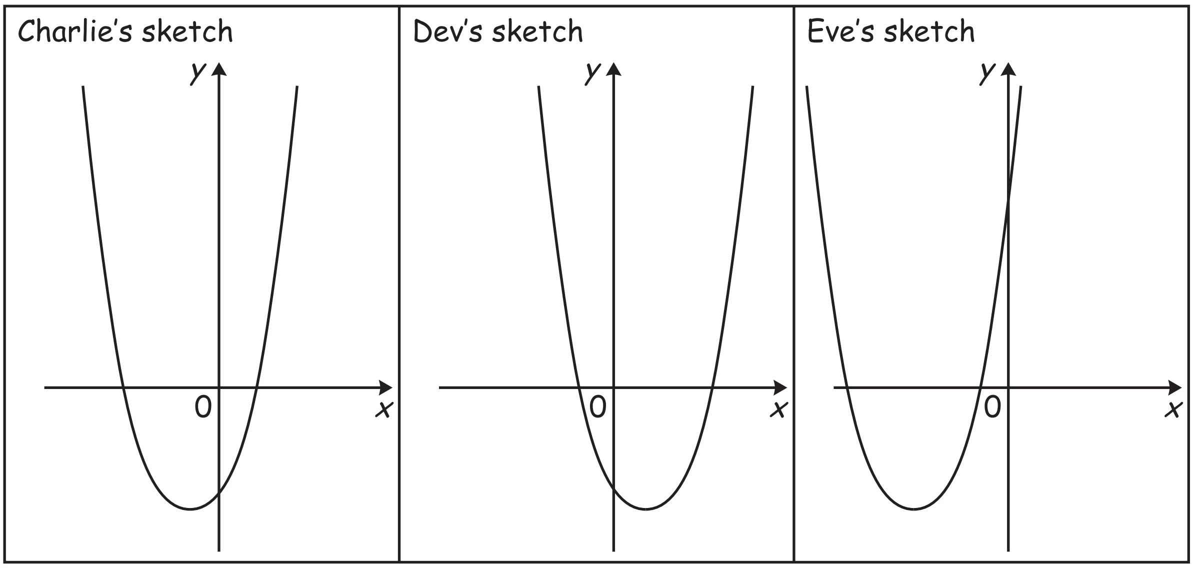
$2x^2+3x-20$ ← Simplifying by collecting like terms

$2(x^2+\frac{3}{2}x)-20$ ← Bringing out 2 as a factor on the first two terms so that it will be able to be put into the completed the square form

$2(x+\frac{3}{4})^2-20-(\frac{3}{4})^2\times 2$ ← Completing the square by halving the coefficient of x and putting this in a bracket with x and squaring it. When expanding out this bracket and multiplying it by the 2, which will give $(3/4)^2$ x 2 too much so this must be subtracted from the -20

(a) $2(x+\frac{3}{4})^2-\frac{169}{8}$ **[5]**

(b) Charlie, Dev and Eve all attempt to sketch the graph of $y = (2x - 5)(x + 4)$.



Whose sketch is the most accurate?
Write down the properties of the graph that you used in making your decision.

Charlie's because The x-coordinate of the turning point is negative and the y-intercept is negative

The turning point can be found using the completed the square form. The minimum the squared bracket can be is 0 and this happens when x = -3/4. The y-intercept is -20 which can be seen from when the brackets were expanded into the quadratic form $ax^2 + bx + c$, where c is the y-intercept

[2]

END OF QUESTION PAPER