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Worked Solutions

H

Monday 13 June 2022 – Morning

GCSE (9–1) Mathematics

J560/06 Paper 6 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space, use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answers should be supported with working. Marks might be given for using a correct method even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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Answer **all** the questions.

1 A student rolls two fair four-sided dice each numbered 1, 2, 3 and 4.
They add the two scores together.

(a) Complete the sample space diagram to show the possible outcomes from the dice.

Second dice

Total	1	2	3	4
First dice 1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

1 + 4 = 5

3 + 4 = 7

[2]

(b) Find the probability that the student gets an even total.

The even totals are 2, 4, 6 and 8. 8 out the of the 16 possible totals are even

(b) $\frac{8}{16}$ **[1]**

(c) Find the probability that the student gets the same score on each dice.

The totals highlighted in green have the save score on each dice. This is 4 out of the 16 possible totals

(c) $\frac{4}{16}$ **[1]**

2 The circumference of a circle is 23 cm.

Show that the area of the circle is 42.1 cm^2, correct to **3** significant figures. **[4]**

$\pi d = 23$ ← π x diameter = circumference

$d = \frac{23}{\pi}$ ← Dividing both sides by π finds the diameter

$r = \frac{23}{2\pi}$ ← Dividing the diameter by 2 finds the radius

$\pi \times \left(\frac{23}{2\pi}\right)^2 = 42.09...$ ← Area of circle = π x radius^2

3 Light from the Sun travels 1 kilometre in 3.3×10^{-6} seconds.
The distance from the Sun to the Earth is 1.5×10^{8} kilometres.

How long does it take light to travel from the Sun to the Earth?
Give your answer in minutes and seconds.

$3.3 \times 10^{-6} \times 1.5 \times 10^{8}$ ← The 1.5×10^{8} km is 1.5×10^{8} times further than 1km. Therefore it will take 1.5×10^{8} times the time taken to do 1km

$495 \div 60$ ← Converting the 495 seconds worked out in the previous calculation into minutes by dividing by 60 as there are 60 seconds in a minute

8.25 minutes is converted into minutes and seconds by using the calculator to express it as a sexagesimal

..........8.......... minutes ....15.......... seconds **[4]**

4 You are given that

$$\frac{10a^k \times a^8}{ma^5} = \frac{2a^7}{5}$$

where k and m are integers.

Find the value of k and the value of m.

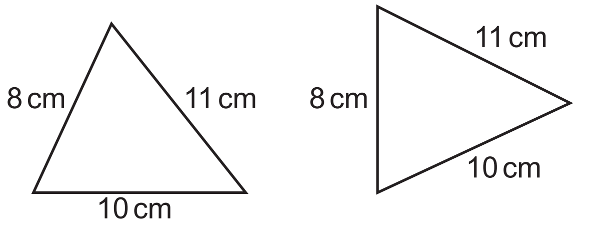
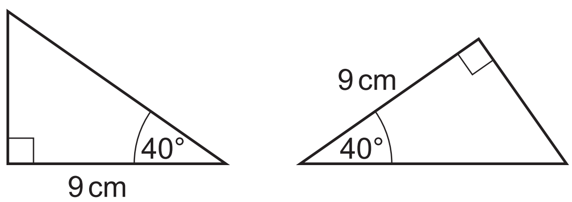
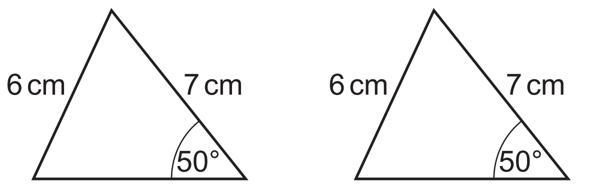
$b^x \times b^y = b^{x+y}$ and $b^x \div b^y = b^{x-y}$. So $a^8/a^5 = a^3$ then this must be multiplied by a^4 to get a^7 on the right. Therefore k is 4. The 10 is divided by 5 to get 2 on the numerator so m must have been divided by 5 to get 5. The opposite of dividing by 5 is multiplying by 5 and 5 x 5 is 25, so this must be m

$k =$ ....4.......... and $m =$ ....25.......... **[4]**

Turn over

5 In each row of the table there are two triangles.

State whether the two triangles are congruent or not.
If they are congruent state a reason from SSS, SAS, ASA or RHS.

Triangles	**Congruent (yes/no)**	**Reason (SSS/SAS/ASA/RHS)**
Not to scale 	Yes	SSS
Not to scale 	Yes	ASA
Not to scale 	No	

[3]

The first two are definitely congruent as all three sides are the same (SSS).

The second two are definitely congruent as there are two angles and a side the same (ASA). The side which is the same is opposite the same angle

The third two could be congruent however it is not certain as none of the reasons have been met as for SAS the angle needs to be between the two sides which are the same. The other sides and angles may not be the same

6 The mass of a stone is 680 g.
The density of the stone is 1.6 g/cm^3.

(a) Work out the volume of the stone.

$d \, {}^{m} \, v$ ← Writing the formula triangle for density, mass, volume

$680 \div 1.6$ ← From the formula triangle, volume = mass ÷ density

(a) ....425........ cm^3 **[2]**

(b) Write 1.6 g/cm^3 in kg/m^3.

$\frac{1.6 \div 1000}{1 \div 100^3}$ ← 1.6g/cm^3 means 1.6g per 1cm^3. There are 1000g in 1kg so dividing the 1.6g by 1000 converts it into kg. There are 100cm in 1m and the unit is cubed so dividing 1cm^3 by 100^3 converts it into m^3. kg/m^3 can be worked out by dividing the mass in kg by the volume in m^3

(b) ....1600........ kg/m^3 **[1]**

7 **(a)** Multiply out and simplify.

$(x-4)(x+5)$

$x^2+5x-4x-20$ ← Expanding the brackets

Collecting like terms to simplify →

(a) ....x^2+x-20........ **[2]**

(b) Factorise.

x^2-25

Difference of two squares can be used: $A^2 - B^2 = (A + B)(A - B)$ →

(b) ....$(x+5)(x-5)$........ **[1]**

8 1600 fish are released into a new lake which has no fish.
The number of fish is expected to increase by 5% each year.

(a) The table shows the expected number of fish in the lake at the end of 1 year and at the end of 2 years.

Complete the table.
Round your answers to the nearest integer.

$1764 \times \frac{100+5}{100}$

$1852 \times \frac{100+5}{100}$

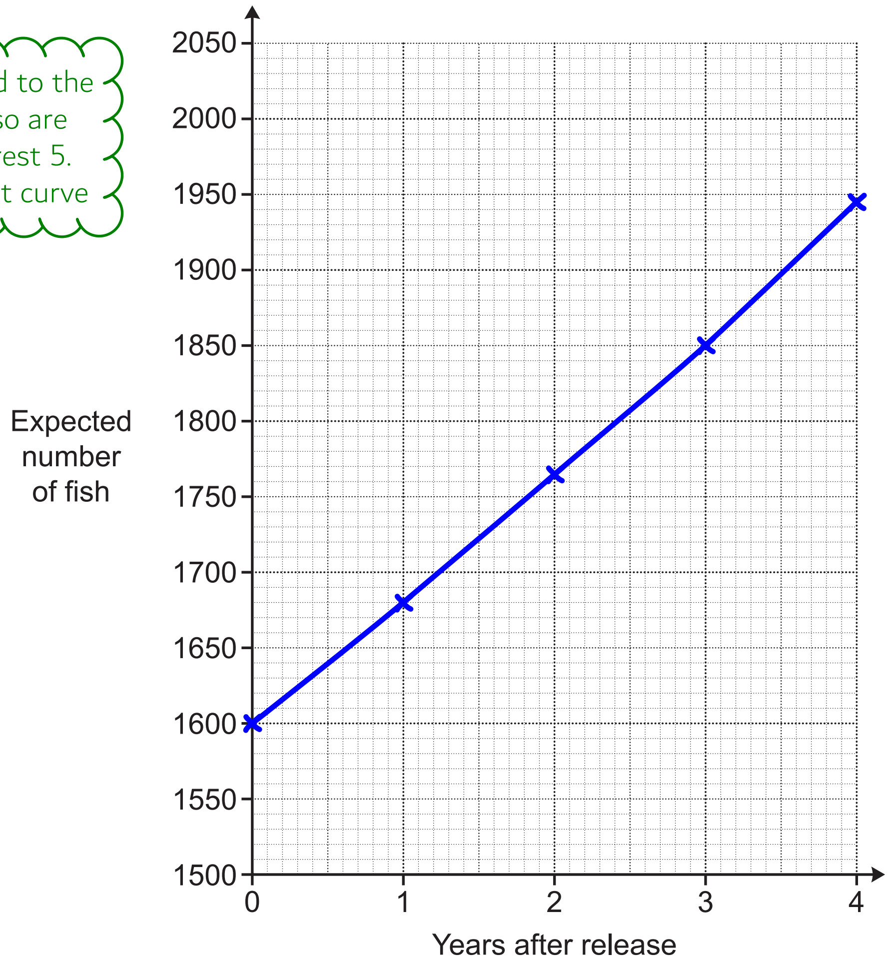
100 + 5 expresses the percentage it will increase to each year. Putting this over 100 converts it into a fraction, which when multiplied by increases by 5%. The value of 1852.2 is rounded to 1852 and the value of 1944.6 is rounded to 1945

Years after release	0	1	2	3	4
Expected number of fish	1600	1680	1764	1852	1945

[3]

(b) Use the table to draw a suitable graph to show the expected number of fish in the lake.

The points are plotted to the nearest half a box, so are rounded to the nearest 5. The graph has a slight curve



[3]

(c) A maximum of 2000 fish can live in the lake.

What effect would you expect this to have on the shape of your graph after 4 years?

Increase up to 2000 and then level off

....

.... **[2]**

9 A garage is trying to sell a car.
The price of the car is normally £18 000.
In a sale, the price of the car is reduced by 30%.
As a special offer, the sale price is then reduced by *r*%.
The special offer price is £9450.

Find the value of *r*.
You must show your working.

$18000 \times \frac{100-30}{100}$

100 - 30 expresses the percentage it decreases to in the sale. Putting this over 100 converts it into a fraction. Multiplying the £18000 by this reduces it by 30% to work out that the sale price is £12600

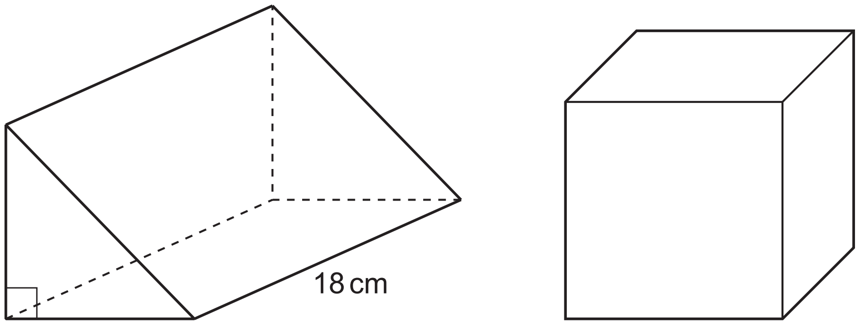
$\frac{9450-12600}{12600} \times 100$

Working out the percentage change between the sale price and the special offer price. Subtracting the sale price from the special offer price expresses the change. Putting this over the sale price expresses the change as a fraction. Multiplying this by 100 converts it into a percentage and finds that the percentage change is -25%

r is the percentage reduction so the negative is ignored

r = 25 **[5]**

10 The diagram shows a triangular prism and a cube.
The ends of the prism are right-angled triangles with base 16 cm and height 12 cm.
The prism is 18 cm long.



The volume of the prism is equal to the volume of the cube.

Find the **surface area of the cube**.
You must show your working.

$\frac{1}{2} \times 16 \times 12 \times 18$ ← Volume of prism = cross sectional area x length. The cross section is a triangle. Area of triangle = 1/2 x base x height. The base is 16cm and the height is 12cm. The length of the prism is 18cm. So this works out that the volume of the prism is 1728cm^3. This is also the volume of the cube

$\sqrt[3]{1728}$ ← Volume of cube = length^3. So cube rooting the volume of the cube works out that its side length is 12cm

12^2 ← Area of square = length^2. So squaring the side length of the cube works out that the area of one of its square faces is 144cm^2

144×6 ← There are 6 square faces on a cube so multiplying the area of one of its square faces by 6 works out the surface area of the cube

.........864......... cm^2 **[6]**

11 Amir, Beth and Charlie work in a cafe.
Customers give spare change as tips.
At the end of each week, Amir, Beth and Charlie share the total amount of tips between them in the ratio matching the number of hours they worked that week.

This week:

- Amir's share of the tips was £25.40.
- Beth worked twice as many hours as Amir.
- Charlie worked 5 more hours than Amir.
- The total hours worked by Amir, Beth and Charlie was 85 hours.

Calculate the total amount of tips received this week.
You must show your working.

$A+2A+A+5=85$ ← Let A be the number of hours Amir worked. Beth worked 2A hours as Beth worked twice as many hours as Amir. Charlie worked A + 5 hours as Charlie worked 5 more hours than Amir. Adding together the expressions for the number of hours worked by Amir, Beth and Charlie must be equal to the 85 hours worked in total

$4A=80$ ← Collecting together the A. A + 2A + A = 4A. Then subtracting 5 from both sides to get the A term on its own

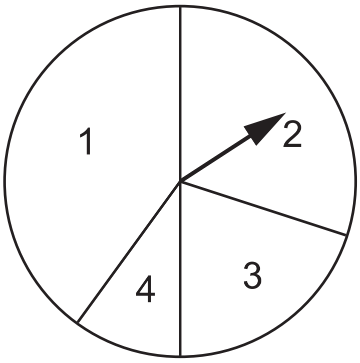
$A=20$ ← Dividing both sides by 4 gets A on its own. So Amir worked 20 hours

$25.40\div20$ ← Dividing the £25.40 Amir got by the 20 hours he worked works out that 1 hour of work would get £1.27

1.27×85 ← Multiplying the £1.27 for each hour of work by the 85 hours worked in total works out the total amount of money from tips

£ ....107.95...................... **[6]**

12 A student has a spinner with sectors numbered 1, 2, 3 and 4.



The table shows the probability of each score.

Score	1	2	3	4
Probability	0.4	0.3	0.2	0.1

The student spins the spinner twice.

Calculate the probability that the student gets the same score on each spin.

0.4×0.4+0.3×0.3+0.2×0.2+0.1×0.1

1 AND 1 OR 2 AND 2 OR 3 AND 3 OR 4 AND 4. AND means to multiply the probabilities. OR means to add the probabilities

.......................0.3....................... **[4]**

13 A car registration plate has two letters, a number from 10–99 and three letters.
For example:

AB56 CDE

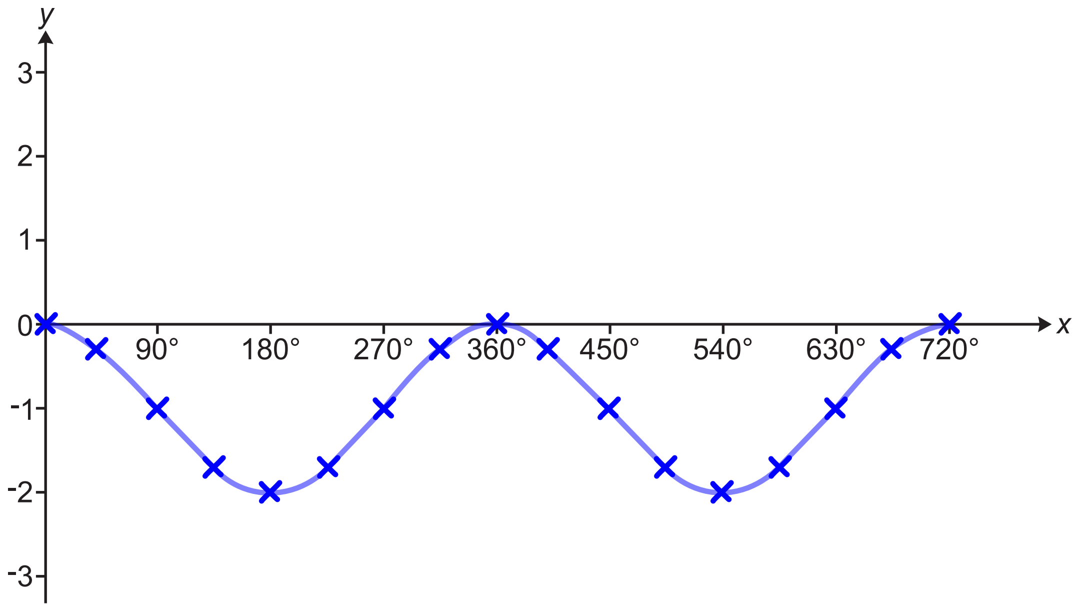
The letters I and O are not used, leaving 24 possible letters.

Show that there are approximately 720 million possible car registration plates of this form. **[4]**

$24 \times 24 \times 9 \times 10 \times 24 \times 24 \times 24 = 716636160$

Using the product rule for counting. Multiplying the number of outcomes for each letter or digit works out the total number of outcomes. There are 24 possible letters for each letter, 9 possibilities for the first digit and 10 possibilities for the second digit of the number from 10-99

14 Sketch the graph of $y = \cos x - 1$ for $0° \leqslant x \leqslant 720°$.



[3]

Using table mode. f(x) = cos(x) - 1. Start: 0. End: 720. Step: 45

This gives a table of values for the graph for values of x from 0 to 720 going up in 45s

15 80 cyclists take part in a race.
A summary of their times is shown in the table.

Time (*t* minutes)	**Frequency**
$20 < t \leqslant 25$	5
$25 < t \leqslant 30$	15
$30 < t \leqslant 35$	24
$35 < t \leqslant 40$	25
$40 < t \leqslant 45$	7
$45 < t \leqslant 50$	4

(a) Complete the cumulative frequency table.

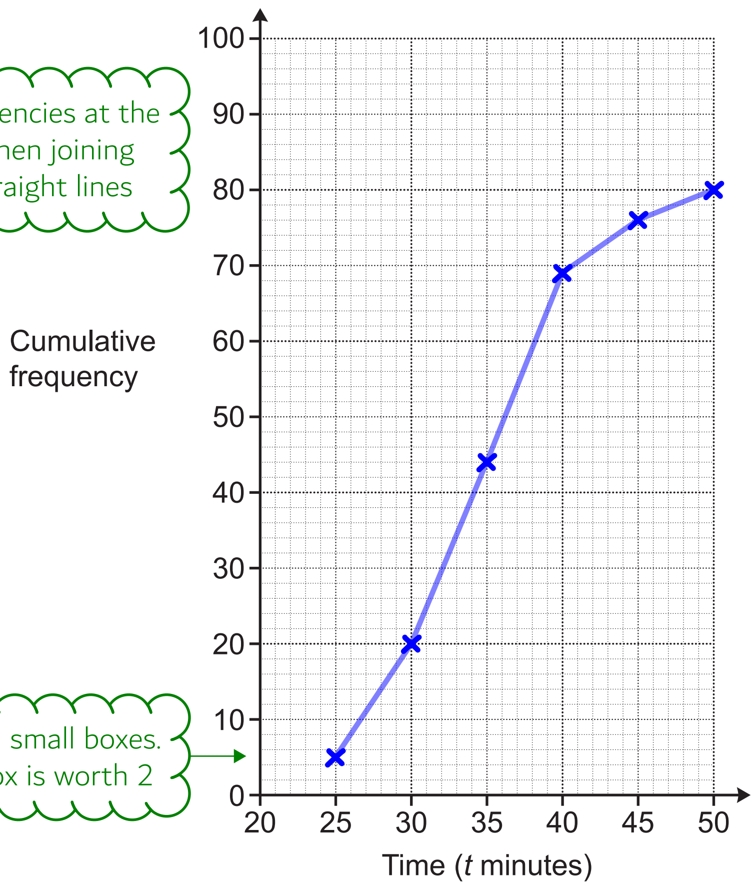
Time (*t* minutes)	**Cumulative frequency**	
$t \leqslant 25$	5	
$t \leqslant 30$	20	5 + 15 = 20
$t \leqslant 35$	44	20 + 24 = 44
$t \leqslant 40$	69	44 + 25 = 69
$t \leqslant 45$	76	69 + 7 = 76
$t \leqslant 50$	80	76 + 4 = 80

[2]

Adding the frequencies up as they go

(b) Draw the cumulative frequency graph to show the information.

Plotting the cumulative frequencies at the end point of each interval then joining them up with a series of straight lines



The scale goes up 10 over 5 small boxes. 10 ÷ 5 = 2 so each small box is worth 2

[3]

(c) Reece makes two comments about the times taken to complete the race.

For each comment, decide if Reece is right or wrong and give a reason for your answer.

(i) $\frac{3}{4}$ of the 80 cyclists took more than 30 minutes to complete the race.

Reece is right because 60 took more than 30 minutes and 60/80 = 3/4

Reading up from 30 minutes to the line then across finds that 20 cyclists took 30 minutes or less. 80 - 20 = 60 so 60 cyclists took more than 30 minutes

[2]

(ii) The longest time that any of the 80 cyclists took to complete the race must have been 50 minutes.

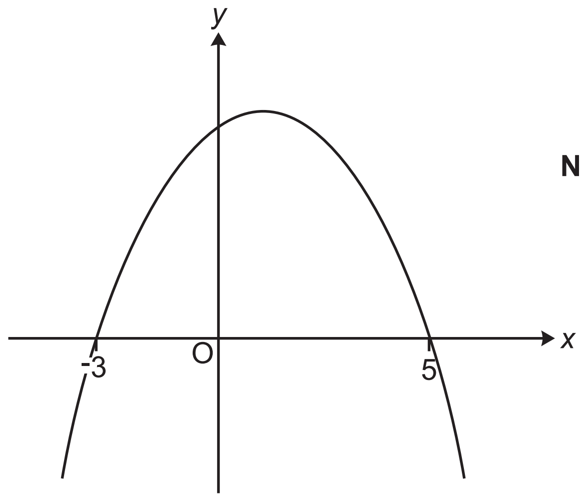
Reece is wrong because it could be less than 50 minutes

The final interval is 45 < t ≤ 50 so the longest time could have been 50 minutes however it could also be less

[1]

Turn over

16 Frankie sketches this quadratic graph.



Not to scale

Frankie says

The *y*-intercept is 15.

(a) Show that what Frankie says could be correct. **[3]**

$y = -(x+3)(x-5)$ ← It could be a $-x^2$ graph. When y = 0, x = -3 or x = 5 so the factorised form could be as shown

$= -(0+3)(0-5)$ ← Substituting x for 0 to find the y-intercept

$= -3 \times -5$

$= 15$ ← This shows that the y-intercept could be 15

(b) Explain why what Frankie says may **not** be correct.

$y = -2(x+3)(x-5)$ ← It could be a $-2x^2$ graph. When y = 0, x = -3 or x = 5 so the factorised form could be as shown

$= -2(0+3)(0-5)$ ← Substituting x for 0 to find the y-intercept

$= -2 \times 3 \times -5$

$= 30$ ← This shows that the y-intercept could be 30

[2]

17 Blake is asked to write 15 552 000 000 as a product of prime factors in index form.
Blake writes

$15552000000 = 2^7 \times 5^6 \times 6^5$.

(a) Explain Blake's mistake.

6 is not prime

Prime numbers only have two factors, themselves and 1. 6 also has 2 and 3 as factors so is not prime

[1]

(b) Write 15 552 000 000 as a product of prime factors in index form.

15552×1000000

$2^6 \times 3^5 \times 2^6 \times 5^6$

The calculator cannot give the number as a product of prime factors. So splitting the number into two smaller numbers then getting the calculator to give both of these as a product of prime factors

$2^6 \times 2^6 = 2^{12}$

(b) $2^{12} \times 3^5 \times 5^6$ **[2]**

(c) You are given that $140000 = 2^5 \times 5^4 \times 7$.

Find the highest common factor (HCF) of 15 552 000 000 and 140 000.

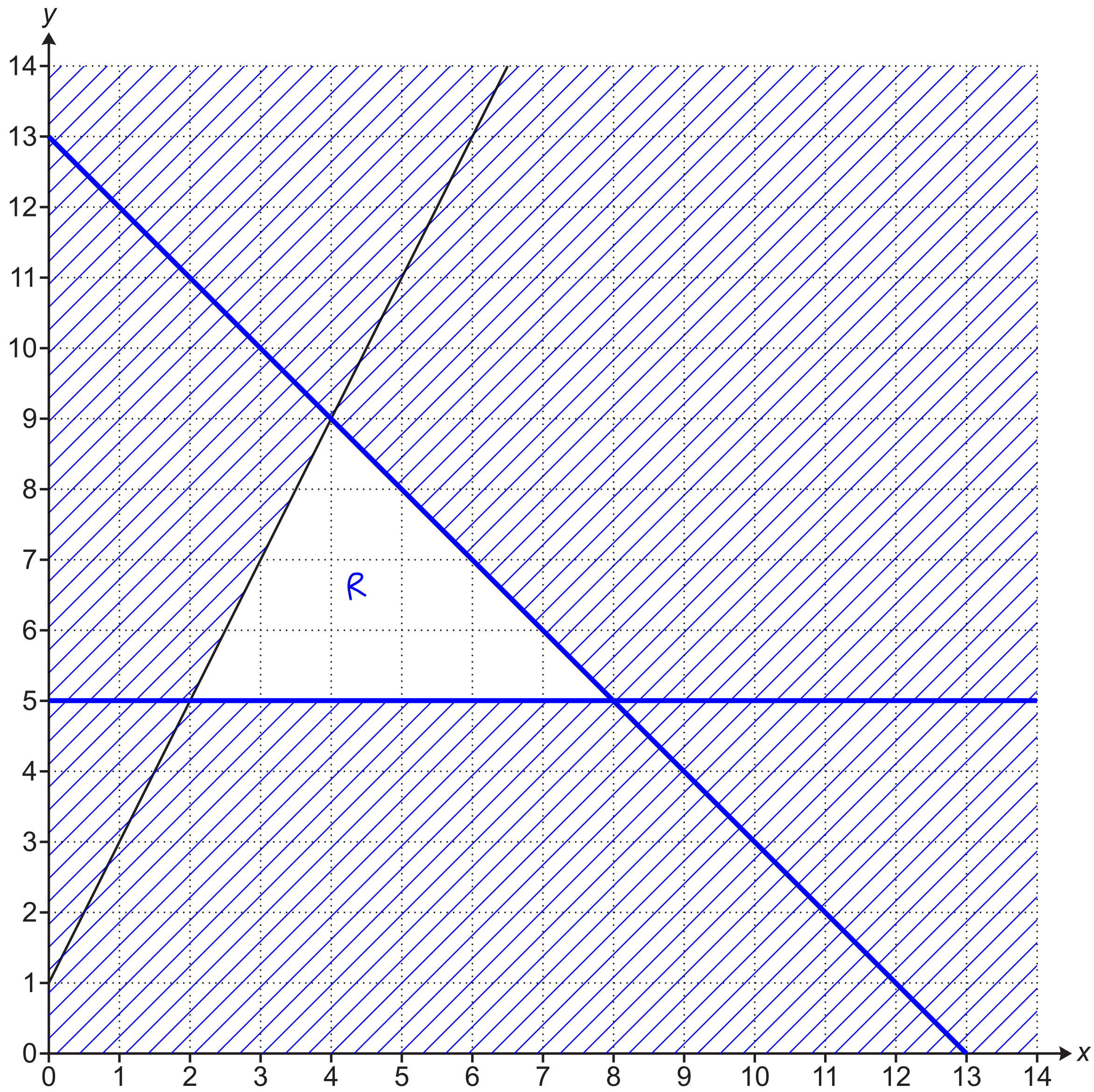
$2^5 \times 5^4$

Multiplying the lowest power of each prime number in both lists give the highest common factor

(c) 20000 **[2]**

Turn over

18 The graph of $y = 2x + 1$ is drawn on this one centimetre grid.



The region **R** satisfies these inequalities.

$y \leqslant 2x + 1$
$y \geqslant 5$
$x + y \leqslant 13$

Drawing the graph of y = 5 and crossing out everything below it as y is greater. Drawing the graph of y = -x + 13 and crossing out everything above it as y is less. The region is indicated with an R. Crossing out everything above the line y = 2x + 1 as y is less

Show that the area of region **R** is $12\,cm^2$. **[6]**

$y \leq -x + 13$ ← Rearranged to make y the subject to make it easier to draw and identify the region

$\frac{1}{2} \times 6 \times 4$ ← Working out the area of the triangle which is the region R. Area of triangle = 1/2 x base x height

19 **(a)** Write $x^2 - 8x + 9$ in the form $(x - a)^2 - b$.

$(x-4)^2+9-16$ ← The form is completed the square form. Completing the square by halving the coefficient of x (which is -8), putting this in a bracket with x, squaring it then subtracting the -4 squared

(a) .... $(x-4)^2-7$ .... **[3]**

(b) Use your answer from part **(a)** to solve.

$x^2 - 8x + 9 = 0$

Give your answers in exact form.
You must show your working.

$(x-4)^2-7=0$ ← Setting the completed the square form equal to 0

$(x-4)^2=7$ ← Adding 7 to both sides

$x-4=\pm\sqrt{7}$ ← Square rooting both sides

Adding 4 to both sides and giving the plus or minus as two separate solutions

(b) $x =$ .... $4+\sqrt{7}$ .... or $x =$ .... $4-\sqrt{7}$ .... **[2]**

20 Two pyramids, A and B, are mathematically similar.

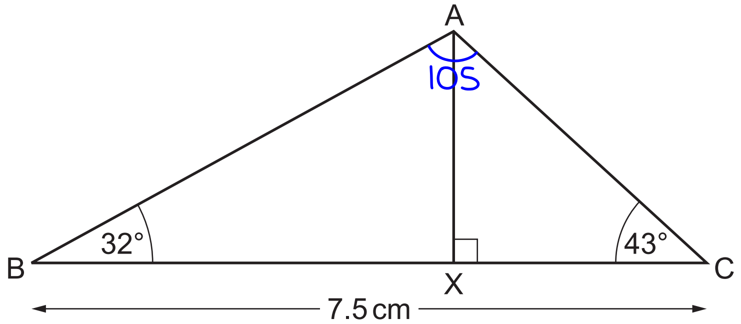
Pyramid A has surface area 12 cm^2 and volume 8 cm^3.
Pyramid B has surface area 75 cm^2.

Work out the volume of pyramid B.
You must show your working.

$8 \times \left(\sqrt{\frac{75}{12}}\right)^3$ ← Dividing the 75cm^2 by the 12cm^2 expresses the area scale factor. Square rooting this expresses the length scale factor. Cubing this expresses the volume scale factor. Multiplying the volume of A by the volume scale factor works out the volume of B

.......125....... cm^3 **[4]**

21 The diagram shows triangle ABC.
X lies on BC such that angle AXC = 90°.



Not to scale

BC = 7.5 cm, angle ABC = 32° and angle ACB = 43°.

Work out length AX.
You must show your working.

180 − 32 − 43 ← There are 180° in total in a triangle so subtracting the other angles from 180 works out that angle BAC is 105°

$\frac{a}{\sin A} = \frac{b}{\sin B}$ ← There are opposite pairs of sides and angles so the sine rule can be used to work out side AC

$AC = \frac{7.5 \sin 32}{\sin 105}$ ← Multiplying both sides by sinA makes side a the subject. Substituting AC for a, 7.5 for b, 32 for B and 105 for A

S O H C A H T O A ← Right-angled trigonometry can be used to work out AX in right-angled triangle AXC. Ticking O as we are looking for the opposite and H as we have the hypotenuse. There are two ticks on the SOH formula triangle so this one can be used

Sin43 × 4.1... ← From the formula triangle: opposite = sin of the angle x hypotenuse. Using the exact value of AC as the hypotenuse

..........2.8.......... cm **[6]**

END OF QUESTION PAPER