OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

Tuesday 1 November 2022 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** the questions.

1 (a) Write 65400 in standard form.

Dividing by 10 4 times gives a number between 1 and 10. So it must be multiplied by 10^4 to keep it equal

(a) .... 6.54×10^4 .... [1]

(b) Write 8.2×10^{-4} as an ordinary number.

8.2 needs to be divided by 10 4 times. This moves the decimal point 4 times to the left

(b) .... 0.00082 .... [1]

2 In 2019, comet A and comet B were both seen from Earth.
Comet A is seen from Earth every 84 years.
Comet B is seen from Earth every 105 years.

Find the next year when both comets will be seen from Earth.

$84 = 2^2 \times 3 \times 7$
$105 = 3 \times 5 \times 7$

Using the calculator to express both 84 and 105 as a product of prime factors

$2^2 \times 3 \times 5 \times 7$

The lowest common multiple of 84 and 105 is the number of years until both comets will be seen from Earth again. The lowest common multiple can be found by multiplying the highest power of each prime in both of the products of prime factors

$2019 + 420$

Adding the number of years until both comets will be seen from Earth again to the year 2019 works out the next year both comets will be seen from Earth

Newer models of Casio calculators can work out the lowest common multiple of two numbers without having to do the method above

.... 2439 .... [4]

3 An examination has three papers.
Paper 1 is marked out of 60.
Paper 2 is marked out of 40.
Paper 3 is marked out of 100.
The three marks are added together to form the total mark out of 200.

A student scored 65% on Paper 1 and 70% on Paper 2.

Find the mark they need to get on Paper 3 to achieve 64% of the total marks.
You must show your working.

$60+40+100$

Adding together the number of marks on Paper 1, 2 and 3 works out that there are 200 marks in total

$\frac{64}{100} \times 200 = 128$

Percentage is out of 100 so putting the 64% over 100 converts it into a fraction. Multiplying the 200 by this fraction finds that 64% of the total marks is 128

$\frac{65}{100} \times 60 = 39$

Putting the 65% over 100 converts it into a fraction. Multiplying the 60 marks on Paper 1 by this fraction finds that 65% of the marks on Paper 1 is 39

$\frac{70}{100} \times 40 = 28$

Putting the 70% over 100 converts it into a fraction. Multiplying the 40 marks on Paper 2 by this fraction finds that 70% of the marks on Paper 2 is 28

$128 - 39 - 28$

Subtracting the 39 marks scored on Paper 1 and the 28 marks scored on Paper 2 from the 128 marks needed to get 64% of the total marks works out that 61 marks are needed on Paper 3

............61............ **[5]**

4 A phone manufacturer records the faults that are reported.
Last week, in a batch of 96 phones, 6 were reported as faulty.

(a) Write down the relative frequency of faulty phones in this batch.

6 out of the 96 were faulty

(a) .... $\frac{6}{96}$ .... **[1]**

(b) In 2020, the manufacturer sold a total of 12 321 phones.

Work out how many of these phones the manufacturer should expect to be reported as faulty.

$\frac{6}{96} \times 12321$

It can be estimated that the relative frequency will be the same for the 12321 phones. Multiplying the 12321 phones by the estimated relative frequency which were faulty works out an estimate of how many should be faulty. The answer of 770.0625 is rounded to the nearest whole number

(b) .... 770 .... **[2]**

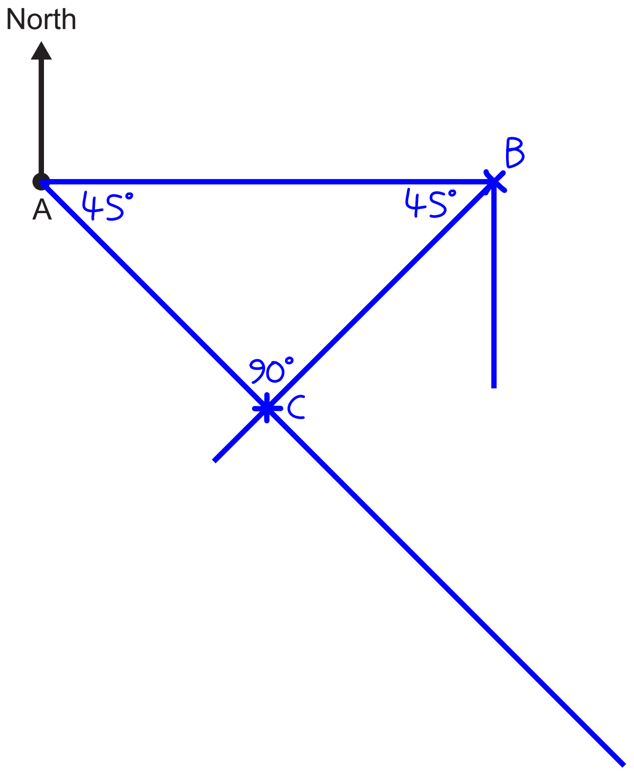
5 B is 12 km due east of A.
C is south-east of A and on a bearing of 225° from B.

Complete the diagram to show the positions of A, B and C.
Show clearly the values of all three angles in triangle ABC.

Scale: 1 cm represents 2 km

$12 \div 2 = 6$ ← Every 2 km is represented by 1 cm. Dividing the 12 km by 2 works out that it is 6 lots of 2 km and therefore should be represented by 6 cm

$225 - 180 = 45$ ← Working out that the 225° is 45° more than 180°



[4]

1. Use a protractor to measure 90° clockwise from north at point A.
2. Draw a 6 cm line from A in this direction.
3. Put a cross for point B at the end of the line.
4. Use a protractor to measure 135° clockwise from north at point A.
5. Draw a line from A in this direction to indicate all points which are south-east from A.
6. Draw a line going straight down from B to indicate which direction is south.
7. Use a protractor to measure 45° clockwise from south at point B.
8. Draw a line from B in this direction to indicate all points which are on a bearing of 225° from B.
9. Put a cross for point C where the two lines meet.
10. Measure the angles in the triangle ABC using a protractor and write in the angles.

6 **(a)** A solid block of wood is a cuboid which measures 3 cm by 4 cm by 5 cm.
Its density is 0.65 g/cm^3.

Work out the mass of the block of wood.

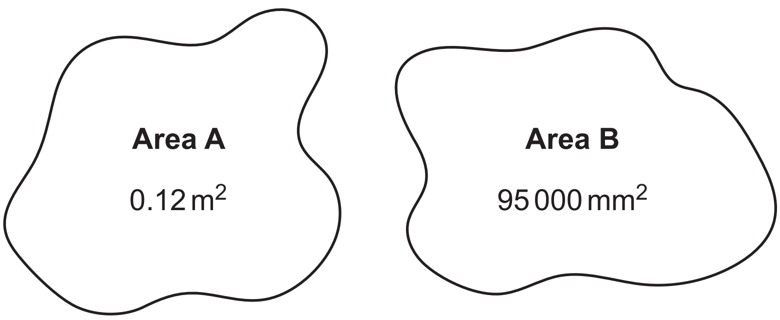
$d\,^{m}\,_{v}$ ← Density = mass/volume. Writing this as a formula triangle

$3 \times 4 \times 5$ ← Volume of cuboid = length x width x height. So the volume of the cuboid is 60 cm^3

0.65×60 ← Covering m in the formula triangle tells that mass = density x volume

(a) ....39............ g **[2]**

(b) Here are two areas.



Not to scale

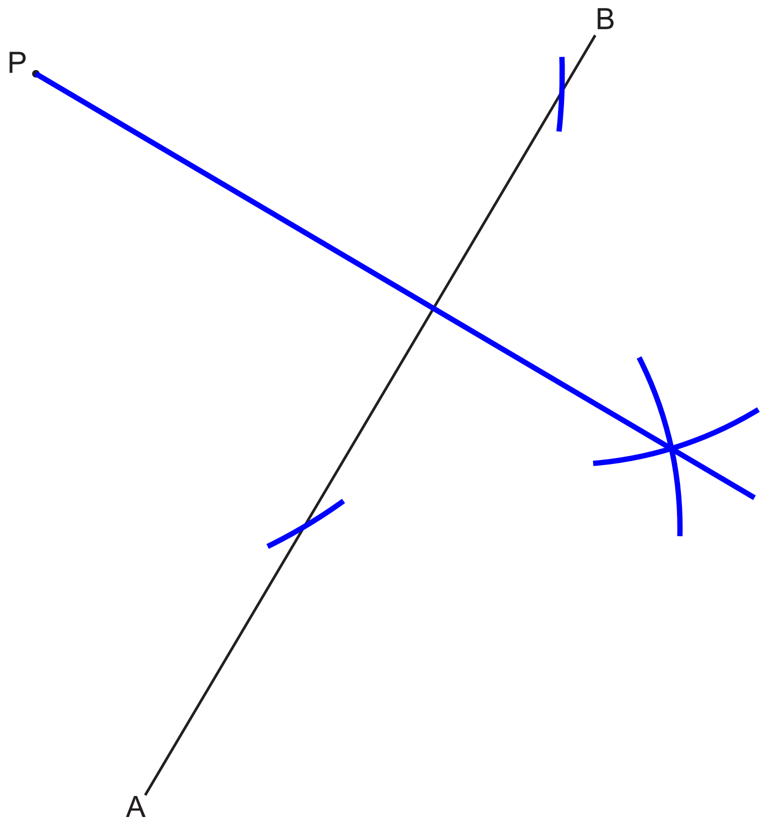
State which area is greater.
Show how you decide.

0.12×1000^2 ← There are 1000 mm in 1 m. As the unit is squared, multiplying the m^2 by 1000^2 converts it into mm^2

Area ....A....... is greater because ....its area is 120000 mm^2....

.... **[2]**

7 **(a)** Construct the perpendicular from the point P to the line AB.

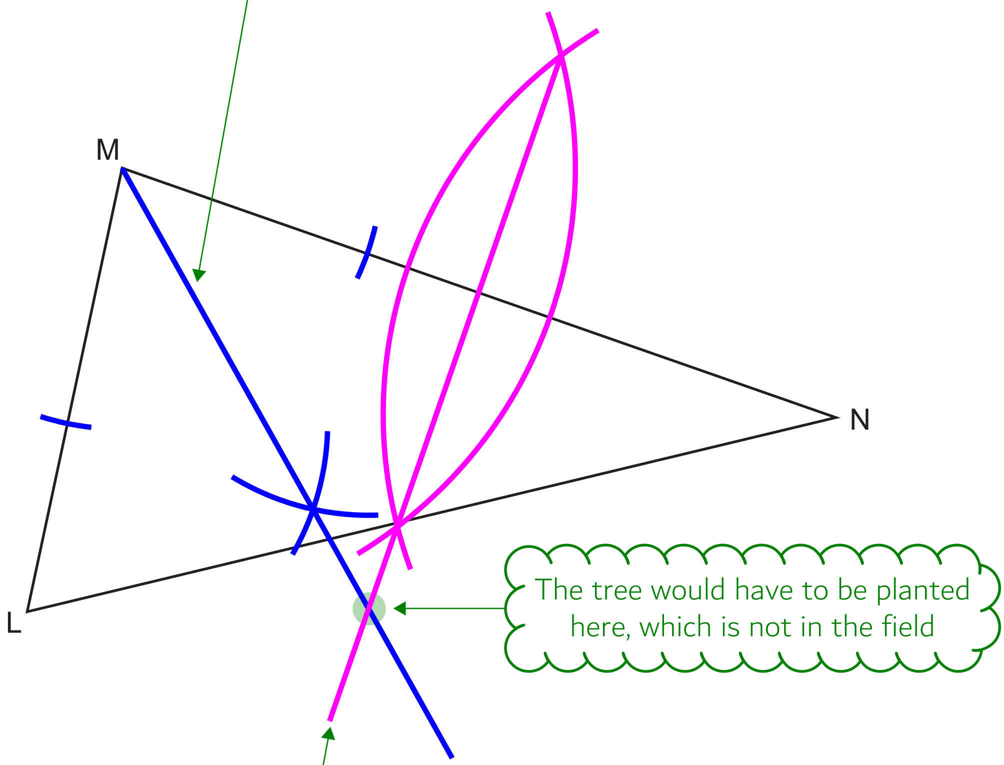


[2]

1. Using a compass, scribe two arcs from P on the line AB.
2. Using a compass, scribe an arc from where the first arc meets line AB and an arc from where the second arc meets AB which meet and form a cross on the right of line AB.
3. Using a ruler, draw a straight line from P through the cross.

(b) The diagram shows a field LMN.

Constructing an angle bisector of angle M (shown in blue) shows all points which are the same distance from MN and ML. To do this:
1. Using a compass, scribe two arcs from point M, one of which is on line MN and the other which is on line ML.
2. Using a compass, scribe an arc from the first arc and scribe an arc from the second arc which meet and form a cross.
3. Using a ruler, draw a straight line from point M through the cross.



Constructing a perpendicular bisector of line MN (shown in pink) shows all points which are the same distance from corner M and corner N. To do this:
1. Using a compass, scribe arcs from points M and N which meet and form two crosses.
2. Using a ruler, draw a straight line through both of the crosses.

A tree is to be planted in the field so that it is

- the same distance from the fences MN and ML

and

- the same distance from corner M as from corner N.

Show, by construction, whether this can be done or cannot be done.

This ....cannot.......... be done. **[5]**

8 A bag contains 35 balls.
Each ball is either red or green.
The ratio of red balls to green balls is 3 : 2.

Work out the smallest number of balls of each colour that have to be added to the bag so that the ratio of red balls to green balls becomes 7 : 3.
You must show your working.

$35 \div 5$ There are 35 balls in total and 5 parts in total in the first ratio (as 3 + 2 = 5). Dividing the 35 balls by the 5 parts works out that each part of the ratio is worth 7 balls

$7 \times 3 = 21$ There are 3 parts in the first ratio for red balls so multiplying the value of 1 part by 3 works out that there were originally 21 red balls

$7 \times 2 = 14$ There are 2 parts in the first ratio for green balls so multiplying the value of 1 part by 2 works out that there were originally 14 green balls

$21 \div 7$
$3 \times 3 = 9$
Checking to see if they are currently in the ratio of 7 : 3 by dividing the 21 red balls by the 7 parts which need to represent it to work out that 1 part of the ratio would be 3 balls. Then multiplying this by the 3 parts for green balls works out that there would need to be 9 green balls. This cannot work as this is less than the original 14 green balls and the question states that balls need to be added

$28 \div 7$
$4 \times 3 = 12$
The 7 : 3 ratio cannot be simplified so the number of red balls needs to be a multiple of 7. Adding another 7 red balls would give 28 red balls. Dividing this by the 7 parts which need to represent it works out that 1 part of the ratio would be 4 balls. Then multiplying this by the 3 parts for green balls works out that there would need to be 12 green balls. This cannot work as this is less than the original 14 green balls and the question states that balls need to be added

Number of red balls added to the bag = ...14........

Number of green balls added to the bag = ...1........ **[5]**

$35 \div 7$
$5 \times 3 = 15$
Adding another 7 red balls would give 35 red balls. Dividing this by the 7 parts which need to represent it works out that 1 part of the ratio would be 5 balls. Then multiplying this by the 3 parts for green balls works out that there would need to be 15 green balls. This works as this is more than the original 14 green balls

$35 - 21$ Subtracting the 21 red balls originally in the bag from the 35 red balls needed in the bag to be in the 7 : 3 ratio while adding balls works out that 14 red balls need to be added

$15 - 14$ Subtracting the 14 green balls originally in the bag from the 15 green balls needed in the bag to be in the 7 : 3 ratio while adding balls works out that 1 green ball need to be added

9 Here are two pieces of work.

For each one, describe the error in the method and give the correct answer.

(a)

Question:

Rearrange $y = 3x + 17$ to make x the subject.

Solution:

$y = 3x + 17$

$y + 17 = 3x$

$x = \dfrac{y + 17}{3}$

Error is 17 needs to be subtracted on the left side

Correct answer $x = \dfrac{y-17}{3}$ **[2]**

(b)

Question:

Rearrange $A = 4x^2$ to make x the subject, where $x > 0$.

Solution:

$A = 4x^2$

$\sqrt{A} = \sqrt{4x^2}$

$\sqrt{A} = 4x$

$x = \dfrac{\sqrt{A}}{4}$

Error is The 4 wasn't square rooted

Correct answer $x = \dfrac{\sqrt{A}}{2}$ **[2]**

10 You may use these kinematics formulae to answer this question.

$v = u + at$

$s = ut + \frac{1}{2}at^2$

A particle has an initial velocity of 3 m/s.
After 20 seconds the particle has a velocity of 11 m/s.

Work out the distance the particle has travelled after 20 seconds.

v = final velocity. u = initial velocity. a = acceleration. t = time. s = distance

$11 = 3 + 20a$ ← Substituting in 11 for v, 3 for u and 20 for t in the first formula

$8 = 20a$ ← Subtracting 3 from both sides eliminates the 3 on the right and gets the term involving a on its own

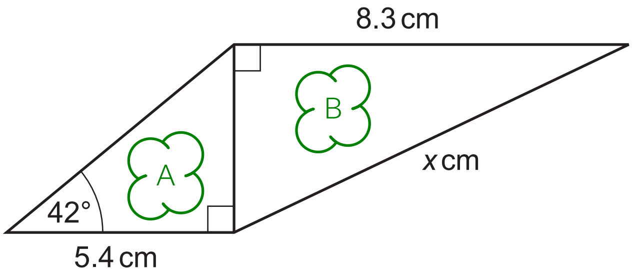
$0.4 = a$ ← Dividing both sides by 20 eliminates the 20 on the right and gets a on its own. So a = 0.4

$3 \times 20 + \frac{1}{2} \times 0.4 \times 20^2$ ← Substituting in 3 for u, 20 for t and 0.4 for a in the right side of the second formula finds s, which is the distance

.......................140....................... m **[4]**

11 The diagram shows two right-angled triangles that are joined together.

All measurements are given accurate to 2 significant figures.



Not to scale

Work out the value of x.
Give your answer correct to an appropriate degree of accuracy.
You must show your working.

S O H C A H T O A

Right-angled trigonometry can be used in triangle A to work out the side joining both triangles. Writing SOH CAH TOA as formula triangles and ticking A as we have the adjacent and O as we are looking for the opposite

$\tan 42 \times 5.4$

There are two ticks on the TOA formula triangle so this one can be used. Covering over O tells us that opposite = tan of the angle x adjacent. The angle is 42° and the adjacent is 5.4 cm. Storing the exact value of 4.862... as A on the calculator

$a^2 + b^2 = c^2$

Pythagoras' Theorem can be used to work out the missing side in triangle B

$\sqrt{4.8...^2 + 8.3^2}$

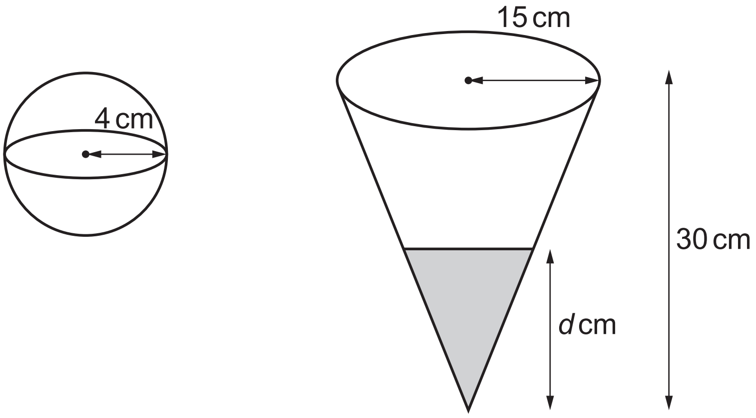
c is the longest side so is x. Square rooting both sides and substituting in the values of shorter sides for a and b works out x. Using the exact value stored as A on the calculator instead of 4.8...

The answer of 9.619... is rounded to 2 significant figures as this was the accuracy of the original data

x = ...9.6........................ **[6]**

12 The diagram shows a sphere and a cone.

Not to scale



The sphere has radius 4 cm.
The cone has radius 15 cm and height 30 cm.

The sphere is completely filled with water.
The same amount of water is poured into the cone.

Work out the depth, *d* cm, of the water in the cone.
You must show your working.

[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

$\frac{4}{3}\pi \times 4^3 = \frac{256}{3}\pi$ ← Working out the volume of the sphere. It's radius is 4 cm

$\frac{1}{3}\pi \times 15^2 \times 30 = 2250\pi$ ← Working out the volume of the cone. It's radius is 15 cm and height is 30 cm

$\dfrac{\frac{256}{3}\pi}{2250\pi}$ ← The volume of the sphere is the same as the volume of the water in the cone. The water takes the shape of a smaller cone with the same angle at the top as the larger cone so it is similar to the larger cone. Expressing the volume of the water as a fraction of the larger cone works out the volume scale factor

$\sqrt[3]{\dfrac{125}{3375}}$ ← Cube rooting the volume scale factor works out the length scale factor

$0.3\ldots \times 30$ ← Multiplying the length scale factor by the height of the larger cone works out the height of the smaller cone, which is the depth of the water

$d =$ 10.1 **[6]**

13 y is directly proportional to $\sqrt{x}$.
$y = 1$ when $x = 16$.

Find a formula for y in terms of x.

$y \propto \sqrt{x}$ ← Writing the proportion

$y = k\sqrt{x}$ ← Converting the proportion into an equation by multiplying the right side by k, which represents a constant value which needs to be found

$k = \frac{y}{\sqrt{x}}$ ← Dividing both sides by $\sqrt{x}$ makes k the subject

$= \frac{1}{\sqrt{16}}$ ← Substituting in the value of x and y given works out that k must be 1/4

Substituting the value of k back into the original equation

$y = \frac{1}{4}\sqrt{x}$ .... **[3]**

14 An estimate for the number of seals on an island is given by the formula

$P = 5200 \times 1.02^{t}$

where P is the number of seals t years after the start of year 2015.

(a) Write down the annual percentage increase in the number of seals on the island.

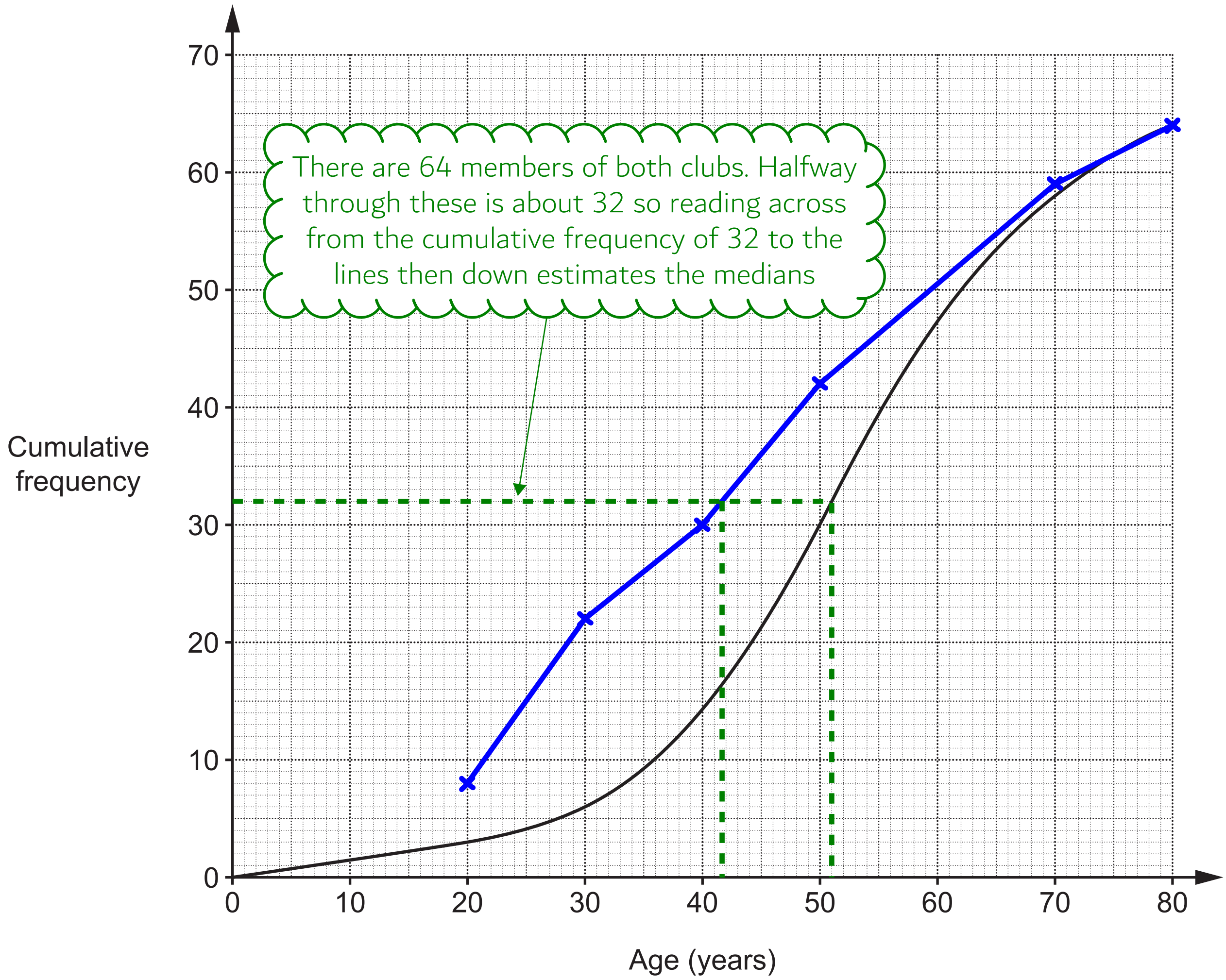
1.02 x 100 = 102, so this means that it is increasing from 100% to 102%

(a) ....2%.............. **[1]**

(b) Use the formula to show that there may have been about 4700 seals on the island at the start of year 2010. **[2]**

$5200 \times 1.02^{-5} = 4710$ ← 2010 is 5 years before 2015 so the value of t is -5. Substituting this into the formula works out that there are an estimated 4709.8... seals, which is rounded to the nearest whole number

15 The cumulative frequency graph shows the distribution of the ages of the members of a **tennis** club.



(a) The table summarises the ages of the members of a **cycling** club.

Age (*a* years)	$0 < a \leqslant 20$	$20 < a \leqslant 30$	$30 < a \leqslant 40$	$40 < a \leqslant 50$	$50 < a \leqslant 70$	$70 < a \leqslant 80$
Frequency	8	14	8	12	17	5
	8	22	30	42	59	64

The cumulative frequencies are worked out by adding the frequencies as they go. The cumulative frequencies are plotted at the end of each interval then can be joined up with straight lines

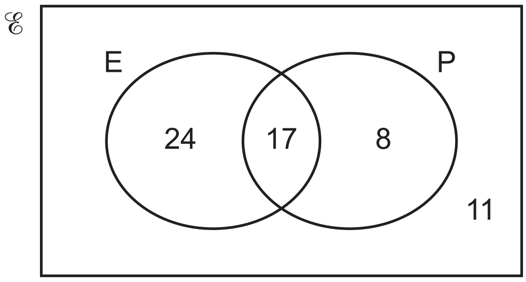
On the graph above, draw the cumulative frequency graph of the ages of the members of the **cycling** club. **[5]**

(b) Find out which club has younger members on average.
Give evidence to support your decision.

Cycling because the estimated median of the cycling club is about 42 and the estimated median of the tennis club is 51 **[2]**

16 A salesroom sells various types of car.
Some cars are electric (E), some are petrol (P), some are both and some are neither.

The Venn diagram below shows the salesroom's stock of cars.



A petrol car is picked at random.

Find the probability that the car is also electric.

$17+8$

Both the 17 and 8 are representing cars which are petrol. Adding these together works out that there are 25 petrol cars in total

17 out of the 25 petrol cars are also electric

$\frac{17}{25}$ .... **[2]**

17 Find the equation of the line through (4, 5) that is perpendicular to $y = 2x - 3$.

$y = -\frac{1}{2}x + c$

The general equation of a straight line is y = mx + c, where m is the gradient. 2 is the gradient of the line y = 2x - 3. The perpendicular gradient is the negative reciprocal of this, which is -1/2

$c = y + \frac{1}{2}x$

Rearranging to find c by adding 1/2 x to both sides

$= 5 + \frac{1}{2}(4)$

Substituting in the x and y-coordinates from (4 , 5)

$= 7$

Substituting the value of c back into the equation

$y = -\frac{1}{2}x + 7$ .... **[3]**

18 **(a)** The next term in a Fibonacci sequence is found by adding together the two previous terms.

(i) The first and second terms of a particular Fibonacci sequence are x and y.

Show that the fourth term of the sequence can be written as $x + 2y$. **[2]**

$x+y$ ← Adding the first and second term expresses the third term

$y+x+y$ ← Adding the second and third term expresses the fourth term

$x+2y$ ← Simplifying the fourth term by collecting like terms

(ii) The fourth term of the same Fibonacci sequence is 7.
The seventh term of the sequence is 31.

Work out the value of x and the value of y.
You must show your working.

Method shown on next page

(a)(ii) $x =$ 3

$y =$ 2 **[6]**

$x+y+x+2y$ ← Adding the third and fourth term expresses the fifth term

$2x+3y$ ← Simplifying the fifth term by collecting like terms

$x+2y+2x+3y$ ← Adding the fourth and fifth term expresses the sixth term

$3x+5y$ ← Simplifying the sixth term by collecting like terms

$2x+3y+3x+5y$ ← Adding the fifth and sixth term expresses the seventh term

$5x+8y=31$ ← Simplifying the seventh term by collecting like terms. Setting it equal to the 31 forms the first equation

$x+2y=7$ ← Setting the fourth term equal to the 7 forms the second equation

$5x+10y=35$ ← Multiplying the second equation by 5 gets the same number of x as the first equation and forms the third equation

$2y=4$ ← Subtracting the first equation from the third equation cancels out the x terms and leaves an equation just in terms of y

$y=2$ ← Dividing both sides by 2 finds that y is 2

$x+2\times 2=7$ ← Substituting the value of y into the second equation

$x=3$ ← Subtracting 2 x 2 from both sides finds that x is 3

(b) Here are the first four terms of a sequence.

1 $\sqrt{3}$ 3 $3\sqrt{3}$

Write an expression for the *n*th term.

This is a geometric sequence as it multiplies by the same amount between each term. The nth term of a geometric sequence is ar^{n-1}, where a is the first term and r is the common ratio. a is 1 as this is the first term. r is $\sqrt{3}$ as this is what it multiplies by between each term

(b) $1\times(\sqrt{3})^{n-1}$ **[2]**

(c) Here are the first four terms of a quadratic sequence.

−1 5 13 23

The *n*th term is $n^2 + bn + c$.

Find the value of *b* and the value of *c*.

n^2: 1 4

Listing out the sequence of n^2. n is 1 on the 1st term so the 1st term is 1^2, which is 1. n is 2 on the 2nd term so the 2nd term is 2^2, which is 4

-2 1 : $3n-5$

Listing out the sequence which needs to be added to the sequence of n^2 to get the original sequence. This is the linear sequence bn + c. It increases by 3 between each term so must be 3n. Going backward in the sequence finds that the 0th term is -5 so the sequence must be 3n - 5

The nth term is $n^2 + 3n - 5$

(c) $b =$ 3

$c =$ -5 **[3]**

19 Describe the **single** transformation that maps the graph of $y = x^2$ onto the graph of $y = (x+3)^2 + 5$.

Translation by vector $\binom{-3}{5}$

3 is added to x so it gets to the same values 3 sooner, so translates 3 to the left. 5 is added to the right side so it translates 5 up

[3]

20 Mrs Sweet has 8 different milk chocolates and 9 different plain chocolates.

Her daughter chooses one of the milk chocolates.
Her son then chooses one of the plain chocolates.
Mrs Sweet then chooses one of the remaining chocolates.

Work out how many different combinations of three chocolates they can choose.

$8 \times 9 \times (7+8)$

Using the product rule for counting. There are 8 possibilities for her daughter. There are 9 possibilities for her son. There are 7 + 8 possibilities for Mrs Sweet as there is 1 less milk chocolate and 1 less plain chocolate when she chooses. Multiplying the number of possible outcomes for each event works out the number of possible outcomes in total

1080 **[3]**

21 60 people each try to solve a puzzle.
The table summarises their recorded times.

$$\frac{F}{C \quad d}$$

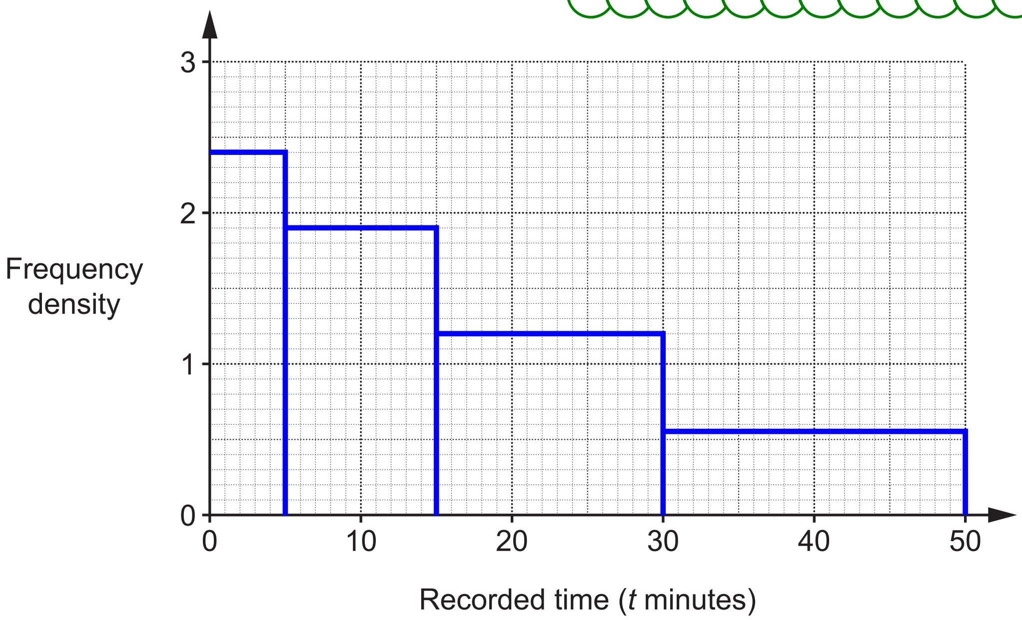
Frequency is represented by the area of each box on a histogram.
Frequency = class width x frequency density.
Writing this as a formula triangle

Recorded time (t minutes)	Frequency
$0 < t \leqslant 5$	12
$5 < t \leqslant 15$	19
$15 < t \leqslant 30$	18
$30 < t \leqslant 50$	11

Fd
2.4 ← 12 ÷ 5 = 2.4
1.9
1.2
0.55

From the formula triangle, frequency density = frequency ÷ class width. The class widths are how wide each interval is and is found by subtracting the lower bound from the upper bound for each interval. Dividing the frequencies by the class widths works out the frequency densities

(a) Draw a histogram to show this information.



[3]

(b) Those people who failed to solve the puzzle within 50 minutes were given a recorded time of 50 minutes.

Nina uses mid-interval values to estimate the mean recorded time of the 60 people.

Explain why Nina's answer is likely to be an under-estimate for the mean of the actual time taken by the 60 people.

The mid-interval value for the final interval should be greater

As the upper bound for the interval would be more than 50 minutes **[1]**

Turn over for Question 22

22 Solve algebraically.

$x^2 + y^2 = 18$ ← First equation

$y = x - 6$ ← Second equation

$(x-6)(x-6)$ ← Squaring the right side of the second equation works out y^2

$x^2 - 6x - 6x + 36$ ← Expanding the brackets

$x^2 + x^2 - 12x + 36 = 18$ ← Simplifying the expansion by collecting like terms and then substituting for y^2 in the first equation

$2x^2 - 12x + 18 = 0$ ← Bringing into the quadratic form $ax^2 + bx + c = 0$

$x = \frac{--12 \pm \sqrt{(-12)^2 - 4 \times 2 \times 18}}{2 \times 2}$ ← $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← Solving using the quadratic formula. Both solutions are x = 3

$y = 3 - 6$ ← Substituting in the value of x into the second equation

$x =$ .... 3 ....

$y =$ .... -3 .... **[5]**

END OF QUESTION PAPER