

Please write clearly in block capitals.

Centre number Candidate number

Surname _____

Forename(s) _____

Candidate signature _____

**GCSE
MATHEMATICS**

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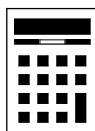
Higher Tier Paper 3 Calculator

Monday 12 November 2018 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
TOTAL	

Advice

In all calculations, show clearly how you work out your answer.



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided

- 1 A shape is translated by the vector $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

In which direction does the shape move?

Circle your answer.

[1 mark]

up

down

left

right

The top number of the vector is the x direction. The bottom number of the vector is the y direction. As it moves 0 in the x direction it doesn't move left or right. As it is positive in the y direction it moves up

- 2 What is 1.75 kilometres as a fraction of 700 metres?

Circle your answer.

[1 mark]

$\frac{5}{2}$

$\frac{1}{4}$

$\frac{4}{1}$

$\frac{2}{5}$

$(1.75 \times 1000)/700 = 5/2$

There are 1000 metres in a kilometre so multiplying the 1.75 by 1000 converts it into metres. This is written as a fraction of the 700

- 3 The first 4 terms of a linear sequence are

3 11 19 27

Circle the expression for the n th term.

[1 mark]

$8 - 5n$

$n + 8$

$8n + 3$

$8n - 5$

Each term increases by 8 so it must involve $8n$. The 0th term (the one which would be before the 1st term) is -5 (as the sequence decreases by 8 each time if followed backwards and $3 - 8 = -5$) so the sequence must be $8n - 5$



- 4 Work out the lowest common multiple (LCM) of 20, 30 and 40
Circle your answer.

[1 mark]

10

120

240

24 000

This is the smallest number listed which is a multiple of 20, 30 and 40

- 5 The length of a table is 110 cm to the nearest cm
Complete the error interval.

[2 marks]

$$\underline{109.5} \text{ cm} \leq \text{length} < \underline{110.5} \text{ cm}$$

The resolution is 1cm. Halving this gives 0.5. Adding and subtracting 0.5 from 110 gives the upper and lower bound

Turn over for the next question

Turn over ►



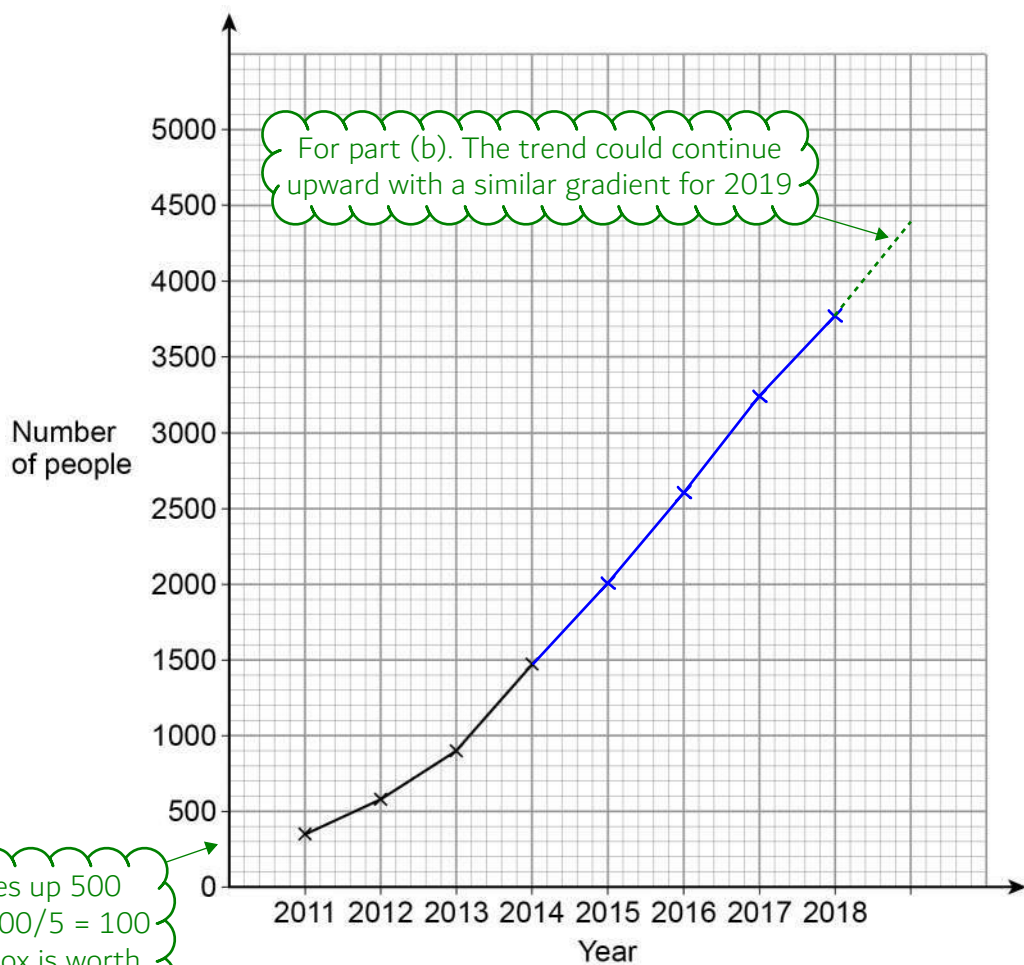
6 A music festival has taken place each year from 2011

The table shows the number of people who attended each year.

Year	2011	2012	2013	2014	2015	2016	2017	2018
Number of people	350	583	906	1471	2023	2612	3251	3780

The festival organisers draw a time series graph to represent the data.

The first four years have been plotted.



6 (a) Complete the graph.

[2 marks]

6 (b) Use the graph to estimate the number of people who will attend the festival in 2019

[2 marks]

Answer 4400

Turn over for the next question



7

$$k = n^2 + 9n + 1$$

Mo says,

“ k will be a prime number for all integer values of n from 1 to 9”

Show that Mo is wrong.

You **must** show that your value of k is **not** prime.

[3 marks]

11, 23, 37, 53, 71, 91, 113, 137, 163

Use table mode by pressing menu then 3. Set $f(x) = x^2 + 9x + 1$. Ignore $g(x)$. Start: 1. End: 9. Step: 1. This lists out all of the values of k needed

$91 = 7 \times 13$

This shows that 91 isn't prime as prime numbers only have two factors, themselves and 1

FACT B
0 9 9

To check if a number is prime by using your calculator, enter the number, press equals, press SHIFT then press FACT (the button on the left). This expresses the number as a product of prime factors. If it comes back as itself, it must be prime



8

Doug owes an amount of £600

He wants to pay off this amount in five months.

He says,

“Each month, I will pay back 20% of the amount I still owe.”

Show working to check if his method is correct.

[3 marks]

$$600 \times 0.8^5 = 196.608$$

100% - 20% = 80%, so paying back 20% leaves 80% of the amount he owes. 80% as a decimal is 0.8 so multiplying by this reduces the amount by 20%. Multiplying by 0.8 5 times reduces it by 20% 5 times. Multiplying by 0.8 5 times is the same as multiplying by 0.8^5

No

The method is wrong as there is still about £196.61 left to pay

Turn over for the next question



9

A motor racing circuit consists of

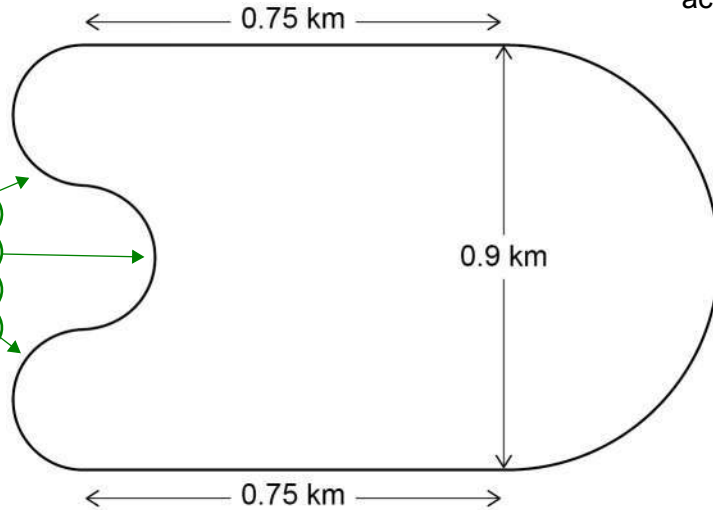
two parallel straight sections, each of length 0.75 km

a semicircle of diameter 0.9 km

three equal, smaller semicircles.

Not drawn
accurately

The diameter of each of the smaller semicircles is 0.3km as 3 of the diameters combined are 0.9km and $0.9/3 = 0.3$



The length of a motor race must be greater than 305 km

What is the lowest number of **full** laps needed at this circuit?

You **must** show your working.

[5 marks]

$$\frac{305}{0.75 \times 2 + \frac{1}{2} \times \pi \times 0.9 + \frac{1}{2} \times \pi \times 0.3 \times 3} = 70.5$$

Dividing the 305km by the distance of one full lap gives the number of laps needed

Two parallel straight sections, each of length 0.75km

A semicircle of diameter 0.9km

Three equal, smaller semicircles

Circumference = $\pi \times$ diameter
As they are semicircles, the circumference is halved to find the curved length

70.5 is rounded up to 71 as the question asks for full laps and 70 laps isn't enough

Answer _____

71



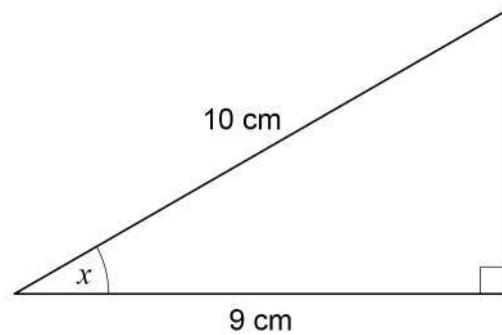
- 10 Solve $8 > 3 - \frac{1}{2}x$ [2 marks]

$$\frac{8-3}{-\frac{1}{2}} < x$$

Rearranged to make x the subject by subtracting 3 then dividing by $-1/2$. When dividing by a negative, the inequality needs to flip

Answer $x > -10$

- 11 Use trigonometry to work out the size of angle x . [2 marks]



Not drawn
accurately

SOH CAH TOA

Listing SOH CAH TOA as formula triangles then ticking what we have. A is ticked as 9cm is the adjacent and H is ticked as 10cm is the hypotenuse

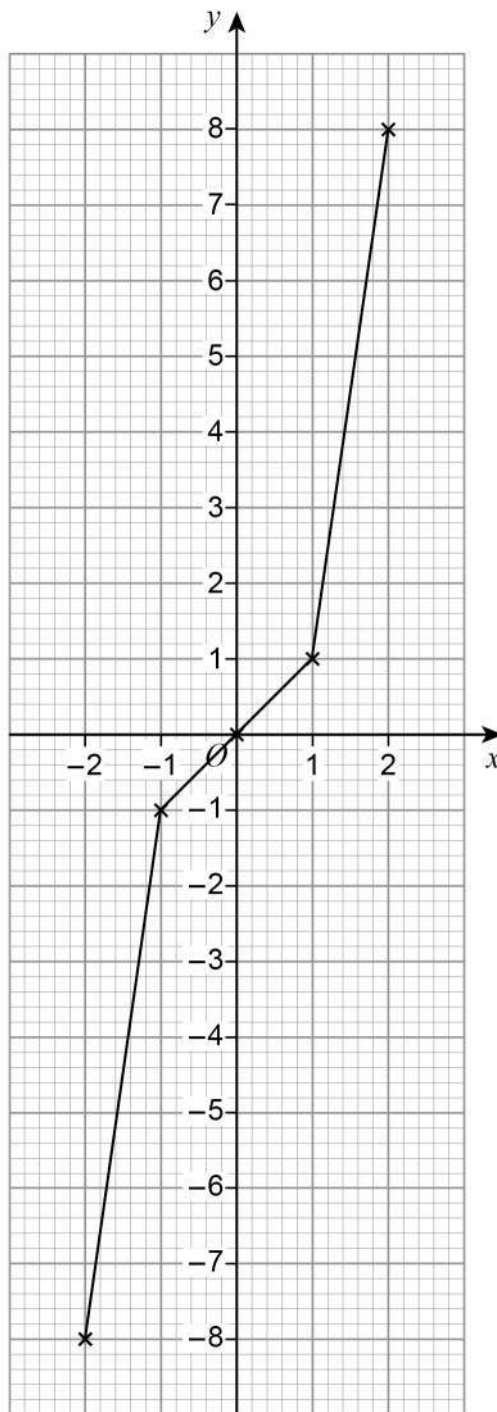
$$x = \cos^{-1}\left(\frac{9}{10}\right)$$

From the formula triangle, $\cos(x) = \text{adjacent/hypotenuse}$. Rearranging this and substituting in the adjacent and hypotenuse

Answer 25.8 degrees



- 12 Lewis wants to draw the graph $y = x^3$ for values of x from -2 to 2 . Here is his graph.



Make **one** criticism of his graph.

[1 mark]

Should be a curve

The plotted points are correct but everything between them is incorrect. For example, 0.5^3 is not 0.5



13 The probability of Heads when a biased coin is thrown is 0.6
The coin is thrown 500 times.

Circle the expected number of Tails.

[1 mark]

20 200 250 300

$$500 - 0.6 \times 500$$

0.6 x 500 works out the expected number of Heads. Subtracting this from 500 gives the expected number of Tails as it can either be Heads or Tails

14 The mean mass of a squad of 19 hockey players is 82 kg
A player of mass 93 kg joins the squad.

Work out the mean mass of the squad now.

[3 marks]

$$\frac{19 \times 82 + 93}{20}$$

Mean = total/number, where total is the total mass of the squad and number is the number of players in the squad. Rearranging the formula gives total = mean x number. So multiplying 82kg by 19 gives the total mass of the 19 players. Adding the 93kg gives the total of all 20 players. Dividing this by 20 gives the new mean

Answer 82.55 kg

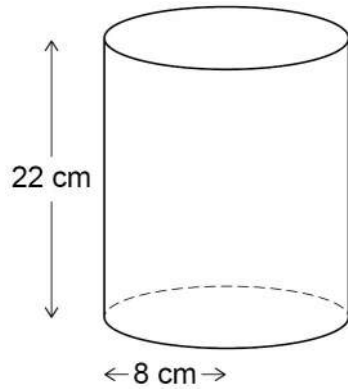


- 15 A company makes two types of lampshade using fabric on wire frames.

Lampshade A

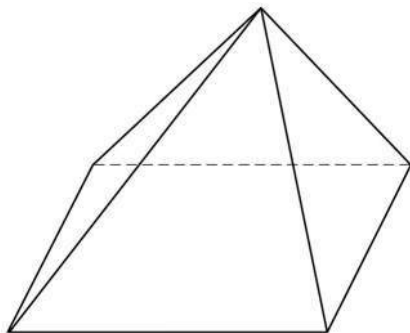
Fabric is used to make the curved surface of a cylinder.

The cylinder has radius 8 cm and height 22 cm

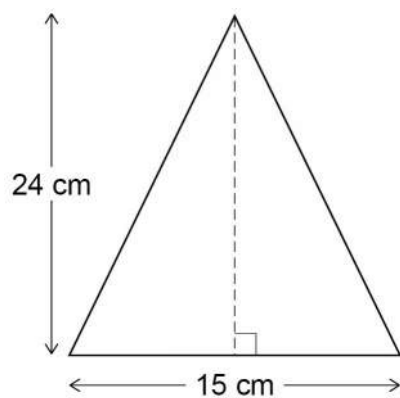


Lampshade B

Fabric is used to make the four triangular faces of a pyramid.



Each triangular face has base 15 cm and perpendicular height 24 cm



Not drawn
accurately



Cost of fabric	£400 per square metre
Other costs for A	£3.50 per lampshade
Other costs for B	£7.50 per lampshade

Work out the ratio cost of one lampshade A : cost of one lampshade B

Give your answer in the form $n : 1$

[5 marks]

$$\frac{\pi \times 0.08 \times 2 \times 0.22 \times 400 + 3.50}{\frac{1}{2} \times 0.15 \times 0.24 \times 4 \times 400 + 7.50}$$

To get 1 on the right side of the ratio, the cost of one lampshade B must be divided by the cost of one lampshade B (dividing a number by itself gives 1). So to work out n , we can divide the cost of one lampshade A by the cost of one lampshade B (as both sides of the ratio need to be divided by the same amount).

The cost of fabric is per square metre so all the measurements in centimetres are converted into metres by dividing them by 100 (as there are 100 centimetres in a metre). This ensures the areas are worked out in square metres rather than square centimetres.

The curved surface area of a cylinder = πdh , where d is the diameter and h is the height. This works as it could be considered as a rectangle folded round to give the cylinder. πd works out the circumference, which would be the base of the rectangle and h is the height. Area of rectangle = base \times height. 0.08×2 gives the diameter as the radius is 0.08m and the diameter is double the radius. Multiplying the curved surface area by the price of £400 per square metre works out the cost of the fabric. Then adding the £3.50 for the other costs for A.

Area of triangle = $\frac{1}{2} \times$ base \times height. The triangle in lampshade B has a base of 0.15m and a height of 0.24m. The area is multiplied by 4 as there are 4 triangular faces. Multiplying the total area by the the price of £400 per square metre works out the cost of the fabric. Then adding the £7.50 for the other costs for B.

Answer 1.31 : 1



16

In a running club there are 50 females and 80 males.

If a female is chosen at random, the probability she has blue eyes is 0.38

If a male is chosen at random, the probability he has blue eyes is 0.6

One person is chosen at random.

Show that the probability the person has blue eyes is **more than** 0.5

[4 marks]

$$\frac{0.38 \times 50 + 0.6 \times 80}{50 + 80} = 0.52$$

Expressing the number of people with blue eyes as a fraction of the number of people then converting the fraction into a decimal, which is more than 0.5.

Multiplying the probability of a female having blue eyes by the number of females gives the number of females with blue eyes. Multiplying the probability of a male having blue eyes by the number of males gives the number of males with blue eyes.

Adding the number of females with blue eyes and males with blue eyes gives the number of people with blue eyes. Adding the females and males gives the number of people

17

$$w = \frac{3}{5\sqrt{x}}$$

Circle the expression for w^2

[1 mark]

$$\frac{6}{10x^2}$$

$$\frac{9}{25x^2}$$

$$\frac{6}{10x}$$

$$\frac{9}{25x}$$

Squaring w also squares the right side of the equation. To square a fraction, square the numerator and the denominator. $3^2 = 9$. $(5\sqrt{x})^2 = 5 \times \sqrt{x} \times 5 \times \sqrt{x} = 5 \times 5 \times \sqrt{x} \times \sqrt{x} = 25x$.

The square root on the x gets cancelled out as it is squared



18

Here is some information about the ages of people at a concert.

Age, x (years)	Frequency
$10 \leq x < 15$	8
$15 \leq x < 25$	24
$25 \leq x < 40$	30
$40 \leq x < 70$	39

C F d

Frequency density = frequency/class width

$$8/(15 - 10) = 1.6$$

$$24/(25 - 15) = 2.4$$

$$30/(40 - 25) = 2$$

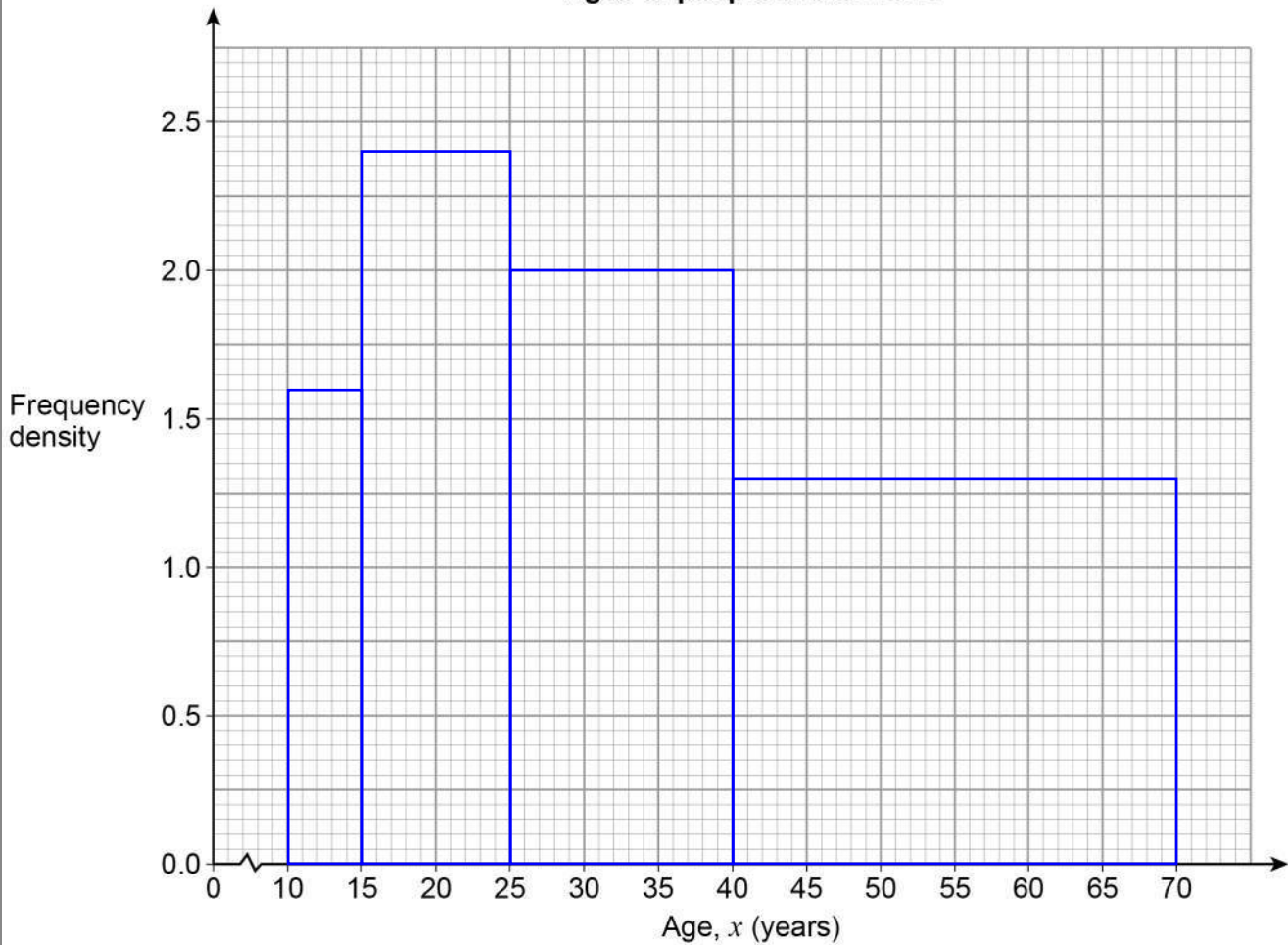
$$39/(70 - 40) = 1.3$$

Working out the frequency densities for each category. The class width is the range of the values of each category

Draw a histogram to represent the information.

[3 marks]

Ages of people at a concert



Turn over ►



19

The length of a roll of ribbon is 30 metres, correct to the nearest half-metre.

A piece of length 5.8 metres, correct to the nearest 10 centimetres, is cut from the roll.

Work out the maximum possible length of ribbon left on the roll.

[3 marks]

$$(30 + \frac{1}{2} \times \frac{1}{2}) - (5.8 - \frac{1}{2} \times \frac{10}{100})$$

The maximum possible length is left when the amount started with is as great as possible and the amount cut from the roll is as small as possible.

The resolution of the 30m is 1/2 m. Halving this and adding it on to 30 gives the upper bound of the length of the roll.

The resolution of the 5.8m is 10cm (which is divided by 100 to convert it into metres). Halving this and subtracting it from 5.8 gives the lower bound of the length cut from the roll

Answer 24.5 metres



20

Curve P has equation $y = 2(x - 1)^2 - 5$ Curve Q is a reflection in the y -axis of curve P.

Work out the equation of curve Q.

Give your answer in the form $y = ax^2 + bx + c$ where a , b and c are integers.**[3 marks]**

$$y = 2(-x - 1)^2 - 5$$

It is a reflection in the y -axis so all the x become negative

$$= 2(x^2 + 2x + 1) - 5$$

Expanding out the square bracket using
'square the first term, double the product
of the two terms, square the last term'

Expanding out the bracket and subtracting 5

Answer

$$y = 2x^2 + 4x - 3$$

Turn over for the next question**Turn over ►**

21

Priya and Joe travel the same 16.8 km route.

Priya starts at 9.00 am and walks at a constant speed of 6 km/h

Joe starts at 9.30 am and runs at a constant speed.

Joe overtakes Priya at 10.20 am

At what time does Joe finish the route?

[5 marks]

 $S^D T$

From the formula triangle, time = distance/speed

 $9:30 + 16.8$

$$\left(\frac{6(10:20 - 9:00)}{10:20 - 9:30} \right)$$

Adding the time taken for Joe to finish the route to 9:30am gives the time he finished. The distance of 16.8km is divided by Joe's speed.

To get Joe's speed, we need to divide the distance he has done by 10:20am by the amount of time taken between 9:30am and 10:20am (which is 10:20 - 9:30). The distance he has done by 10:20am is the same distance as Priya has done by 10:20am (as Joe overtakes Priya at this time).

The distance Priya has done is found by multiplying her speed by the amount of time taken (which is $6(10:20 - 9:00)$)

Answer

11:15am

To put time into the calculator, enter the hours, press the button on the left, enter the minutes then press the button on the left again. For example, 10:20 should be entered as 10°20° into the calculator.

To convert the result the calculator gives (11.25) into time, press the button on the left and it should show 11°15'0'', which means 11 hours, 15 minutes and 0 seconds



22 An approximate solution to an equation is found using the iterative formula

$$x_{n+1} = \frac{(x_n)^3 - 2}{10} \quad \text{with } x_1 = -1$$

22 (a) Work out the values of x_2 and x_3

[2 marks]

Enter -1 then press =. Enter (Ans³ - 2)/10
and press = to get x_2 . Press = again to get x_3

$$x_2 = \underline{\quad -0.3 \quad}$$

$$x_3 = \underline{\quad -0.2027 \quad}$$

22 (b) Work out the solution to 5 decimal places.

[1 mark]

Following on from what was done in part (a), keep pressing = until all
the decimal places do not change. It should show -0.2008097565

$$x = \underline{\quad -0.20081 \quad}$$

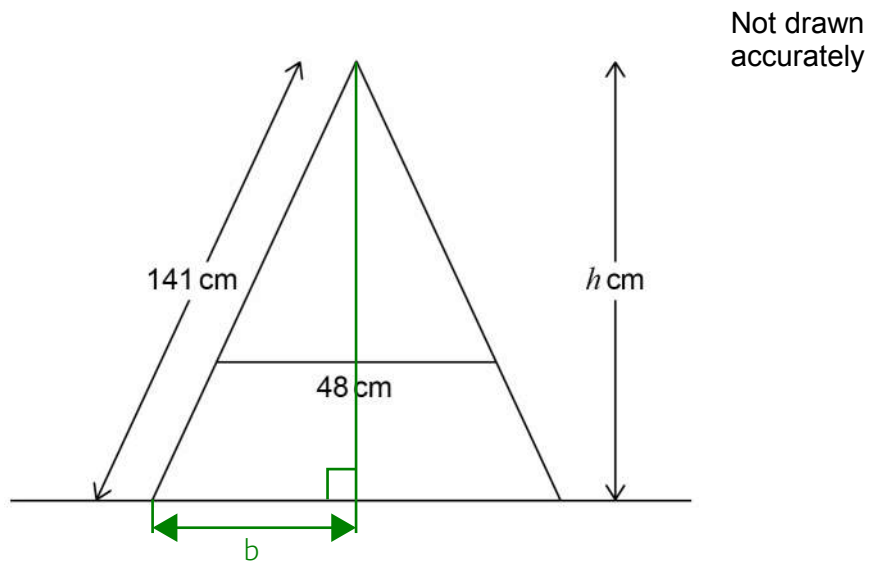


23

The diagram shows the side view of a step ladder with a horizontal strut of length 48 cm

The strut is one third of the way up the ladder.

The symmetrical cross section of the ladder shows two similar triangles.



Work out the vertical height, h cm, of the ladder.

[5 marks]

$$\sqrt{141^2 - \left(\frac{48 \div \frac{2}{3}}{2}\right)^2}$$

Drawing a line down the line of symmetry divides the shape in two and gives a right angled triangle. The missing side is h and this can be found using Pythagoras' Theorem.

$a^2 + b^2 = c^2$, where c is the longest side and a and b are the shorter sides. Rearranging to make a (which is representing h) the subject gives $a = \sqrt{c^2 - b^2}$.

b is found by using the fact the triangles are similar. The strut is $\frac{1}{3}$ of the way up the ladder, so is $\frac{2}{3}$ of the way down. $\frac{2}{3}$ of the base of the ladder is 48cm so dividing 48 by $\frac{2}{3}$ gives the base. Dividing the base by 2 gives length b

Answer 136.3 cm



24

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

Volume of a cone = $\frac{1}{3}\pi r^2 h$ where r is the radius and h is the perpendicular height

A sphere has radius $2x$ cm

A cone has

radius $3x$ cm

perpendicular height h cm

The sphere and the cone have the same volume.

Work out radius of cone : perpendicular height of cone

Give your answer in the form $a : b$ where a and b are integers.

[4 marks]

$$\frac{4}{3}\pi(2x)^3 = \frac{1}{3}\pi(3x)^2 h$$

Setting the volume of the sphere equal to the volume of the cone as they have the same volume. Substituting $2x$ for r in the volume of the sphere and $3x$ for r in the volume of the cone

$$\frac{\frac{4}{3}\pi \times 8x^3}{\frac{1}{3}\pi \times 9x^2} = h$$

Rearranging to find h and expanding the cube and square brackets

$$3x : \frac{32}{9}x$$

Radius of cone : perpendicular height of cone

$$x^3/x^2 = x$$

The coefficient of x is found by typing $(4/3 \pi \times 8)/(1/3 \pi \times 9)$ into the calculator, which gives $32/9$

Dividing both sides of the ratio by x then multiplying by 9 to get rid of the fraction and give integers

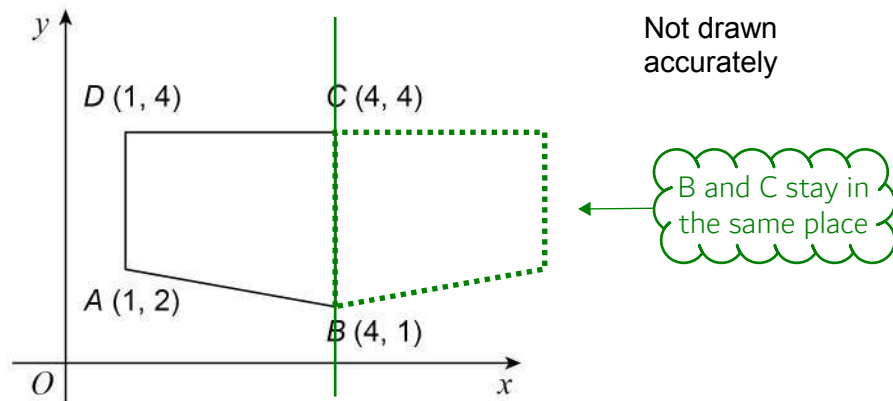
Answer 27 : 32

9

Turn over ►



25

 $ABCD$ is a quadrilateral.The quadrilateral is reflected in the line $x = 4$

Which vertices are invariant?

Circle your answer.

[1 mark]

A and D

C and D

B and C

B and D



26

$$f(x) = \frac{2x+3}{x-4}$$

Work out $f^{-1}(x)$

$$x = \frac{2y+3}{y-4}$$

$f(x)$ is basically y . The inverse function is when x and y are swapped so doing this then rearranging to make y the subject finds the inverse function

[4 marks]

$$xy - 4x = 2y + 3$$

Multiplying both sides by $y - 4$ gets rid of the denominator on the right. Expanding out $x(y - 4)$ on the left

$$xy - 2y = 4x + 3$$

Bringing all the terms involving y to the left and everything else to the right. Subtracting $2y$ and adding $4x$ to both sides

$$y(x-2) = 4x+3$$

Factorising to get y out of both the terms

Answer

$$\frac{4x+3}{x-2}$$

Dividing both sides by $x - 2$ makes y the subject. This is the inverse function

Turn over for the next question

- 27 The line $y = 3x + p$ and the circle $x^2 + y^2 = 53$ intersect at points A and B .
 p is a positive integer.

- 27 (a) Show that the x -coordinates of points A and B satisfy the equation

$$10x^2 + 6px + p^2 - 53 = 0$$

[3 marks]

$$x^2 + (3x + p)^2 = 53$$

Substituting $3x + p$ for y in the second equation

$$x^2 + 9x^2 + 6px + p^2 - 53 = 0$$

Expanding out the square bracket using 'square the first term, double the product of the two terms, square the last term'. Subtracting 53 from both sides

$$10x^2 + 6px + p^2 - 53 = 0$$

Collecting the like terms



27 (b) The coordinates of A are (2, 7)

Work out the coordinates of B.

You **must** show your working.

[5 marks]

$$7 = 3(2) + p$$

A satisfies the equation $y = 3x + p$ so substituting in the x and y-coordinates into the equation allows us to rearrange and find p

$$1 = p$$

$$10x^2 + 6x - 52 = 0$$

Substituting in 1 for p in $10x^2 + 6px + p^2 - 53 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 10 \times -52}}{2 \times 10}$$

Using the quadratic formula to solve x. It is possible to simplify and factorise but this is quicker and easier on a calculator paper

$$x = 2, x = -2.6$$

The first solution is the x-coordinate of point A. The second solution is the x-coordinate of point B

$$y = 3(-2.6) + 1$$

Substituting -2.6 for x and 1 for p in the equation $y = 3x + p$ finds y for point B

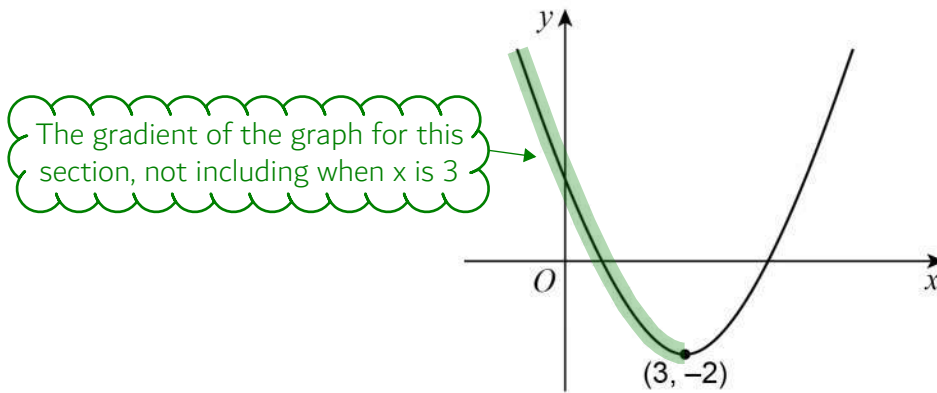
Answer (-2.6 , -6.8)

Turn over for the next question



28

Here is a sketch of a quadratic curve.

The turning point is $(3, -2)$ Not drawn
accuratelyCircle the correct statement about the gradient of the curve for $x < 3$

[1 mark]

gradient is positive

gradient is negative

gradient is zero

gradient could be any value

END OF QUESTIONS

