Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

GCSE MATHEMATICS

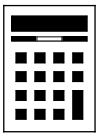
H

Higher Tier Paper 3 Calculator

Wednesday 8 November 2017 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.



Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice
- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

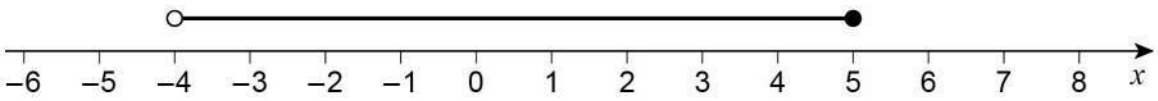
Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

.CG Maths.

Answer **all** questions in the spaces provided

1 Circle the inequality shown by the diagram.



[1 mark]

$-4 \leqslant x < 5$ $-4 \leqslant x \leqslant 5$ $-4 < x < 5$ $-4 < x \leqslant 5$

The closed dot means it can be equal to the value it is above.
The open dot means it cannot be equal to the value it is above

2 y is 100% **more** than x.

Circle the ratio $x : y$

[1 mark]

1 : 100 100 : 1 1 : 2 2 : 1

Increasing by 100% makes it 200%. So the ratio would be 100 : 200, which simplifies to 1 : 2

3 The first four terms of a sequence are –10 –8 –6 –4

Circle the expression for the nth term of the sequence.

[1 mark]

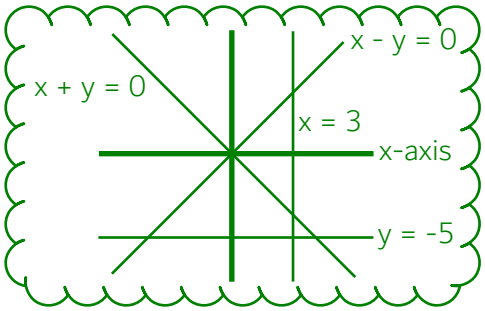
$-12 - 2n$ $-8 - 2n$ $n + 2$ $2n - 12$

The sequence increases by 2 between each term so it must involve 2n

4 Circle the equation of the line that is parallel to the x-axis.

[1 mark]

$y = -5$ $\quad x - y = 0$ $\quad x = 3$ $\quad x + y = 0$



5 Multiply out and simplify $(x - 8)^2$

[2 marks]

$(x-8)(x-8)$

$x^2-8x-8x+64$

Answer $x^2-16x+64$

Turn over for the next question

6

6 Show that 268 can be written as the sum of a power of 3 and a square number.

[2 marks]

Enter table mode by pressing MENU then 3. f(x) = 268 - 3^x. Ignore g(x). Start: 1. End: 30. Step: 1

Subtracting the powers of 3 from 268 until the result is a square number. 268 - 3^5 = 25, which is a square number

Answer 3^5+25

7 Here is some information about the times taken by 40 people to fill in a form.

Time, t minutes	Number of people
$0 < t \leqslant 5$	3
$5 < t \leqslant 10$	9
$10 < t \leqslant 15$	11
$15 < t \leqslant 20$	17

3
12
23

In which class interval is the median?

Circle your answer.

[1 mark]

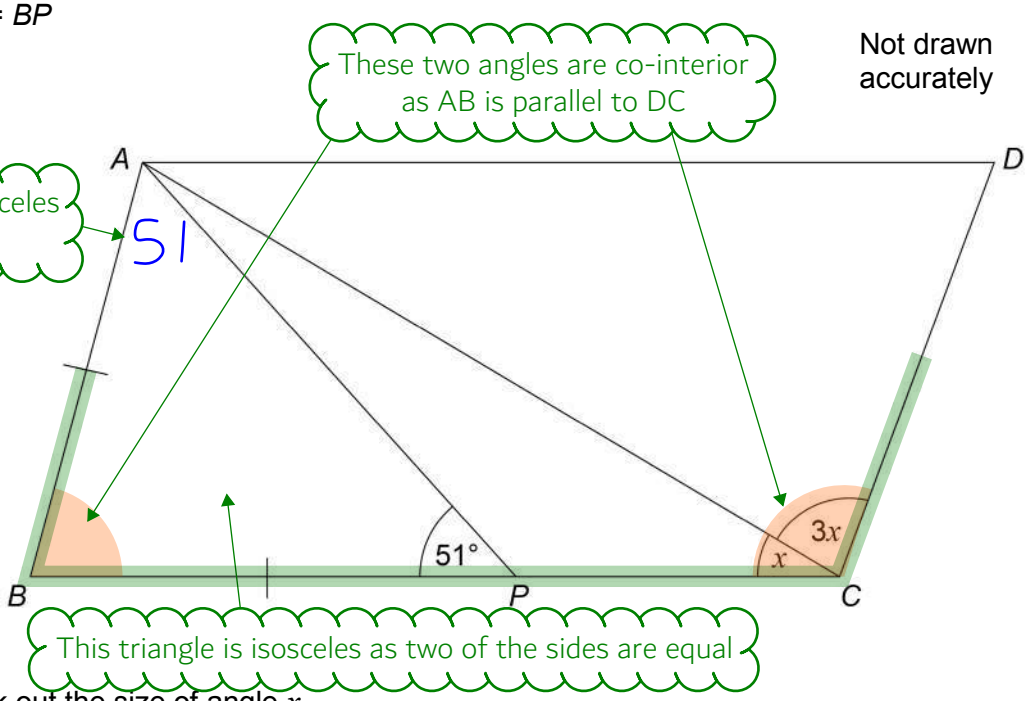
$0 < t \leqslant 5$ $\quad$ $5 < t \leqslant 10$ $\quad$ (circled) $10 < t \leqslant 15$ $\quad$ $15 < t \leqslant 20$

$\frac{40+1}{2} = 20.5$

The formula (n + 1)/2, where n is the number of people, works out that the median is the 20.5th value. Listing a cumulative frequency until it goes over 20.5. The median is in the category where it first goes over

8 *ABCD* is a parallelogram.

AB = *BP*



Not drawn accurately

These two angles are co-interior as AB is parallel to DC

Base angles of an isosceles triangle are equal

51

This triangle is isosceles as two of the sides are equal

Work out the size of angle x.

[4 marks]

$180-51\times2+x+3x=180$ ← Co-interior angles add up to 180

Expressing angle ABP by subtracting the other two angles in the triangle. The angles in a triangle add up to 180

Expressing angle PCD, which is co-interior to ABP

$x=\frac{180-(180-51\times2)}{4}$ ← Rearranged the equation to find x. Subtracting (180 - 51 x 2) from both sides. x + 3x = 4x so dividing both sides by 4 makes x the subject

Answer 25.5 degrees

Turn over for the next question

7

9 (a) Rearrange $v = u + at$ to make t the subject of the formula.

[2 marks]

$v - u = at$ ← Subtracting u from both sides gets the term involving t on its own

Answer $\frac{v-u}{a} = t$

Dividing both sides by a gets t on its own

9 (b) Complete this table with consistent metric units.

[2 marks]

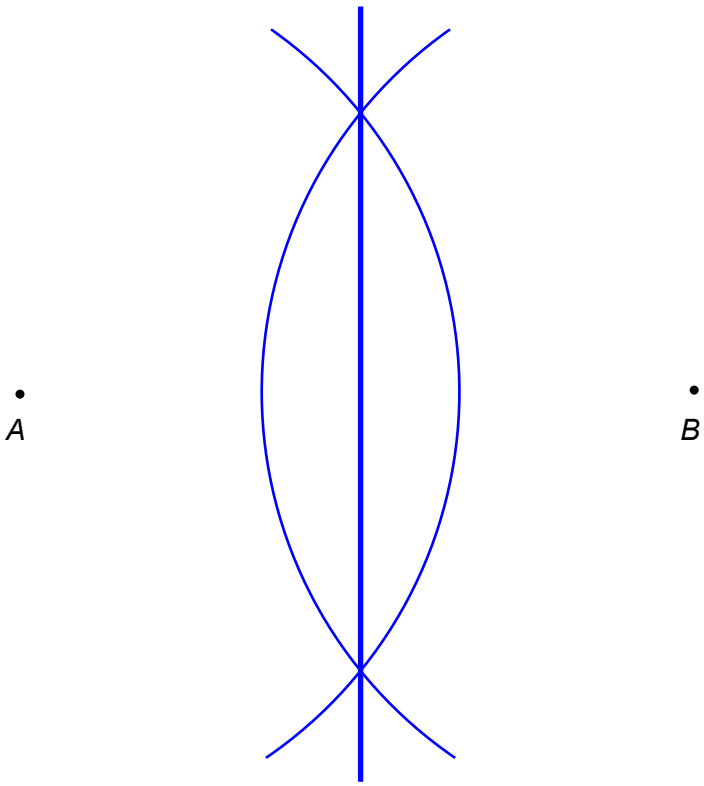
Distance	Time	Speed	Acceleration
m	s	m/s	m/s^2

Speed = distance/time. Dividing the metres by seconds give the unit of speed.

Acceleration = (change in speed)/(change in time). Dividing the unit of speed by seconds gives the unit of acceleration

10 Construct a locus of points that are the same distance from points *A* and *B*.

[2 marks]



Creating a perpendicular bisector of line AB

Scribe an arc from A which is at least halfway between A and B.
Scribe an arc from B which is at least halfway between A and B.
Draw a straight line through the points the two arcs cross

Turn over for the next question

6

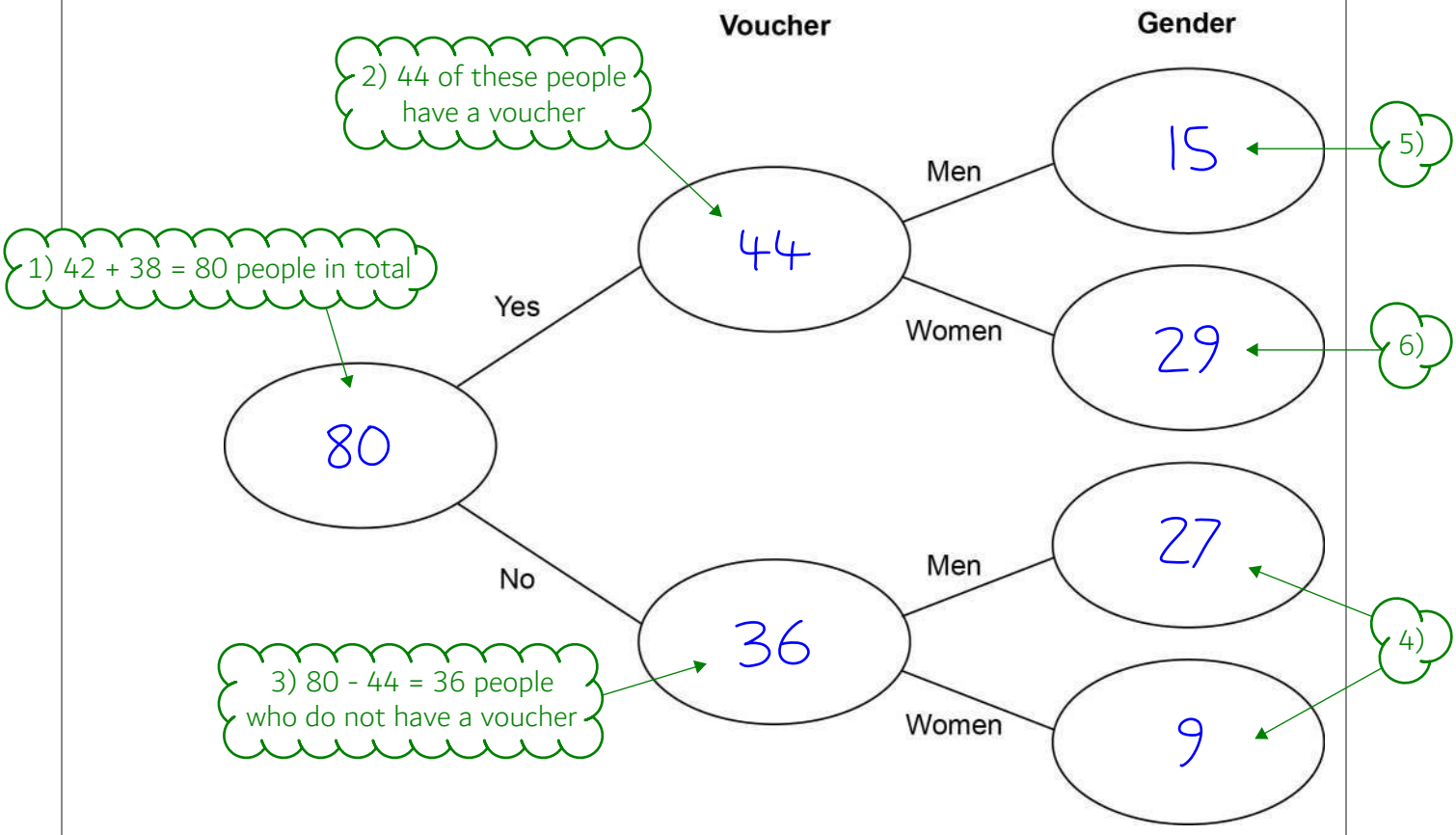
11 42 men and 38 women visit a restaurant.

44 of these people have a voucher.

Three times as many men as women do **not** have a voucher.

11 (a) Complete the frequency tree.

[4 marks]



4) The ratio of men who do not have a voucher to women who do not have a voucher is 3 : 1. Dividing the 36 who do not have a voucher into this ratio. There are 4 parts in total so 36/4 = 9, which is the value of 1 part of the ratio and is how many women do not have a voucher. 9 x 3 works out that the value of the 3 parts of the ratio is 27.

5) There are 42 men in total and 27 of these do not have a voucher. So 42 - 27 works out that 15 men do have a voucher.

6) There are 38 women in total and 9 of these do not have a voucher. So 38 - 9 works out that 29 women do have a voucher

11 (b) A voucher takes **15% off** the bill.

After using the voucher, the bill for a meal is £27.20

How much was the bill before using the voucher?

[3 marks]

$\frac{27.20}{100-15} \times 100$

100 - 15 works out the percentage of the price of the meal it has reduced to. Dividing by this percentage works out 1%. Multiplying by 100 works out 100%, which is the price before the voucher was used

Answer £ 32

Turn over for the next question

7

12 The distance by road from Newport to London is 140 miles.

Tom travels by coach from Newport to London.
The coach leaves Newport at 1.30 pm

12 (a) He assumes the coach will travel at an average speed of 50 mph

Use his assumption to work out the arrival time in London.

[3 marks]

$s\ ^{d}\ t$ ← This is a speed, distance, time problem so writing the formula triangle

$1:30 + \frac{140}{50}$ ← From the formula triangle, time = distance/speed. 140/50 works out the time taken in hours. Adding this to the original time works out the arrival time

FACT B ° ' " Enter 1:30 by pressing 1, then the button on the left, then 30, then the button on the left again. It should appear as 1°30°

Answer 4:18pm

FACT B ° ' " Convert the answer of 4.3 hours into time by pressing the button on the left. This gives the answer as 4°18'0"

12 (b) In fact, the coach has a lower average speed.

How does this affect the arrival time?

[1 mark]

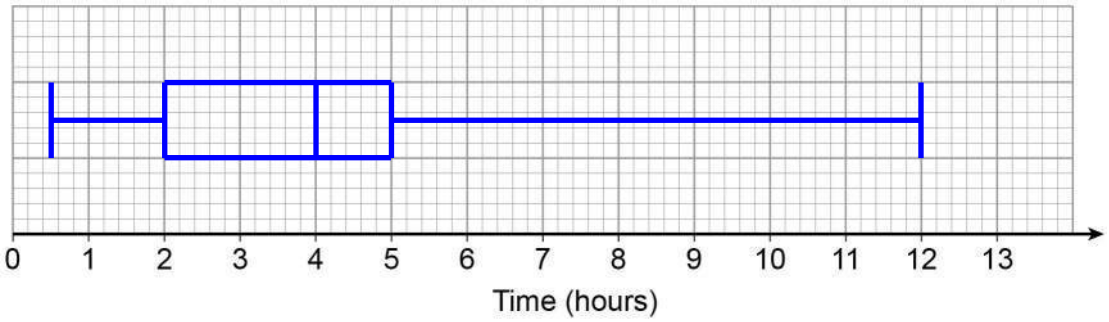
It will be later ← Time = distance/speed so having a lower speed will mean dividing by less, which increases the time

13 Here is some information about the length of time cars stayed in a car park.

Shortest time	30 minutes	Lower quartile	2 hours
Longest time	12 hours	Interquartile range	3 hours
		Median time	4 hours

Draw a box plot to show this information.

[3 marks]



Turn over for the next question

Drawing vertical lines for the lowest, lower quartile, median, upper quartile and highest. Connecting the lowest value to the lower quartile and the upper quartile to the highest value using horizontal lines. Drawing a box around the quartiles and median. The upper quartile is found by adding the interquartile range to the lower quartile. 2 + 3 = 5

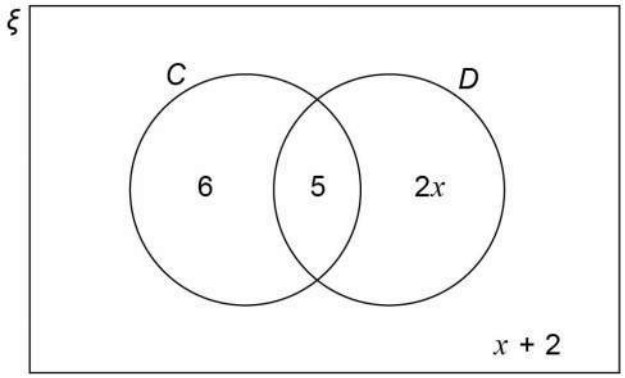
7

14 In the Venn diagram

ξ represents 31 students in a class

C is students who have a cat

D is students who have a dog



14 (a) One student from the class is picked at random.

Work out the probability that the student has a dog.

[3 marks]

$6+5+2x+x+2$

$3x+13=31$

Expressing the total number of students in the class in terms of x. Simplifying the expression by collecting like terms then setting it equal to the 31

$x=\frac{31-13}{3}=6$

Rearranging to find x by subtracting 13 from both sides then dividing both sides by 3

Answer $\frac{17}{31}$

The number who have a dog is 5 + 2x = 5 + 2(6) = 5 + 12 = 17.
17 out of the 31 total students has a dog

14 (b) One of the students who has a cat is picked at random.

Work out the probability that this student has a dog.

[1 mark]

Answer $\frac{5}{11}$

11 have a cat as 6 + 5 = 11. Out of these, 5 also have a dog

15 Circle the highest common factor (HCF) of $6xy^2$ and $4x^3y$

[1 mark]

$2xy^2$ $\quad$ (2xy) $\quad$ $12x^3y^2$ $\quad$ $24x^4y^3$

2 is the highest common factor of 6 and 4. x is the highest common factor of x and x^3. y is the highest common factor of y^2 and y

16 $f(x) = x^2 - x^3$

Circle the value of f(–3)

[1 mark]

18 $\quad$ –18 $\quad$ (36) $\quad$ –36

$(-3)^2 - (-3)^3 = 36$

Negatives must go into brackets when raised to a power. f(-3) basically means to substitute -3 for x in f(x)

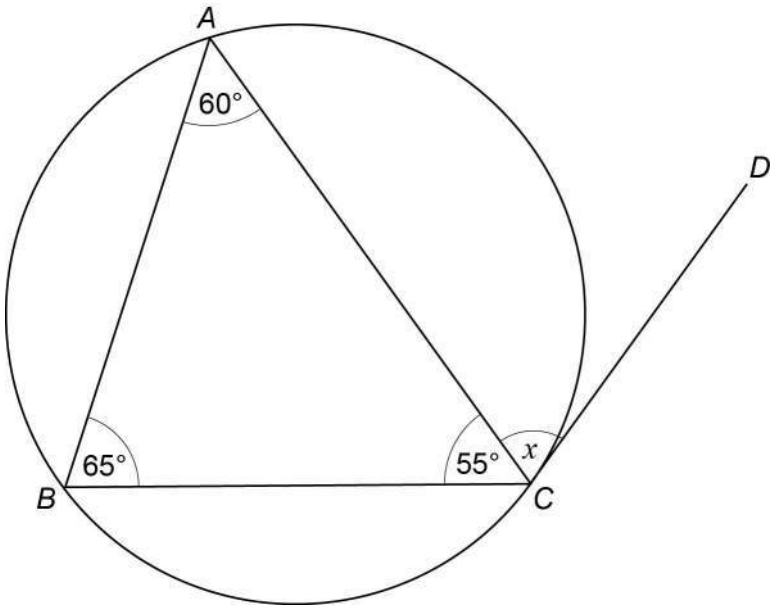
Turn over for the next question

6

19 *A*, *B* and *C* are points on a circle.

CD is a tangent to the circle.

Not drawn accurately



Write down the size of angle x.

Give a reason for your answer.

[2 marks]

Answer 65 degrees

Reason Alternate segment theorem

Turn over for the next question

9

20 w is a positive number.

x is 10% more than w.

y is 10% less than x.

Let w = 100. x = 1.1 x 100 = 110. y = 0.9 x 110 = 99

Which statement is true?

Tick **one** box.

[1 mark]

$w < x$ and $w < y$ ☐

$w < x$ and $w = y$ ☐

$x > y$ and $w > y$ ☑

$x > y$ and $w = y$ ☐

21 N is a number.

As a product of prime factors in index form $N = 2 \times 3^4 \times y^3$

Work out $3N^2$ as a product of prime factors in index form.

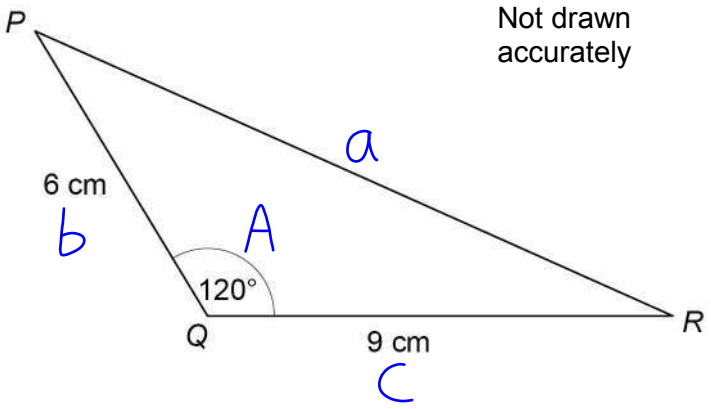
Give your answer in terms of y.

[3 marks]

$3N^2 = 3(2 \times 3^4 \times y^3)^2$. Everything in the bracket is raised to the power of 2 first. This doubles the power of each of them as $(a^x)^y = a^{xy}$. Then multiplying by 3 adds 1 to the power of the 3^8 as $a^x \times a^y = a^{x+y}$

Answer $2^2 \times 3^9 \times y^6$

22 Here is a triangle.



Work out the length *PR*.

[3 marks]

$a^2 = b^2 + c^2 - 2bc\cos A$

The sine rule can't be used as there are not opposite pairs of sides and angles. So the cosine rule could be used

$PR = \sqrt{6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 120}$

Rearranged to make a the subject and substituted in the values of b, c and A

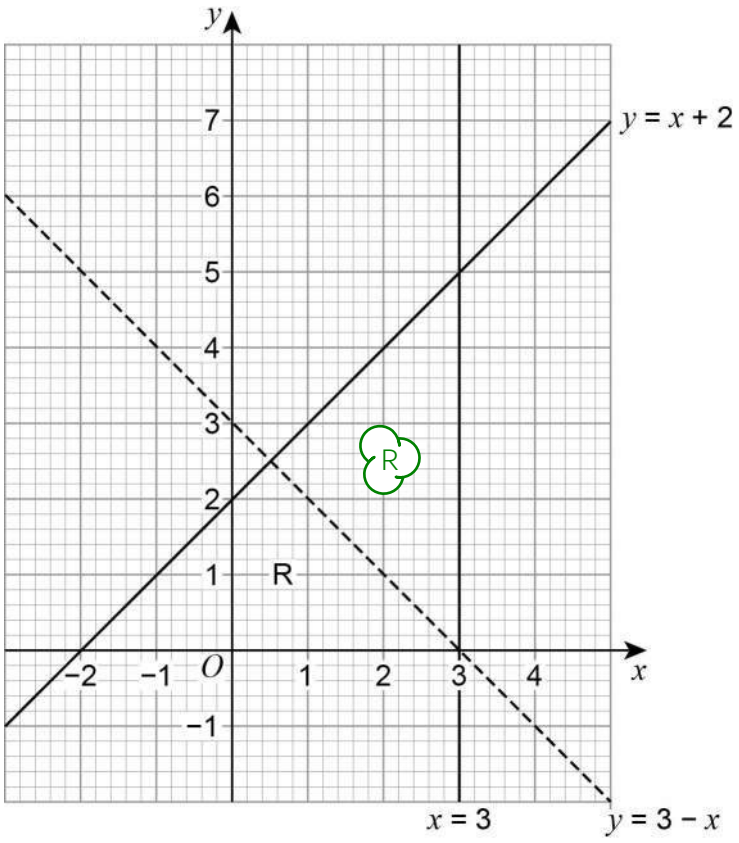
Answer $3\sqrt{19}$ cm

Turn over for the next question

7

23 Joe draws this graph to identify the region R represented by

$y \leqslant x + 2$ and $y > 3 - x$ and $x < 3$



Make **two** criticisms of his graph.

[2 marks]

Criticism 1 x = 3 should be dashed As x cannot be equal to 3

Criticism 2 R is in the wrong place It should go in the place indicated

24 $a : b = 9 : 4$ and $10b = 7c$

Work out $a : c$ in its simplest form.

[3 marks]

a	b	c
9	4	
	7	10
63	28	40

In the equation 10b = 7c, b could be 7 and c could be 10

28 is a common multiple of 4 and 7. The 4 must be multiplied by 7 to get 28 so the 9 must also be multiplied by 7. The 7 must be multiplied by 4 to get 28 so the 10 must also be multiplied by 4. Now the ratios can be combined as they both have b in common and they have the same number of parts for it

Answer 63 : 40

63/40 = 63/40

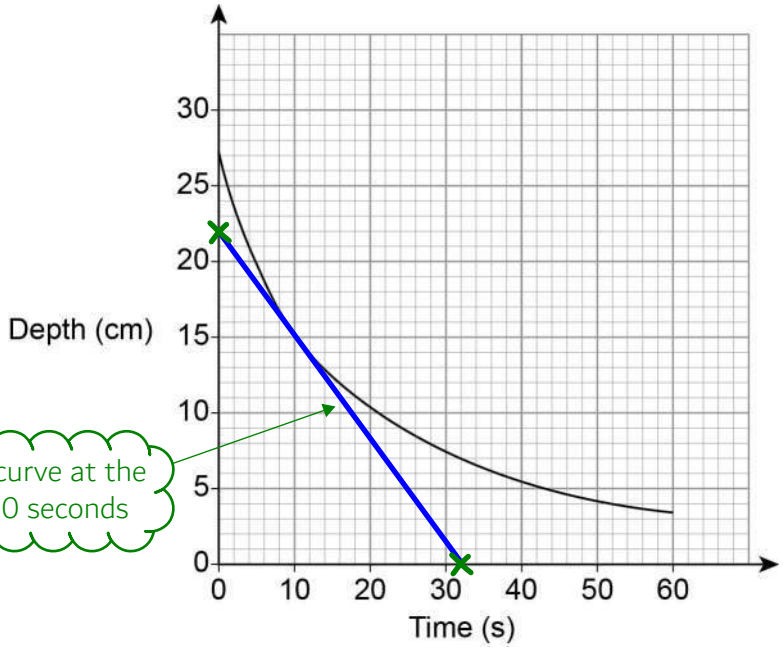
The fraction 63/40 does not simplify so the ratio 63 : 40 will not either

Turn over for the next question

5

25 Liquid is leaking out of a container.

The graph shows the depth of the liquid for 60 seconds.



Drawing a tangent to the curve at the point where the time is 10 seconds

Use the graph to work out an estimate of the rate of decrease of depth at 10 seconds.

You **must** show your working.

[3 marks]

$\frac{0-22}{32-0}$ ← The rate of change of depth is the gradient, which can be found using (change in y)/(change in x)

Answer 0.7 cm/s

The rate of change of depth is negative but as it asks for the rate of decrease the negative is ignored

26 $a^2 - b^2 \equiv (a + b)(a - b)$

a and b are positive whole numbers with $a > b$

$a^2 - b^2$ is a **prime** number.

Why are a and b consecutive numbers?

[2 marks]

(a + b) or (a - b) must be 1

As (a + b)(a - b) is the factorised form and prime numbers only have two factors: themselves and 1. (a + b) and (a - b) are factors of a^2 - b^2

(a + b) cannot be 1 so (a - b) must be 1

The smallest b could be is 1. The smallest a could be is 2. So the smallest (a + b) could be is 3. Therefore (a - b) must be 1. Subtracting b from a works out the difference and as this difference is 1, they must be consecutive (next to each other)

Turn over for the next question

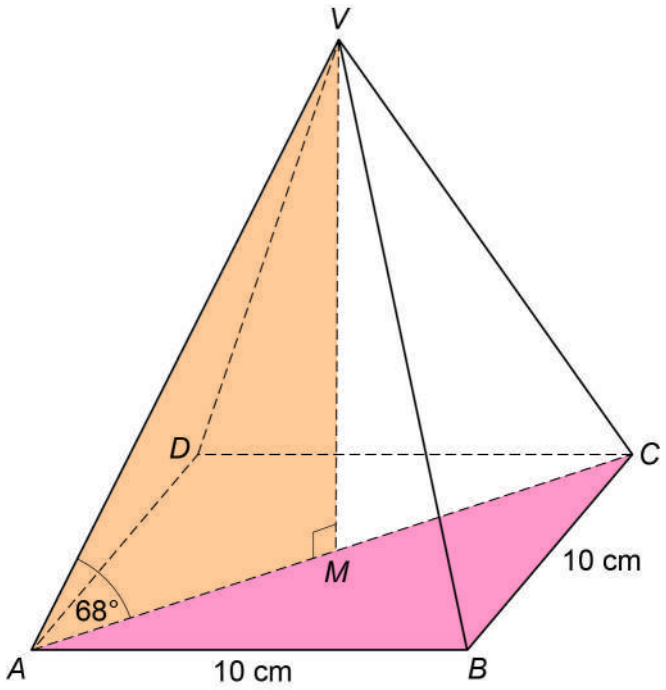
5

27 *VABCD* is a square-based pyramid.

The horizontal base *ABCD* has side length 10 cm and centre *M*.

Angle *VMA* is 90°

Angle *VAM* is 68°



Volume of pyramid = $\frac{1}{3}$ × area of base × perpendicular height

Work out the volume of the pyramid.

[6 marks]

SOHCAHTOA

The perpendicular height, VM, can be found using right angled trigonometry in the orange triangle. VM is the opposite so O is ticked and AM is the adjacent, which can be found, so A is ticked

$a^2 + b^2 = c^2$

AM is half of AC, which is the hypotenuse of the pink triangle. This can be found using Pythagoras' Theorem

$\frac{1}{3} \times 10 \times 10 \times \tan 68 \times \frac{1}{2}\sqrt{10^2 + 10^2}$

Area of the base. The base is a square and area of a square = length x width

Perpendicular height. The TOA formula triangle can be used to find the opposite, VM, as there are two ticks on it. Covering over O finds that opposite = (tan of the angle) x adjacent. The angle is 68°. The adjacent AM is half of AC as M is the centre of the base. AC is found using Pythagoras' Theorem. Rearranging it gives that $c = \sqrt{a^2 + b^2}$. a is 10 and b is 10

Answer 583.4 cm^3

Turn over for the next question

6

28 $y = p \times q^{x-1}$ where p and q are numbers.

$y = 10$ when $x = 1$

$y = 0.3125$ when $x = 6$

Work out the value of y when $x = 3$

[5 marks]

$10 = p \times q^{1-1}$ ← Substituting 10 for y and 1 for x in the equation. 1 - 1 = 0 and anything to the power of 0 is 1. So 10 = p x 1 meaning p = 10

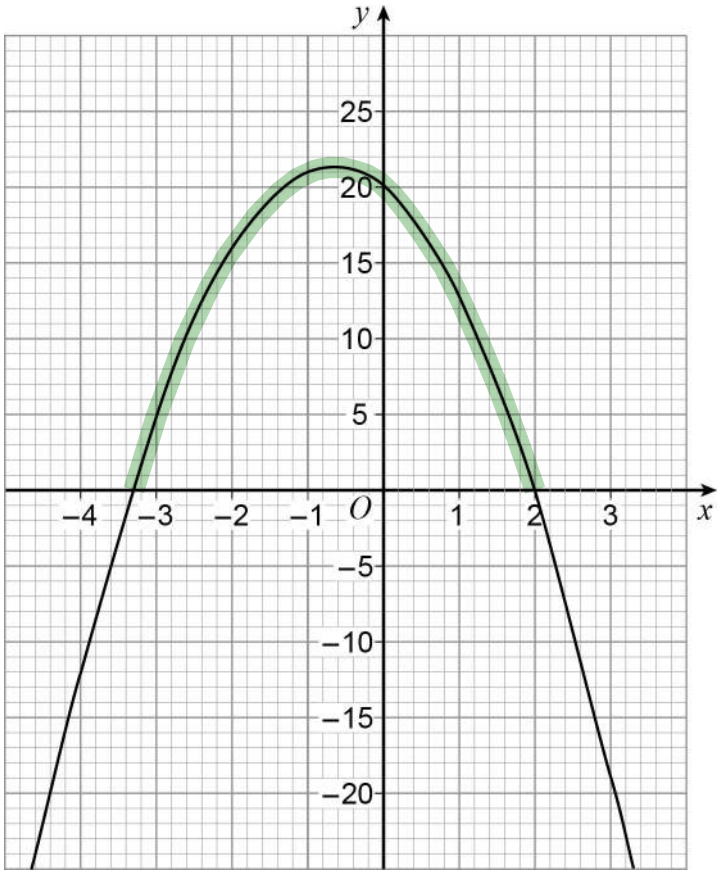
$0.3125 = 10 \times q^{6-1}$ ← Substituting 0.3125 for y, 6 for x and 10 for p in the equation

$\sqrt[5]{\frac{0.3125}{10}} = q$ ← Rearranged to make q the subject. q = 0.5

$y = 10 \times 0.5^{3-1}$ ← Substituted 10 for p, 0.5 for q and 3 for x to find the value of y

Answer 2.5

29 Here is the graph of $y = f(x)$ where $f(x)$ is a quadratic function.



Write down all the **integer** solutions of $f(x) \geqslant 0$

[2 marks]

The highlighted region of the line is where f(x) ⩾ 0. It is basically asking what the integer values of x are when the line is above or on the x axis

Answer -3, -2, -1, 0, 1, 2

Turn over for the next question

7

30 $f(x) = \frac{x}{3} + 4$ for all values of x.

$g(x) = 6x^2 + 3$ for all values of x.

Work out $fg(x)$.

Give your answer in the form $ax^2 + b$ where a and b are integers.

[2 marks]

$\frac{6x^2+3}{3} + 4$ ← Substituting g(x) for x in f(x)

$6x^2/3 = 2x^2$. $3/3 = 1$. $1 + 4 = 5$

Answer $2x^2+5$

END OF QUESTIONS

2