

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

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Forename(s)

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Candidate signature

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# GCSE MATHEMATICS

# H

Higher Tier

Paper 1 Non-Calculator

Thursday 24 May 2018

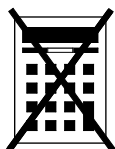
Morning

Time allowed: 1 hour 30 minutes

## Materials

For this paper you must have:

- mathematical instruments



You must **not** use a calculator.

## Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use

Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
<b>TOTAL</b>	

## Advice

- In all calculations, show clearly how you work out your answer.



JUN1883001H01

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to [curtis@cgmaths.co.uk](mailto:curtis@cgmaths.co.uk)

Answer **all** questions in the spaces provided

- 1 Work out  $\sqrt[3]{64 \times 1000}$   $= \sqrt[3]{64} \times \sqrt[3]{1000}$   
Circle your answer. [1 mark]

40

80

400

4000

The numbers can be cube rooted separately. The cube root of 64 is 4 as  $4^3 = 64$  and the cube root of 1000 is 10 as  $10^3 = 1000$ .  $4 \times 10 = 40$

- 2 The vector  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$  translates A to B.  $\leftarrow$  The vector means 2 to the left and 3 up

Circle the vector that translates B to A.

[1 mark]

 $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$  $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$  $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ 

The exact opposite is 2 to the right and 3 down

- 3 Circle the expression that is equivalent to  $3a - a \times 4a + 2a$  [1 mark]

 $8a^2 + 2a$  $12a^2$  $5a - 4a^2$  $3a - 6a^2$ 

The order of operations, BIDMAS, should be followed so multiplication is done first.  $-a \times 4a = -4a^2$ . Then the addition and subtraction can be done in any order so the like terms can be collected



- 4 Circle the number that is closest in value to  $\frac{9.8}{0.0195}$  [1 mark]

5                      50                      500                      5000

$$\frac{10}{0.02} = \frac{1000}{2}$$

Using estimation by rounding each number to 1 significant figure works out an estimation of the value. To eliminate the decimal on the denominator the numerator and denominator of the fraction are multiplied by 100.  $1000/2 = 500$

- 5 Solve  $5(x + 3) < 60$  [2 marks]

$$x + 3 < 12$$

Dividing both sides by 5

Subtracting 3 from both sides

Answer

$$x < 9$$

Turn over for the next question



6

The height of Zak is 1.86 metres.

The height of Fred is 1.6 metres.

Write the height of Zak as a fraction of the height of Fred.

Give your answer in its simplest form.

**[3 marks]**

$$\frac{1.86}{1.6} = \frac{186}{160}$$

Putting the height of Zak over the height of Fred expresses the fraction. Multiplying the numerator and denominator by 100 eliminates the decimals and makes it simpler

$$2 \overline{) \begin{array}{r} 093 \\ 186 \end{array}}$$

Both the numerator and denominator are even so they can both be divided by 2 to get smaller whole numbers

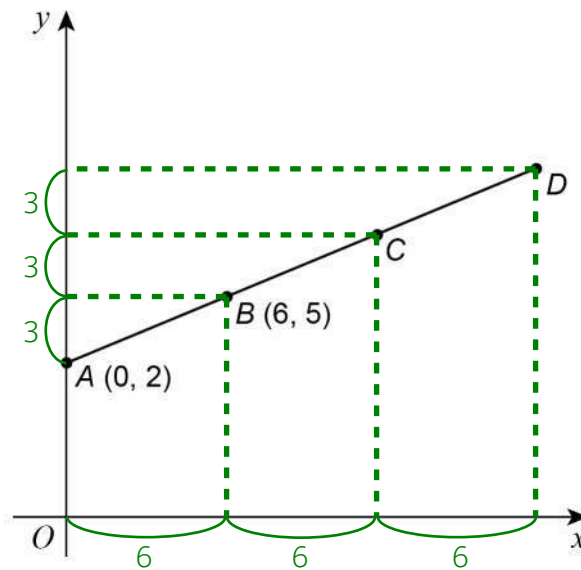
$$2 \overline{) \begin{array}{r} 080 \\ 160 \end{array}}$$

Answer  $\frac{93}{80}$

93 and 80 cannot be divided by the same amount to get smaller whole numbers so the fraction does not go any simpler



7  $A(0, 2)$  and  $B(6, 5)$  are points on the straight line  $ABCD$ .



Not drawn  
accurately

$AB = BC = CD$

Therefore all of the points are equally spaced out

Work out the coordinates of  $D$ .

[3 marks]

As all the points are equally spaced out and are on a straight line, the change in  $x$  and  $y$  between each point must be the same. From  $A$  to  $B$  the  $x$  coordinate changed by 6 from 0 to 6. So it changes by another 6 twice to get from  $B$  to  $D$ .  $6 + 6 + 6 = 18$ . From  $A$  to  $B$  the  $y$  coordinate changed by 3 from 2 to 5. So it changes by another 3 twice to get from  $B$  to  $D$ .  $5 + 3 + 3 = 11$

Answer ( 18 , 11 )

Turn over for the next question





9 The range of a set of numbers is  $15\frac{1}{4}$

The smallest number is  $-2\frac{7}{8}$

Work out the largest number.

[3 marks]

$$\frac{61}{4} - \frac{23}{8}$$

Converting both mixed fractions into improper fractions by multiplying the whole numbers by the denominators then adding the results to the numerators. The range is the distance between the largest and smallest so adding the range to the smallest number works out the largest number.  $-23/8 + 61/4$  is the same as  $61/4 - 23/8$

$$\begin{array}{r} 61 \\ \times 2 \\ \hline 122 \end{array}$$

Multiplying the numerator and denominator of  $61/4$  by 2 to get the denominators the same. Once this is done the numerators can be subtracted

$$\frac{122}{8} - \frac{23}{8}$$

Answer  $\frac{99}{8}$

10  $y$  is inversely proportional to  $x$ .

Complete the table.

[2 marks]

$x$	12	6	3
$y$	2	4	8

$x$  doubled from 6 to 12 so  $y$  must half from 4 to 2.  
 $y$  doubled from 4 to 8 so  $x$  must half from 6 to 3

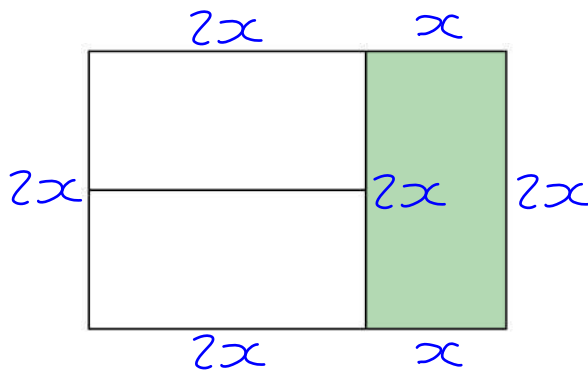
Turn over for the next question





- 11 A large rectangle is made by joining three identical small rectangles as shown.

Let  $x$  be the shorter edge of each small rectangle. The longer edge on each small rectangle must be  $2x$



Not drawn accurately

The perimeter of one small rectangle is 15 cm

Work out the perimeter of the large rectangle.

[4 marks]

$$6x = 15$$

The perimeter of one small rectangle is  $x + x + 2x + 2x = 6x$  which must equal to 15

$$x = \frac{15}{6}$$

Dividing both sides by 6 works out  $x$

$$6 \overline{) 150} \begin{array}{r} 025 \\ 120 \\ \hline 30 \\ 30 \\ \hline 0 \end{array}$$

The perimeter of the large rectangle is  $2x + x + 2x + x + 2x + 2x = 10x$ .  $10 \times 15/6 = 150/6$ . Dividing 150 by 6 works out the perimeter

Answer 25 cm



- 12 Put these numbers in order from smallest to largest.

$$8 \times 10^{-4} \quad 4 \times 10^{-2} \quad 6 \times 10^{-4} \quad 0.07$$

$0.0008, 0.04, 0.0006$

[2 marks]

$\times 10^{-n}$  means to divide by  $10^n$  times. Converting the standard form into ordinary form allows the numbers to be easily compared

Smallest  $6 \times 10^{-4}$

$$8 \times 10^{-4}$$

$$4 \times 10^{-2}$$

Largest  $0.07$

- 13 Circle the volume that is the same as  $15 \text{ cm}^3$

[1 mark]

$$15\,000 \text{ mm}^3$$

$$1.5 \text{ mm}^3$$

$$0.0015 \text{ mm}^3$$

$$150 \text{ mm}^3$$

There are 10mm in 1cm so multiplying by 10 converts the centimetres to millimetres. But as the unit is cubed the 15 should be multiplied by  $10^3$

**Turn over for the next question**

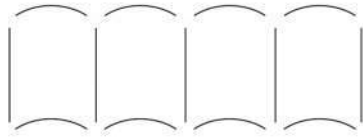
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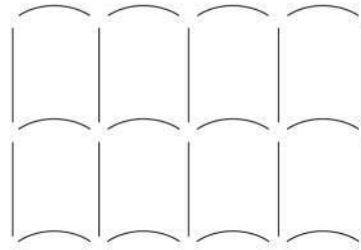
14 Patterns are made using straight lines and arcs.

14 (a)

Pattern A (one row)



Pattern B (two rows)



More rows are added to **Pattern B** so that

$$\text{number of straight lines} : \text{number of arcs} = 10 : 9$$

How many rows are added?

[2 marks]

- 15:16 ← Pattern C (three rows)
- 20:20 ← Pattern D (four rows)
- 25:24 ← Pattern E (five rows)
- 30:28 ← Pattern F (six rows)
- 35:32 ← Pattern G (seven rows)
- 40:36 ← Pattern H (eight rows)

5 more straight lines and 4 more arcs are added to get the next pattern in the sequence. Expressing the ratio of the number of straight lines : number of arcs in each pattern until the ratio simplifies to 10 : 9. Both sides of 40 : 36 can be divided by 4 to get 10 : 9

Answer \_\_\_\_\_

6



14 (b) A different pattern is made using 20 straight lines and 16 arcs.

The straight lines and arcs are made from metal.

20 straight lines cost £12

cost of one straight line : cost of one arc = 2 : 3

Work out the **total** cost of the metal in the pattern.

[3 marks]

$$\begin{array}{r} 00.6 \\ 20 \overline{)12.0} \end{array}$$

Dividing the £12 by the 20 works out the cost of 1 straight line

$$\begin{array}{r} 0.3 \\ 2 \overline{)0.6} \\ \times 0.3 \\ \hline 0.9 \end{array}$$

2 parts of the ratio represent the cost of 1 straight line. Dividing the cost of 1 straight line by 2 works out what 1 part of the ratio is worth. Multiplying this by 3 works out what the 3 parts which represent the cost of 1 arc are worth

$$\begin{array}{r} 16 \\ \times 0.9 \\ \hline 14.4 \end{array}$$

Multiplying the cost of 1 arc by the 16 works out the cost of the 16 arcs

Answer £

26.40

Adding the cost of the straight lines to the cost of the arcs works out the total cost of the pattern.  $14.4 + 12 = 26.4$ , which is £26.40

**Turn over for the next question**



15

A biased dice is thrown.

Here are the probabilities of each score.

<b>Score</b>	1	2	3	4	5	6
<b>Probability</b>	0.25	0.05	0.15	0.05	0.3	0.2

The dice is thrown 200 times.

Work out the expected number of times the score will be odd.

**[3 marks]**

$$\begin{array}{r}
 0.25 \\
 +0.15 \\
 +0.3 \\
 \hline
 0.70 \\
 \times 200 \\
 \hline
 140.00
 \end{array}$$

1 OR 3 OR 5. OR means to add the probabilities. 0.7 is the probability of getting an odd number. Multiplying this by the 200 works out the expected number of times it will be odd

Answer \_\_\_\_\_

140



- 16 The value of  $y$  is 20% more than the value of  $x$ .

Circle the ratio  $x : y$

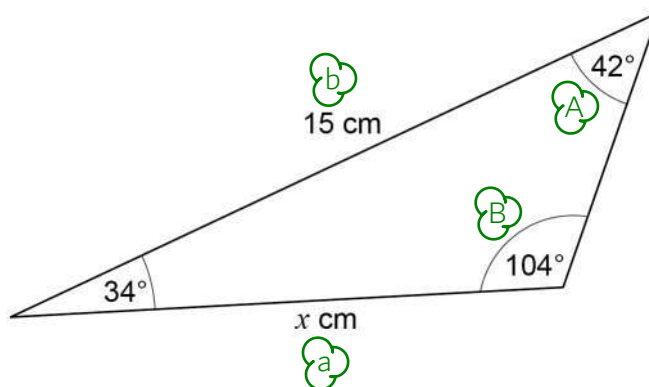
[1 mark]

5 : 6     
  6 : 5     
  4 : 5     
  5 : 4

6 is 20% more than 5

- 17 Here is a triangle.

Not drawn  
accurately



Circle the correct equation.

[1 mark]

$$\frac{\sin x}{42} = \frac{\sin 15^\circ}{104}$$

$$\frac{x}{\sin 42^\circ} = \frac{15}{\sin 104^\circ}$$

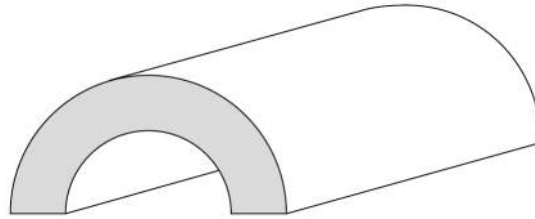
$$\frac{\sin x}{34} = \frac{\sin 15^\circ}{104}$$

$$\frac{x}{\sin 42^\circ} = \frac{15}{\sin 34^\circ}$$

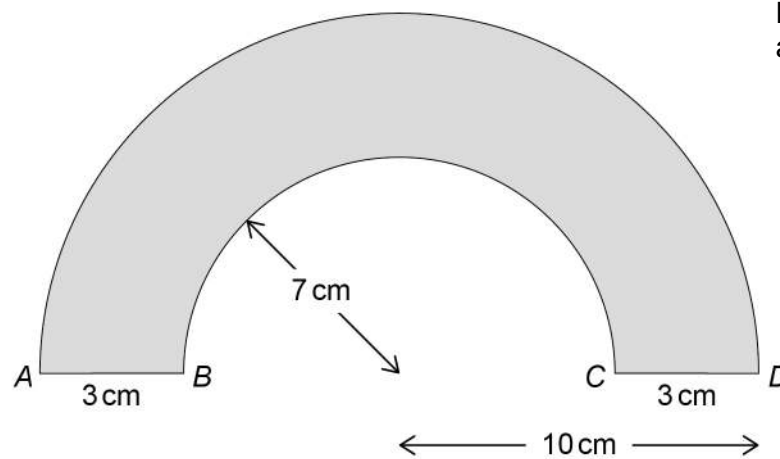
The sine rule is  $a/\sin A = b/\sin B$



- 18 Here is a tunnel for a toy train.



The diagram below shows the cross section of the tunnel.



$AD$  is a semicircular arc of radius 10 cm

$BC$  is a semicircular arc of radius 7 cm

The length of the tunnel is 30 cm

Work out the total area of all **six** faces of the tunnel.

Give your answer in terms of  $\pi$ .

**[5 marks]**



$$2\left(\frac{\pi \times 10^2}{2} - \frac{\pi \times 7^2}{2}\right) + \frac{20\pi}{2} \times 30 + \frac{14\pi}{2} \times 30 + 2 \times 3 \times 30$$

The area of the front and back face of the tunnel. Area of circle =  $\pi \times \text{radius}^2$ . The radius of the cross section is 10cm so  $\pi \times 10^2$  expresses the area of the whole circle then dividing by 2 expresses the area of the semicircle from A to D. Subtracting the semicircle from B to C works out the area of the shaded cross section

The area of the two rectangular bottom faces. Area of rectangle = length  $\times$  width

The area of the top curved face of the tunnel. The face is basically a curved rectangle and the area of a rectangle = length  $\times$  width. The length of the face is 30cm. The width, which is the curved edge, is found by finding half of the circumference of the circle with diameter 20cm

The area of the under-side curved face of the tunnel. The face is basically a curved rectangle and the area of a rectangle = length  $\times$  width. The length of the face is 30cm. The width, which is the curved edge, is found by finding half of the circumference of the circle with diameter 14cm

Adding the area of all of the faces gives the total area of all six faces of the tunnel. It is worth expressing all the areas before simplifying as this gets 4 out of the 5 marks for the question

$$51\pi + 300\pi + 210\pi + 180$$

The dividing by 2 inside the bracket cancels out with the multiplying by 2 outside of the bracket.  $10^2 = 100$ .  $7^2 = 49$ .  $100 - 49 = 51$  so the area of the front and back of the tunnel simplifies to  $51\pi$

$$2 \times 3 = 6. 6 \times 30 = 180$$

$20/2 = 10$ .  $10 \times 30 = 300$  so the area of the top curved face simplifies to  $300\pi$

$14/2 = 7$ .  $7 \times 30 = 210$  so the area of the under-side curved face simplifies to  $210\pi$

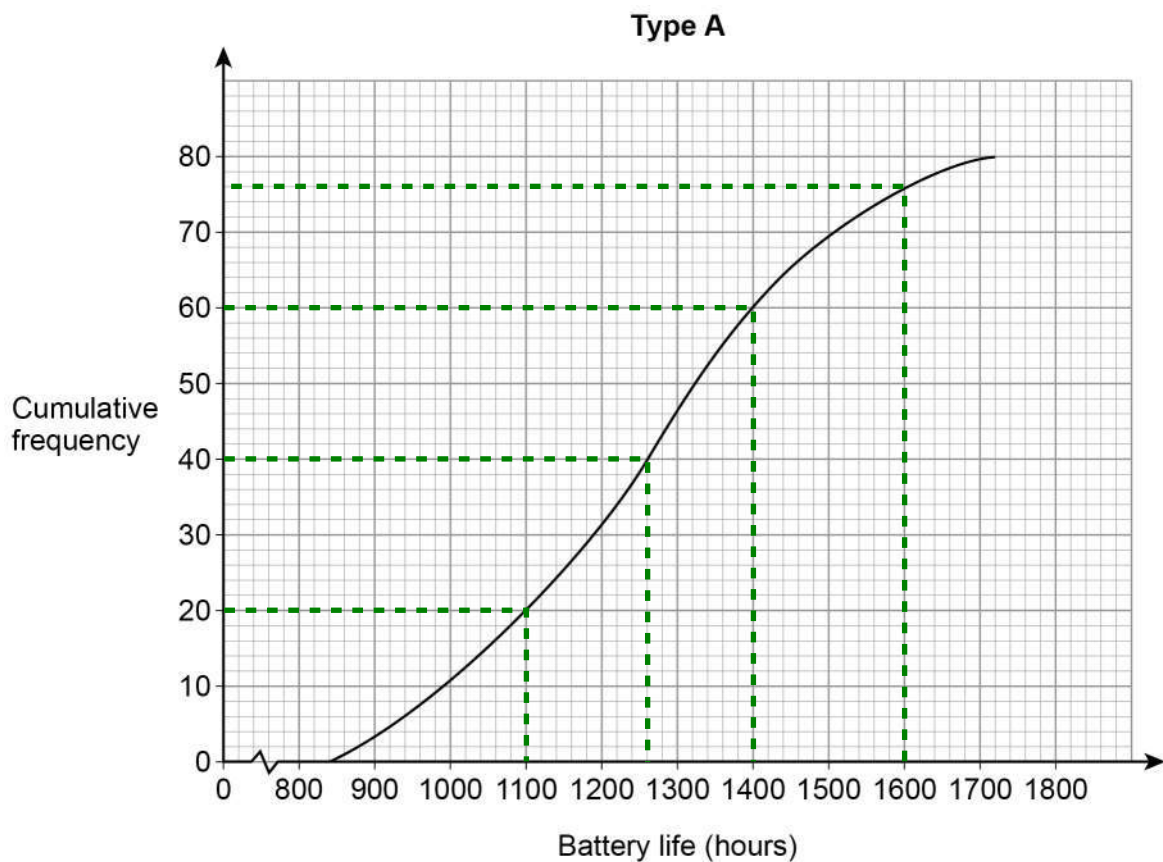
$$51 + 300 + 210 = 561$$

Answer            $561\pi + 180$             $\text{cm}^2$





- 19 Type A batteries and type B batteries were tested.  
The cumulative frequency diagram shows information about the battery life of type A.



- 19 (a) Estimate the interquartile range for type A.

[2 marks]

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Answer 300 hours

There were 80 type A batteries. The lower quartile is  $\frac{1}{4}$  of the way through these so is about the 20th. Drawing a line from the cumulative frequency of 20 to the line then down works out an estimate of the lower quartile, which is 1100. The upper quartile is  $\frac{3}{4}$  of the way through the 80 so is about the 60th. Drawing a line from the cumulative frequency of 60 to the line then down works out an estimate of the upper quartile, which is 1400. Interquartile range = upper quartile - lower quartile.  $1400 - 1100 = 300$



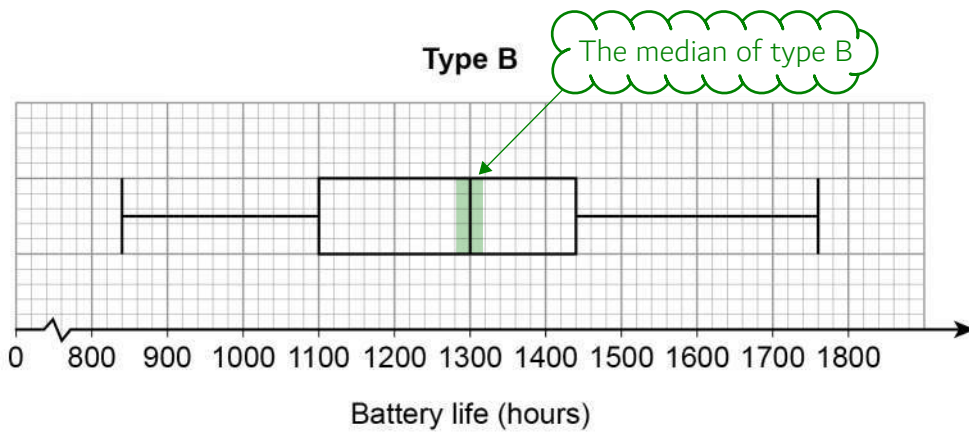
19 (b) Estimate the number of type A batteries that had a battery life of more than 1600 hours.

[1 mark]

Drawing a line up from 1600 to the line and across works out an estimate of how many had a battery life of 1600 hours or less. This is 76 so the rest of the 80 batteries must have had more than 1600.  $80 - 76 = 4$

Answer 4

19 (c) The box plot shows information about the battery life of type B.



On average, which type had the greater battery life?

Tick a box.

type A

type B

Using data from **both** diagrams, state how you chose your answer.

[2 marks]

The median of type B is 1300. The median of type A is 1260

The median for type A is halfway through the 80 batteries so is about the 40th. Drawing a line across from 40 on the cumulative frequency to the line then down works out an estimate of the median of type A



20

A linear sequence starts

$$a + 2b \quad a + 6b \quad a + 10b \quad \dots\dots \quad \dots\dots$$

The 2nd term has value 8

The 5th term has value 44

Work out the values of  $a$  and  $b$ .**[4 marks]**

$$a + 6b = 8$$

The 2nd term has value 8

$$a + 18b = 44$$

The sequence increases by  $4b$  between each term.  $10b + 4b + 4b = 18b$  so the 5th term is  $a + 18b$ , which has a value of 44

$$12b = 36$$

Solving the equations simultaneously. Subtracting the first equation from the second equation eliminates the  $a$  terms.  $18b - 6b = 12b$ .  $44 - 8 = 36$

$$a = 8 - 6 \times 3$$

Subtracting  $6b$  from both sides in the first equation makes  $a$  the subject and gives  $a = 8 - 6b$ . As  $b = 3$ , 3 can be substituted for  $b$

$$a = \underline{\quad -10 \quad}$$

$$b = \underline{\quad 3 \quad}$$

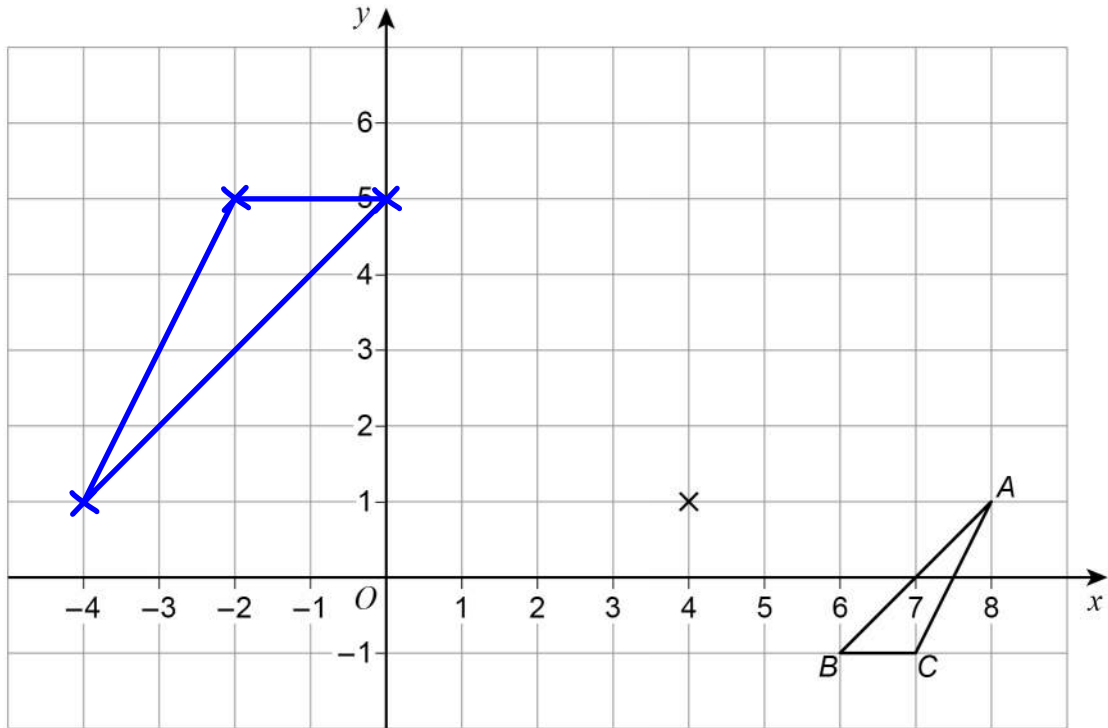
Dividing both sides of  $12b = 36$  by 12 works out that  $b = 3$ .  $a = 8 - 6 \times 3 = 8 - 18 = -10$



21 Enlarge triangle  $ABC$  by scale factor  $-2$ , centre  $(4, 1)$

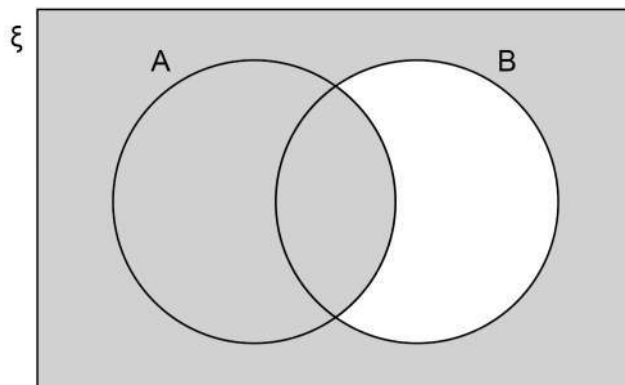
[2 marks]

$\begin{pmatrix} 4 \\ 0 \end{pmatrix} \times -2 = \begin{pmatrix} -8 \\ 0 \end{pmatrix}$        $\begin{pmatrix} 2 \\ -2 \end{pmatrix} \times -2 = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$        $\begin{pmatrix} 3 \\ -2 \end{pmatrix} \times -2 = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$



Expressing the vectors from the centre of enlargement to points A, B and C then multiplying each of them by  $-2$  works out the new vectors from the centre of enlargement

22



Which of these represents the shaded region?

Circle your answer.

[1 mark]

$A \cap B'$        $B'$        $A \cup B'$        $A' \cup B'$

7

Turn over ►





24 (a) Work out the value of  $2^{14} \div (2^9)^2$

Give your answer as a fraction in its simplest form.

[3 marks]

$$2^{14} \div 2^{18} \leftarrow (2^9)^2 = 2^{9 \times 2} = 2^{18}$$

$$2^{-4} \leftarrow 2^{14} \div 2^{18} = 2^{14-18} = 2^{-4}$$

Answer  $\frac{1}{16}$

$2^4 = 16$ . The negative power means to do the reciprocal

24 (b) Work out the value of  $25^{\frac{3}{2}}$

[2 marks]

$$5^3 \leftarrow \text{The over 2 as a power means to do the positive square root}$$

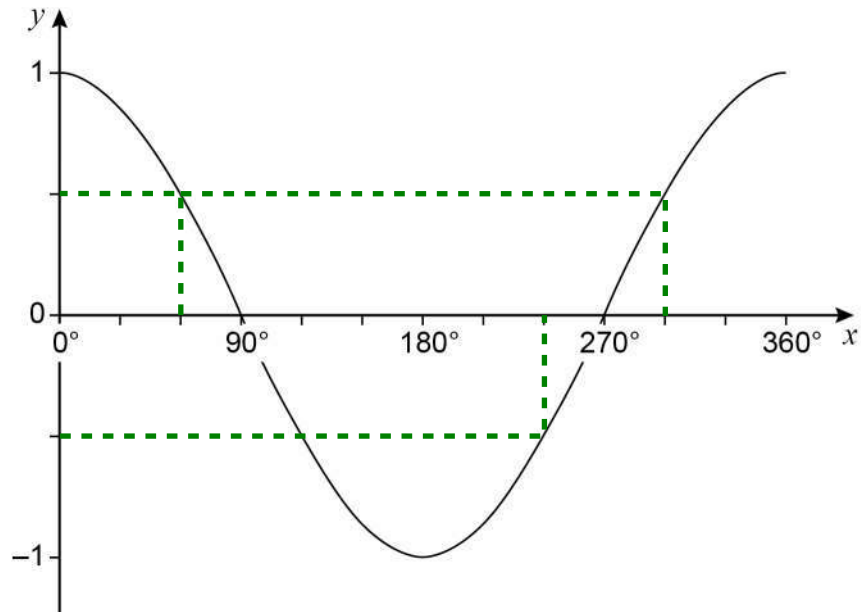
Answer 125

The numerator of 3 as a power means to cube

Turn over for the next question



25 Here is a sketch of the graph of  $y = \cos x$  for values of  $x$  from  $0^\circ$  to  $360^\circ$



25 (a)  $\cos x = \cos 60^\circ$

Work out the value of  $x$  when  $90^\circ \leq x \leq 360^\circ$

[1 mark]

Answer 300 degrees

From the graph,  $\cos 60 = 1/2$ .  $\cos 300$  is also equal to  $1/2$

25 (b)  $\cos x = -\cos 60^\circ$

Work out the value of  $x$  when  $180^\circ \leq x \leq 360^\circ$

[1 mark]

Answer 240 degrees

$\cos 60 = 1/2$  so  $-\cos 60 = -1/2$ .  $\cos 240$  is also equal to  $-1/2$  and is within  $180^\circ \leq x \leq 360^\circ$



26

 $b$  is two thirds of  $c$ .

$$5a = 4c$$

Work out the ratio  $a : b : c$ Give your answer in its simplest form where  $a$ ,  $b$  and  $c$  are integers.**[3 marks]**

$$4 : \frac{10}{3} : 5$$

From the equation  $5a = 4c$ ,  $a$  could be 4 and  $c$  could be 5. If  $c$  is 5,  $b$  is  $\frac{2}{3} \times 5 = \frac{10}{3}$

Multiplying all sides of the ratio by 3 eliminates the denominator to get integers. They cannot be divided by anything to get smaller whole numbers

Answer 12 : 10 : 15

**Turn over for the next question****Turn over ►**



27 (a) Jo wants to work out the solutions of  $x^2 + 3x - 5 = 0$

She says,

“The solutions **cannot** be worked out because  $x^2 + 3x - 5$  does **not** factorise to  $(x + a)(x + b)$  where  $a$  and  $b$  are integers.”

Is Jo correct?

Tick a box.

Yes

No

Give a reason for your answer.

[1 mark]

Could use the quadratic formula

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27 (b) **Without** expanding any brackets,

show how to work out the **exact** solutions of  $9(x + 3)^2 = 4$

Give the solutions.

[3 marks]

$$(x + 3)^2 = \frac{4}{9} \leftarrow \text{Dividing both sides by 9 to eliminate the 9 on the left}$$

$$x + 3 = \pm\frac{2}{3} \leftarrow \text{Square rooting both sides to eliminate the square on the left}$$

$$x = \pm\frac{2}{3} - \frac{9}{3} \leftarrow \text{Subtracting 3 from both sides to eliminate the 3 from the left and make } x \text{ the subject. Converting the 3 into } \frac{9}{3} \text{ so that it can be subtracted from the other fraction}$$

$$\begin{aligned} x &= \frac{-7}{3} \\ x &= \frac{-11}{3} \end{aligned} \leftarrow \text{These are the solutions of } x$$



28

Simplify  $\sqrt{80} + \sqrt{2\frac{2}{9}}$ Give your answer in the form  $\frac{a\sqrt{5}}{b}$  where  $a$  and  $b$  are integers.

[3 marks]

$$\sqrt{4} \times \sqrt{20} = \sqrt{4} \times \sqrt{4} \times \sqrt{5} = 4\sqrt{5}$$

Simplifying  $\sqrt{80}$  by using  $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$  in reverse.  $\sqrt{4} \times \sqrt{4} = 4$

$$\sqrt{\frac{20}{9}} = \frac{2\sqrt{5}}{3}$$

Expressing the mixed number as an improper fraction then square rooting the numerator and denominator. Simplifying  $\sqrt{20}$

$$\frac{12\sqrt{5}}{3} + \frac{2\sqrt{5}}{3}$$

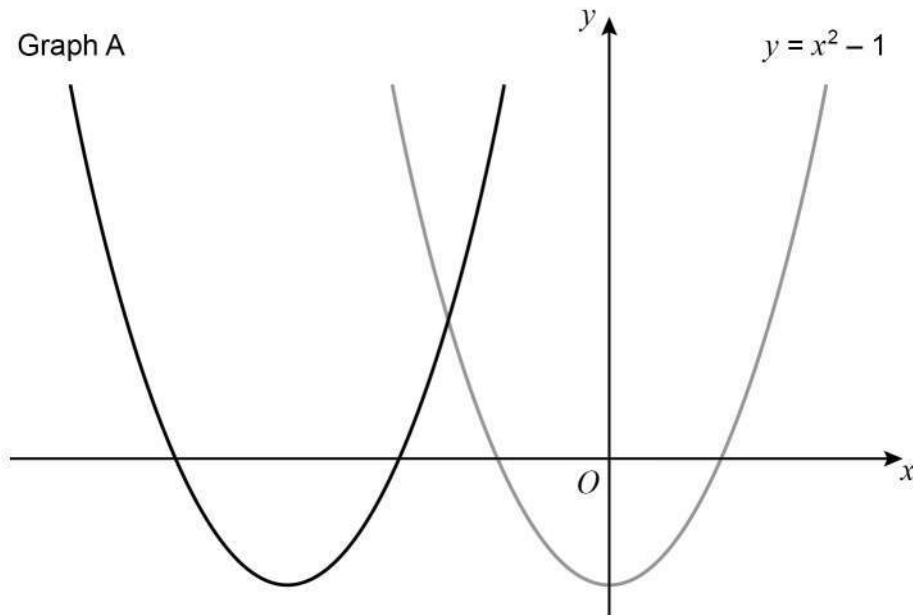
Multiplying  $4\sqrt{5}$  by 3 and putting it over 3 to convert it into a fraction with the same denominator so they can be added

Answer  $\frac{14\sqrt{5}}{3}$

Turn over for the next question



- 29 Here are sketches of two graphs.



The graph of  $y = x^2 - 1$  is translated 3 units to the left to give graph A.

- 29 (a) The equation of graph A can be written in the form  $y = x^2 + bx + c$

Work out the values of  $b$  and  $c$ .

[3 marks]

$$(x+3)^2 - 1$$

Adding 3 to  $x$  translates the graph 3 units to the left

$$x^2 + 6x + 9 - 1$$

Expanding the square bracket by squaring the first term, doubling the product of the two terms and squaring the last term

$$b = \underline{\quad 6 \quad}$$

$$c = \underline{\quad 8 \quad}$$

$$y = x^2 + 6x + 8$$



29 (b) The graph of  $y = x^2 - 1$  is reflected in the  $x$ -axis to give graph B.

Work out the equation of graph B.

[1 mark]

Changing the sign of all of the terms on the right reflects the graph in the  $x$ -axis

Answer  $y = -x^2 + 1$

30 Show that the value of  $\cos 30^\circ \times \tan 60^\circ + \sin 30^\circ$  is an integer.

[3 marks]

0	30	45	60	90
0	$\frac{1}{2}$	2	$\frac{\sqrt{3}}{2}$	4
4	$\frac{\sqrt{3}}{2}$	2	$\frac{1}{2}$	0
			$\sqrt{3}$	

Working out the trig values needed by writing out the angles we need to remember and writing 0, 1, 2, 3, 4 under these for the sin values and 4, 3, 2, 1, 0 for the cos values. Square rooting them and putting them over 2 works out the trig values for the sin and cos. Dividing the sin value by the cos value works out the tan value

$$\frac{\sqrt{3}}{2} \times \sqrt{3} + \frac{1}{2} = 2 \quad \leftarrow \quad \sqrt{3} \times \sqrt{3} = 3 \text{ so it becomes } 3/2 + 1/2 = 4/2 = 2$$

END OF QUESTIONS

