Write your name here

Surname	Other names

Pearson Edexcel
Level 1/Level 2 GCSE (9–1)

Centre Number

Candidate Number

Mathematics

Paper 3 (Calculator)

Higher Tier

Wednesday 8 November 2017 – Morning
Time: 1 hour 30 minutes

Paper Reference
1MA1/3H

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may be used**.
- If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

Please note that these worked solutions have neither been provided nor approved by Pearson Education and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

Working out the cumulative frequencies. Once it goes above 40.5, that category contains the median

1 The table shows information about the heights of 80 children.

Height (h cm)	**Frequency**	
$130 < h \leqslant 140$	4	4
$140 < h \leqslant 150$	11	15
$150 < h \leqslant 160$	24	39
$160 < h \leqslant 170$	22	61
$170 < h \leqslant 180$	19	

(a) Find the class interval that contains the median.

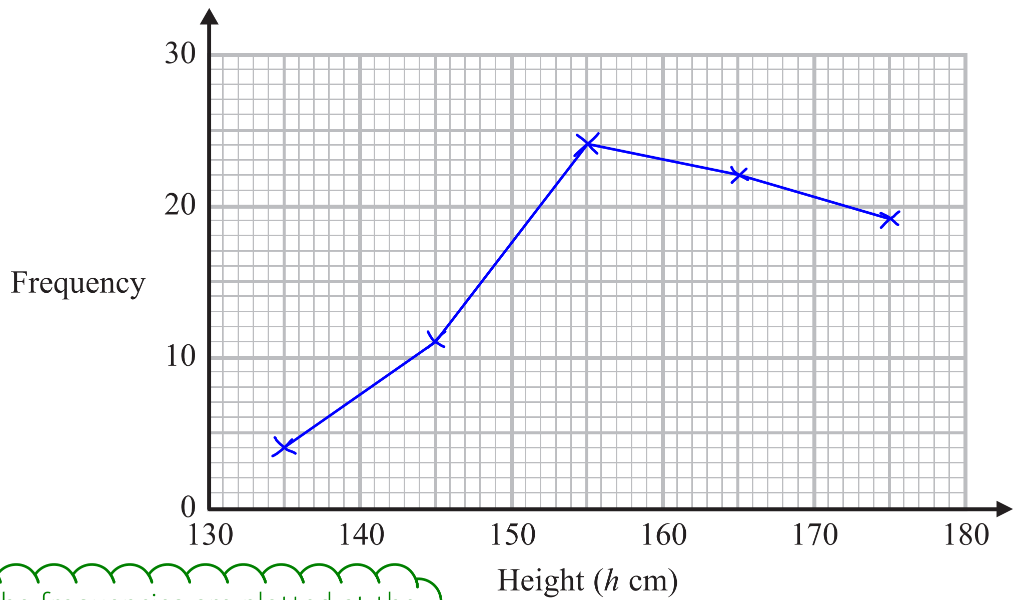
$$\frac{80+1}{2} = 40.5$$

Using the formula (n + 1)/2 works out that the median is halfway between the 40th and 41st value

$160 < h \leq 170$

(1)

(b) Draw a frequency polygon for the information in the table.



The frequencies are plotted at the midpoint of each class interval

(2)

(Total for Question 1 is 3 marks)

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2 In London, 1 litre of petrol costs 108.9p
In New York, 1 US gallon of petrol costs \$2.83

1 US gallon = 3.785 litres
£1 = \$1.46

In which city is petrol better value for money, London or New York?
You must show your working.

$$\frac{\left(\frac{2.83}{1.46}\right)}{3.785} \times 100 = 51$$

Every \$1.46 is £1 so dividing \$2.83 by \$1.46 converts it into pounds. Dividing the result by 3.785 finds the price of 1 litre from New York in pounds. Multiplying the result by 100 converts the pounds into pence so it can be compared to the London price per litre

New York

As the cost per litre in New York is just over 51p and this is less than the 108.9p per litre in London

(Total for Question 2 is 3 marks)

---

3 A gold bar has a mass of 12.5 kg.

The density of gold is 19.3 g/cm^3

Work out the volume of the gold bar.
Give your answer correct to 3 significant figures.

m
d V

From the formula triangle for density, mass and volume, volume = mass/density

$$\frac{12.5 \times 1000}{19.3}$$

There are 1000g in 1kg so multiplying the mass by 1000 converts it into grams. This needs to be done as the unit of density is in terms of grams, not kilograms

648 cm^3

(Total for Question 3 is 3 marks)

4 There are only blue pens, green pens and red pens in a box.

The ratio of the number of blue pens to the number of green pens is 2 : 5
The ratio of the number of green pens to the number of red pens is 4 : 1

There are less than 100 pens in the box.

What is the greatest possible number of red pens in the box?

B	G	R
2	5	
	4	1
8	20	5

Both ratios have green in common. A common multiple of 5 and 4 is 20 so multiplying the 2:5 by 4 and the 4:1 by 5 makes the ratios both have 20 parts for green and therefore they can be combined

$\frac{100}{33} \approx 3$

The combined ratio of 8:20:5 can't be simplified so the fewest total amount of pens is 33 as there are 33 parts in total in the ratio (8 + 20 + 5). If there were 100 pens in the box, just over 3 lots of 33 would go into it. As there are less than 100, it gets rounded down to 3 lots of 33

5×3

As the total amount of pens is multiplied by 3 (to get 99 pens in total in the box) the number of red pens needs to multiplied by 3 to keep the same ratio of pen colours

15

(Total for Question 4 is 3 marks)

---

5 (a) Find the value of the reciprocal of 1.6
Give your answer as a decimal.

1/1.6

0.625

(1)

Jess rounds a number, x, to one decimal place.
The result is 9.8

(b) Write down the error interval for x.

The resolution of one decimal place is 0.1. Dividing this by 2 gives 0.05. Adding and subtracting this from 9.8 gives the upper and lower bounds for x

$9.75 \leq x < 9.85$

(2)

(Total for Question 5 is 3 marks)

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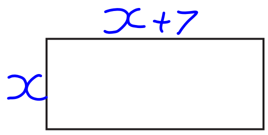
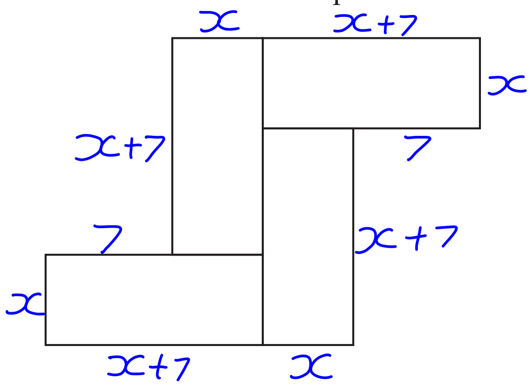
6 Here is a rectangle.



We don't know the width of the rectangle so we can label this as x. The length is 7cm longer than this so it is x + 7

The length of the rectangle is 7 cm longer than the width of the rectangle.

4 of these rectangles are used to make this 8-sided shape.



The perimeter of the 8-sided shape is 70 cm.

Work out the area of the 8-sided shape.

$8x + 42 = 70$ ← Adding together all of the sides gives the perimeter in terms of x. This is equal to 70cm

$8x = 28$ ← Subtracting 42 from both sides

$x = 3.5$ ← Dividing both sides by 8

$3.5 + 7 = 10.5$ ← The width of each rectangle is 3.5cm. Adding 7cm finds the length of each rectangle

$3.5 \times 10.5 \times 4$ ← Area of one of the rectangles is length x width. There are 4 of these rectangles so the area of one is multiplied by 4

.......147....... cm^2

(Total for Question 6 is 5 marks)

7 Work out $(13.8 \times 10^{7}) \times (5.4 \times 10^{-12})$
Give your answer as an ordinary number.

Typing it into the calculator gives 7.452×10^{-4}

Divide 7.452 by ten 4 times to convert into an ordinary number

0.0007452

(Total for Question 7 is 2 marks)

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8 When a drawing pin is dropped it can land point down or point up.

Lucy, Mel and Tom each dropped the drawing pin a number of times.

The table shows the number of times the drawing pin landed point down and the number of times the drawing pin landed point up for each person.

	Lucy	**Mel**	**Tom**
point down	31	53	16
point up	14	27	9

Rachael is going to drop the drawing pin once.

(a) Whose results will give the best estimate for the probability that the drawing pin will land point up?
Give a reason for your answer.

Mel as she dropped the pin the most times

(1)

Stuart is going to drop the drawing pin twice.

(b) Use all the results in the table to work out an estimate for the probability that the drawing pin will land point up the first time and point down the second time.

$31 + 53 + 16 = 100$

$14 + 27 + 9 = 50$

Adding together the total amount of point down and point up results

$100 + 50 = 150$

Working out how many results there are altogether

$\frac{50}{150} \times \frac{100}{150}$

Point up AND point down, so the probabilities should be multiplied together. The fraction of results which were point up multiplied by the fraction of results which were point down

$\frac{2}{9}$

(2)

(Total for Question 8 is 3 marks)

9 Jack bought a new boat for £12 500

The value, £V, of Jack's boat at the end of n years is given by the formula

$$V = 12\,500 \times (0.85)^n$$

(a) At the end of how many years was the value of Jack's boat first less than 50% of the value of the boat when it was new?

$0.85^4 = 0.5...$

$0.85^5 = 0.4...$

Keep increasing n by 1 until 0.85^n is less than 0.5. If the multiplier is less than 0.5, the value of the boat will be less than 50% of the original

Use table mode on the calculator (press menu then 3) and put in $f(x) = 0.85^x$. Start: 1. End: 30. Step: 1. Scroll down until the value of f(x) is less than 0.5. x should be 5 when this happens

5

(2)

A savings account pays interest at a rate of R% per year.
Jack invests £5500 in the account for one year.

At the end of the year, Jack pays tax on the interest at a rate of 40%.
After paying tax, he gets £79.20

(b) Work out the value of R.

$60\% = 79.20$

100% - 40% = 60% so the amount he gets is 60% of the total interest

$1\% = 1.32$

$100\% = 132$

Dividing the amount he gets by 60 works out 1% then multiplying by 100 works out the full 100% of the interest

$\frac{132}{5500} \times 100$

Expressing the amount of interest as a fraction of the amount invested then converting it into a percentage by multiplying by 100

2.4

(3)

(Total for Question 9 is 5 marks)

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10 There are only blue counters, yellow counters, green counters and red counters in a bag.
A counter is taken at random from the bag.

The table shows the probabilities of getting a blue counter or a yellow counter or a green counter.

Colour	blue	yellow	green	red
Probability	0.2	0.35	0.4	

(a) Work out the probability of getting a red counter.

$\boxed{1 - 0.2 - 0.35 - 0.4}$

It is certain that one of the colours will be chosen so the probabilities must add up to 1. Subtracting the other probabilities from 1 leaves the probability for red

0.05

(1)

(b) What is the least possible number of counters in the bag?
You must give a reason for your answer.

0.2 : 0.35 : 0.4 : 0.05

20 : 35 : 40 : 5

4 : 7 : 8 : 1

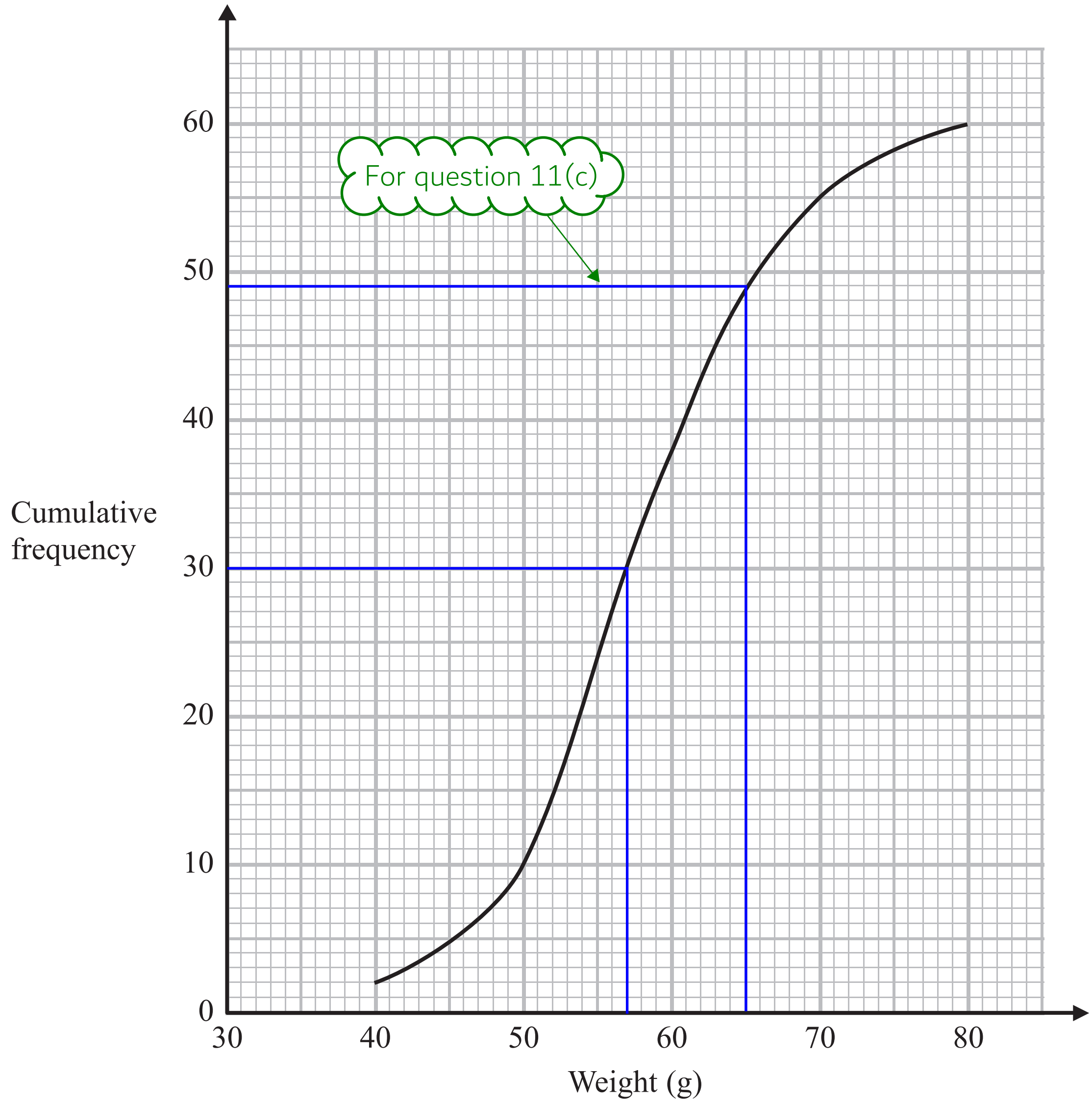
Expressing the probabilities, which are equal to the relative frequencies, as a ratio then simplifying the ratio by multiplying all parts by 100 then dividing by 5

20, as there has to be a whole number of counters for each colour and this is the number of parts in the simplified ratio

(2)

(Total for Question 10 is 3 marks)

11 The cumulative frequency graph shows information about the weights of 60 potatoes.



(a) Use the graph to find an estimate for the median weight.

The total frequency is 60 and the median is roughly halfway through the data so the 30th value is an estimate for the median

57 g

(1)

Jamil says,

"80 − 40 = 40 so the range of the weights is 40 g."

(b) Is Jamil correct?
You must give a reason for your answer.

Maybe not as the minimum value could be less than 40g

(1)

(c) Show that less than 25% of the potatoes have a weight greater than 65 g.

$$\frac{60-49}{60} \times 100 = 18.\dot{3}\%$$

49 potatoes had a mass equal to or less than 65g.
Subtracting this from the total number of potatoes gives the number which are greater than 65g.
Putting this over 60 (the total number of potatoes) gives the fraction of the potatoes greater than 65g.
Multiplying by 100 converts this into a percentage

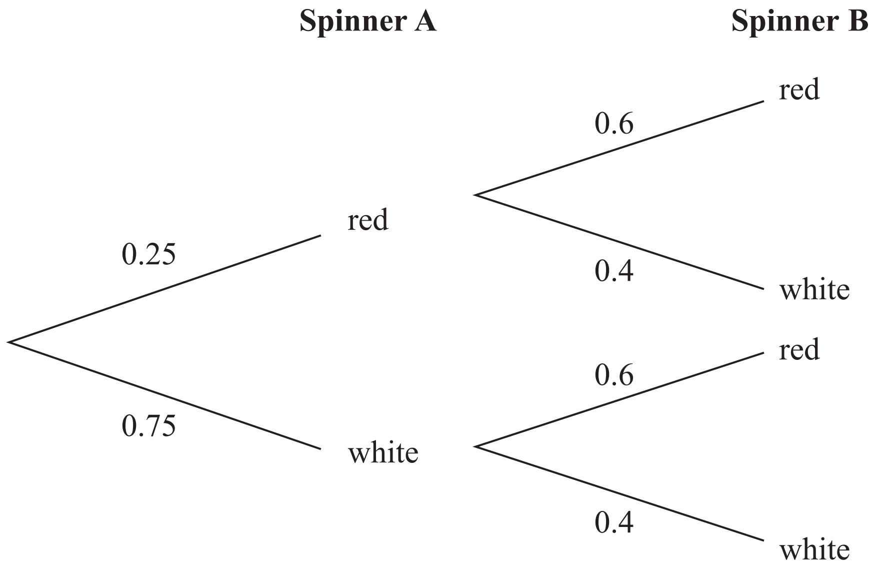
(2)

(Total for Question 11 is 4 marks)

12 Alan has two spinners, spinner **A** and spinner **B**.
Each spinner can land on only red or white.

The probability that spinner **A** will land on red is 0.25
The probability that spinner **B** will land on red is 0.6

The probability tree diagram shows this information.



Alan spins spinner **A** once and he spins spinner **B** once.
He does this a number of times.

The number of times **both** spinners land on red is 24

Work out an estimate for the number of times **both** spinners land on white.

$0.25 \times 0.6 = 0.15$

This works out the probability of getting red AND red. AND means to multiply the probabilities together

$x \times 0.15 = 24$

$x = 160$

The total number of spins, x, multiplied by the probability for red and red gives the number of times both spinners should land on red. Dividing both sides by 0.15 finds x

$0.75 \times 0.4 = 0.3$

This works out the probability of getting white AND white. AND means to multiply the probabilities together

160×0.3

The total number of spins multiplied by the probability for white and white gives the number of times both spinners should land on white

48

(Total for Question 12 is 3 marks)

13 Write $x^2 + 6x - 7$ in the form $(x + a)^2 + b$ where a and b are integers.

$(x+3)^2 - 7 - 9$

The coefficient of x is 6. Halving this gives 3. Put it in a bracket with x. Expanding out $(x + 3)^2$ gives $x^2 + 6x + 9$ so the 9 needs to be taken away

$(x+3)^2 - 16$

(Total for Question 13 is 2 marks)

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14 Cone **A** and cone **B** are mathematically similar.
The ratio of the volume of cone **A** to the volume of cone **B** is 27 : 8

The surface area of cone **A** is 297 cm^2

Show that the surface area of cone **B** is 132 cm^2

3 : 2

Cube rooting both sides of the ratio of the volumes gives the ratio of the lengths

9 : 4

Squaring both sides of the ratio of the lengths gives the ratio of the surface areas

$\frac{297}{9} \times 4 = 132$

9 parts of the ratio represents the surface area of cone A so dividing by 9 works out the value of 1 part of the ratio. Multiplying by 4 works out that 4 parts, which represents the surface area of cone B, is 132

(Total for Question 14 is 3 marks)

15 (a) Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$

$(0)^3 + 7(0) - 5 = -5$

$(1)^3 + 7(1) - 5 = 3$

The equation is a continuous curve and it goes from a negative value to a positive value. Therefore the curve must go through 0 somewhere between

Therefore there must be a solution here as there is a change of sign

(2)

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \dfrac{5}{x^2 + 7}$

$x^3 + 7x = 5$ ← Add 5 to both sides

$x(x^2 + 7) = 5$ ← Factorise to get the $x^2 + 7$

$x = \dfrac{5}{x^2+7}$ ← Divide both sides by $x^2 + 7$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \dfrac{5}{x_n^{\,2} + 7}$ three times to find an estimate for the solution of $x^3 + 7x - 5 = 0$

$x_1 = \dfrac{5}{(1)^2 + 7}$ $\boxed{= 5/8}$

$x_2 = \dfrac{5}{(x_1)^2 + 7}$ $\boxed{= 320/473}$

$x_3 = \dfrac{5}{(x_2)^2 + 7}$ $\boxed{= 0.6704483001}$

Press 1 then = to set 1 as the answer. Type $5/(\text{ANS}^2 + 7)$ then press = three times to get these results

0.67

(3)

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(d) By substituting your answer to part (c) into $x^3 + 7x - 5$,
comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

$$0.67^3 + 7(0.67) - 5 = -9 \times 10^{-3}$$

It is accurate as it gives a value very close to 0

(2)

(Total for Question 15 is 9 marks)

---

16 The petrol consumption of a car, in litres per 100 kilometres, is given by the formula

$$\text{Petrol consumption} = \frac{100 \times \text{Number of litres of petrol used}}{\text{Number of kilometres travelled}}$$

Nathan's car travelled 148 kilometres, correct to 3 significant figures.
The car used 11.8 litres of petrol, correct to 3 significant figures.

Nathan says,

"My car used less than 8 litres of petrol per 100 kilometres."

Could Nathan be wrong?
You must show how you get your answer.

The third significant figure of 11.8 is in the 1st decimal place, which has a resolution of 0.1. Halving the resolution then adding this to 11.8 calculates the upper bound for the amount of petrol used

$$\frac{100 \times 11.85}{147.5} = 8.03$$

The numerator of the fraction should be as large as possible and the denominator should be as small as possible (as this means dividing by less) to get the upper bound for the petrol consumption

The third significant figure of 148 is in the units place, which has a resolution of 1. Halving the resolution then subtracting this from 148 calculates the lower bound for distance travelled

Yes

As the upper bound of the petrol consumption is more than 8 litres

(Total for Question 16 is 3 marks)

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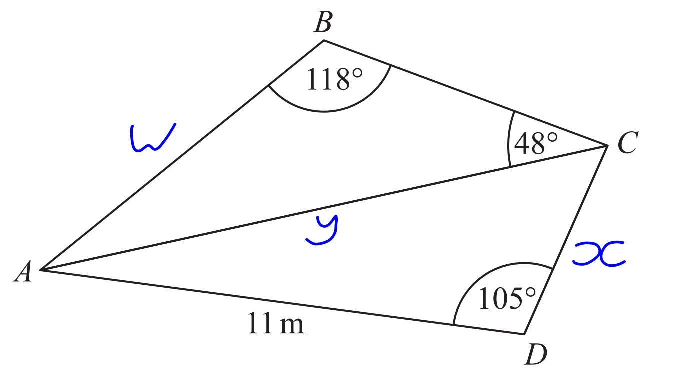
.CG Maths.

17 *ABC* and *ADC* are triangles.



The area of triangle *ADC* is 56 m^2

Work out the length of *AB*.
Give your answer correct to 1 decimal place.

$\frac{1}{2} \times x \times 11 \times \sin 105 = 56$

Substituting a, b and C (from triangle ADC) into the formula for the area of a triangle 1/2 ab x sinC. This must be equal to 56 as this is the area of the triangle

$x = \frac{56}{\frac{1}{2} \times 11 \times \sin 105}$

Rearranging to find x (side CD)

Store the answer of 10.54099384 as x on the calculator

$y = \sqrt{11^2 + x^2 - 2(11)(x)\cos 105}$

Rearranging the cosine rule ($a^2 = b^2 + c^2 - 2bc\cos A$) to make a the subject (as AC is opposite the angle, it must be a) by square rooting both sides. Then substituting in the values from triangle ADC

Store the answer of 17.09190364 as y on the calculator

$w = \frac{y \sin 48}{\sin 118}$

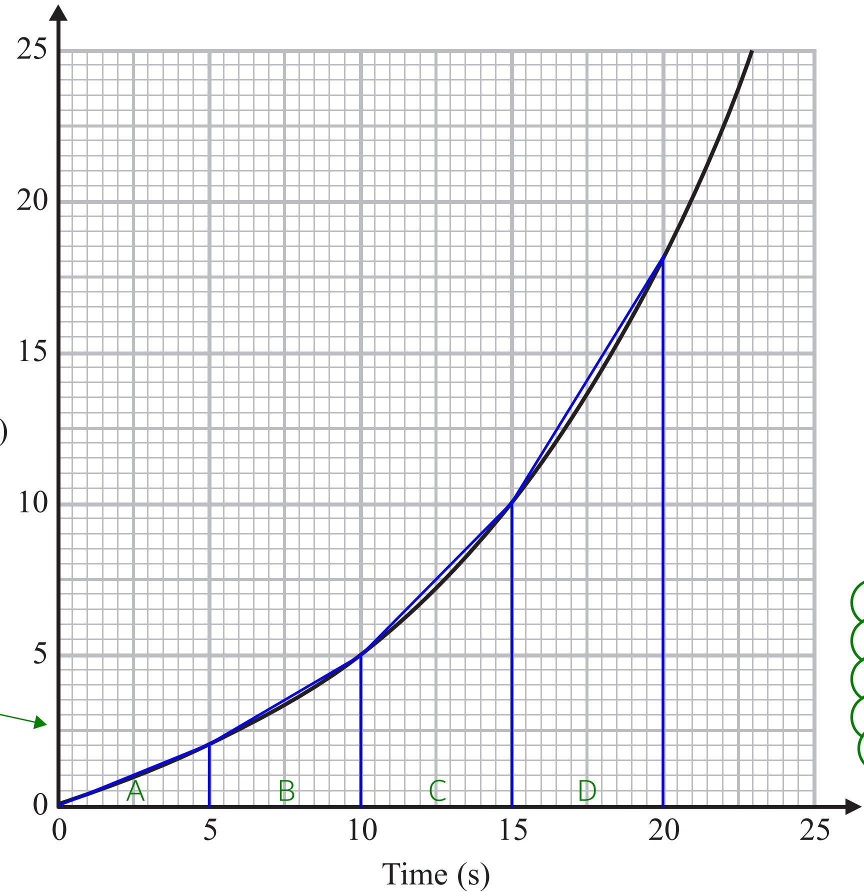
Rearranging the sine rule (a/sinA = b/sinB) to make a the subject by multiplying both sides by sinA. Then substituting in the values from triangle ABC

The answer of 14.38563268 is rounded to 1 decimal place

14.4 m

(Total for Question 17 is 5 marks)

18 Here is a speed-time graph for a train.



The difference between 5 and 0 is 5. Dividing this by the 10 boxes between them tells us that the scale is going up in 0.5

The first 20 seconds divided by the 4 equal strips is 5 so this is the width of each strip

(a) Work out an estimate for the distance the train travelled in the first 20 seconds.
Use 4 strips of equal width.

$$\underline{\frac{1}{2}\times5\times2}+\underline{\frac{1}{2}(2+5)\times5}+\underline{\frac{1}{2}(5+10)\times5}+\underline{\frac{1}{2}(10+18)\times5}$$

Area of triangle A = 1/2 x base x height

Area of a trapezium (shapes B, C and D) = 1/2 (a + b) x h where a and b are the parallel sides and h is the distance between them

130 m

(3)

(b) Is your answer to (a) an underestimate or an overestimate of the actual distance the train travelled?
Give a reason for your answer.

Overestimate as the area of the curve is less than what was calculated

(1)

(Total for Question 18 is 4 marks)

19 Prove algebraically that the straight line with equation $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$

$x = 10 + 2y$

Rearranging to make x the subject by adding 2y to both sides

$(10 + 2y)^2 + y^2 = 20$

Substituting x for (10 + 2y) in the equation of the circle to eliminate a variable

$100 + 40y + 4y^2 + y^2 - 20 = 0$

Expanding the square bracket (using 'square the first term, double the product, square the last term'. Also subtracting 20 from both sides to make the equation equal to zero so it can be solved

$5y^2 + 40y + 80 = 0$

Collecting like terms to simplify and bring into the quadratic form

$y^2 + 8y + 16 = 0$

Dividing all terms by 5 to make the quadratic easier to solve

$(y + 4)^2 = 0$

Solving by factorisation. 4 and 4 multiply to give 16 and add to give 8 so both brackets are the same

$y = -4$

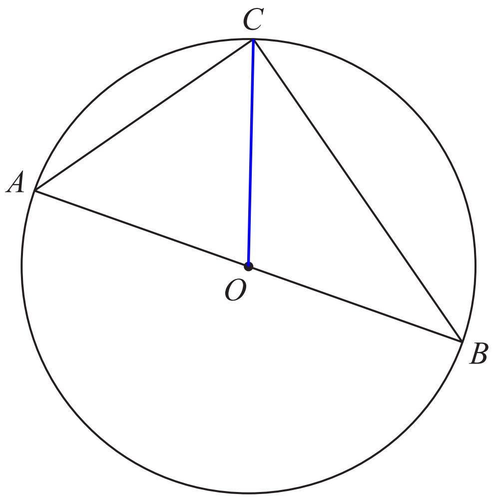
Square rooting both sides gives y + 4 = 0. Subtracting 4 from both sides gives y = -4

There is no need to solve for x as the straight line can't cross the circle at more than one x coordinate at a single y coordinate

Therefore the line is a tangent as there is only one point of intersection

(Total for Question 19 is 5 marks)

20



A, *B* and *C* are points on the circumference of a circle, centre *O*.
AOB is a diameter of the circle.

Prove that angle *ACB* is 90°
You must **not** use any circle theorems in your proof.

Let angles ACO = x and OCB = y

Triangles ACO and OCB are isosceles as sides AO, OC and OB are radii and are equal

Angle CAO = x and angle CBO = y as the base angles of isosceles triangles are equal

2x + 2y = 180 as angles in a triangles add up to 180

Adding angles CAO, ACO, OCB and CBO gives 2x + 2y and this gives the total number of degrees in triangle ACB

x + y = 90

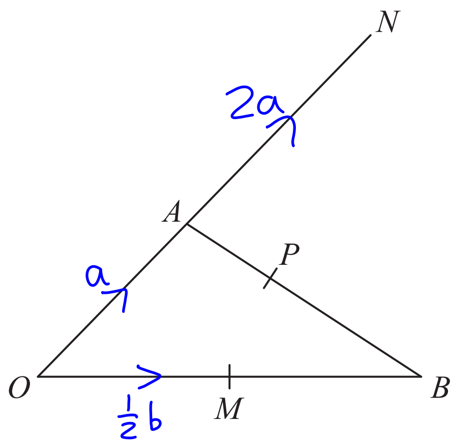
Dividing both sides of the equation by 2 gives this

Therefore angle ACB is 90

Adding angles x and y gives angle ACB

(Total for Question 20 is 4 marks)

21



OAN, *OMB* and *APB* are straight lines.
$AN = 2OA$.
M is the midpoint of OB.

$\overrightarrow{OA} = \mathbf{a}$ $\qquad \overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{AP} = k\overrightarrow{AB}$ where k is a scalar quantity.

Given that *MPN* is a straight line, find the value of k.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b$

$\overrightarrow{AP} = k(-a+b)$

$\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OA} + \overrightarrow{AP} = -\frac{1}{2}b + a + k(-a+b)$

$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON} = -\frac{1}{2}b + 3a = \underline{x(-\frac{1}{2}b + a + k(-a+b))}$

MPN is a straight line so $\overrightarrow{MN}$ must be something multiplied by $\overrightarrow{MP}$

$= -\frac{1}{2}xb + xa - kxa + kxb$

$= (x - kx)a + (kx - \frac{1}{2}x)b$

$3 = x - kx$

$-\frac{1}{2} = kx - \frac{1}{2}x$

Equating the coefficients of a and b gives these two equations which can be solved simultaneously

$3 = x(1-k)$

$x = \frac{3}{1-k}$

$-\frac{1}{2} = k\left(\frac{3}{1-k}\right) - \frac{1}{2}\left(\frac{3}{1-k}\right) = \frac{3k}{1-k} - \frac{\left(\frac{3}{2}\right)}{1-k} = \frac{3k - \frac{3}{2}}{1-k}$

$-\frac{1}{2}(1-k) = 3k - \frac{3}{2}$

$-\frac{1}{2} + \frac{1}{2}k = 3k - \frac{3}{2}$

$3k - \frac{1}{2}k = -\frac{1}{2} + \frac{3}{2}$

$\frac{5}{2}k = 1$

$k = 1 \div \frac{5}{2} = 1 \times \frac{2}{5}$

$\frac{2}{5}$

(Total for Question 21 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS