OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

Thursday 3 November 2022 – Morning

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- geometrical instruments
- tracing paper

Do not use:
- a calculator

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

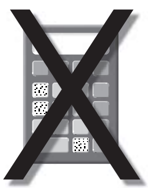
First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.



Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

.CG Maths.

Answer **all** the questions.

1 Jamie was paid £14.00 per hour.
Jamie receives a pay increase of 20%.

Work out how much Jamie is now paid per hour.

1.40
× 2
2.80
+14.00
16.80

10% as a fraction is 1/10. Dividing the £14 by 10 gives £1.40, which is 10% of £14. Multiplying this by 2 works out that 20% of £14 is £2.80. Adding this on to the £14 works out that Jamie is now paid £16.80 per hour

£ ....16.80........ **[3]**

2 Find all the possible integer values that satisfy the inequality $-4 \leqslant x - 3 < 1$.

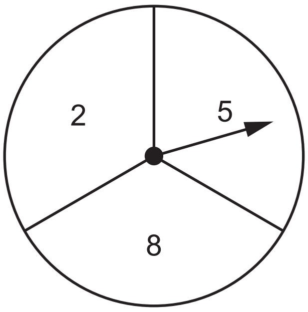
$-1 \leq x < 4$

Adding 3 to all sides eliminates the -3 in the middle and gets x on its own

x can be greater or equal to -1 but less than 4

........-1, 0, 1, 2, 3........ **[3]**

3 Azmi has a fair spinner numbered 2, 5 and 8.



Azmi spins the spinner twice and adds the two scores to get a total.

(a) Complete the table to show all of the possible totals.

First spin

Total	2	5	8
2	4	7	10
5	7	10	13
8	10	13	16

Second spin

2 + 8 = 10

5 + 8 = 13

8 + 2 = 10

[1]

(b) Find the probability that the total is a square number.

The possible totals which are square numbers are 4 (as $2^2 = 2 \times 2 = 4$) and 16 (as $4^2 = 4 \times 4 = 16$). This is 2 out of the 9 possible totals

(b) ....$\frac{2}{9}$.................. **[2]**

4 Layla and Jamal open a box of sweets.
Layla and Jamal share all of the sweets in the ratio 2 : 3.

(a) Write down the fraction of the sweets that Layla receives.

2 + 3 = 5 and this is the total amount of parts. Layla gets 2 out of the 5 parts

$\frac{2}{5}$

(a) .... **[1]**

(b) Layla eats some of **her** sweets.
She is then left with 18% of the sweets that were in the box.

Work out the percentage of **her** sweets that Layla has eaten.

$100 \div 5 = 20$ ← Percentage is out of 100 so dividing 100 by 5 works out that 1/5 is 20%

$20 \times 2 = 40$ ← Multiplying 20% by 2 works out that 2/5 is 40%

$40 - 18 = 22$ ← Layla had 40% and now has 18% of the sweets that were in the box. Subtracting the 18% from the 40% works out that she had eaten 22% of the sweets in the box

$\frac{22}{40}$ ← Expressing the 22% which she had eaten as a fraction of the 40% she started with

$\frac{11}{20}$ ← Simplifying the fraction by dividing both the numerator and denominator by 2

$\frac{55}{100}$ ← Making the denominator of the fraction 100 by multiplying both the numerator and denominator by 5

Percentage is out of 100 so 55/100 must be 55%

(b) ....55.............................. % **[4]**

5 Ashley goes on a journey.

She travels by taxi for $\frac{1}{8}$ of the journey.

She travels by train for $\frac{4}{5}$ of the journey.

She walks for the remaining 900 m of the journey.

Find the length of this journey in kilometres.
You must show your working.

$\frac{1}{8}+\frac{4}{5}$ ← Adding the 1/8 of the journey which is done by taxi and the 4/5 which is done by train works out the fraction of the journey which wan not done by walking

$\frac{5}{40}+\frac{32}{40}$ ← To add fractions the denominators need to be the same. Multiplying both the numerator and denominator of the 1/8 by 5 and multiplying both the numerator and denominator of the 4/5 by 8 converts both fractions so that they have the same denominator of 40

$\frac{40}{40}-\frac{37}{40}$ ← 1 represents the whole journey. Subtracting the 37/40 which was not done by walking from the whole journey leaves the fraction which was done by walking. 1 is converted into 40/40 so that the denominators are the same and they can be subtracted

$\frac{3}{40}x=900$ ← Let x be the length of the whole journey in metres. 3/40 of x must be 900 m

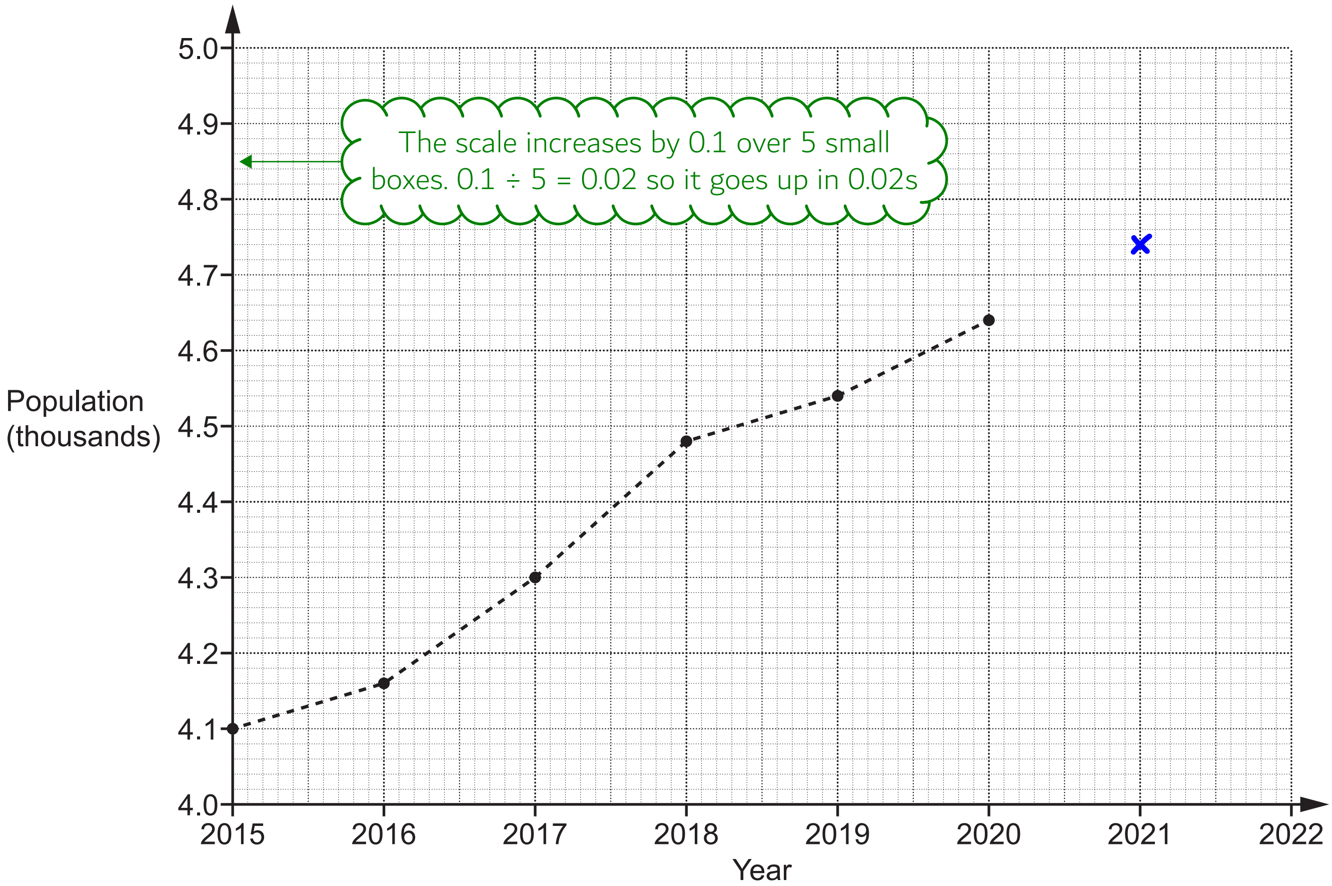
$900 \div 3 = 300$ ← Dividing the 900 m by 3 works out 1/40 of the journey

$300 \times 40 = 12000$ ← Multiplying 1/40 of the journey by 40 works out 40/40 of the journey, which is the whole journey

There are 1000 m in 1 km so dividing the 12000 m by 1000 converts it into 12 km

12 km **[6]**

6 The graph shows information about the population of a village.



(a) The population of the village in 2021 was 4740.
Plot this point on the graph.

4.74 thousand

[1]

(b) Work out the increase in the population of the village between 2016 and 2018.

4.48
− 4.16
0.32

The population in 2016 was 4.16 thousand. The population in 2018 was 4.48 thousand. Subtracting these works out the increase

0.32 x 1000 = 320

(b) .... 320 .... **[2]**

(c) Rowan says that there was a huge increase in the population of the village between 2015 and 2020.

Describe how Rowan may have been misled by the graph.

The vertical scale does not start at 0

So it looks like a huge increase but it has only increased from 4100 to 4640 **[1]**

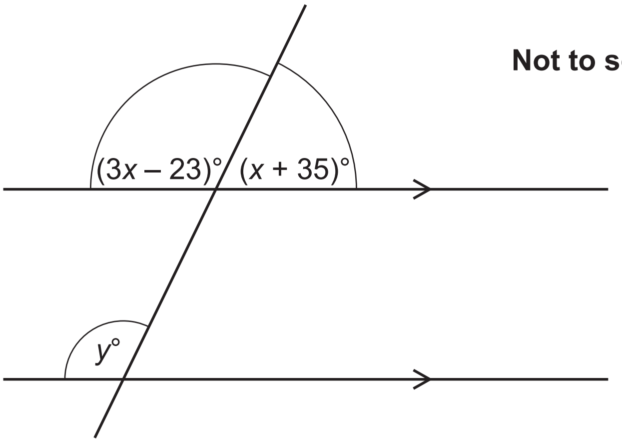
(d) Blake says that the population of the village will be greater than 4800 in 2022.

Write down an assumption Blake has made.

The population will continue to increase at a similar rate

The rate is how quickly it increases **[1]**

7 The diagram shows a straight line crossing a pair of parallel lines.



Not to scale

Find the value of y.
You must show your working.

$3x - 23 + x + 35$ ← Adding both the angles around the point on the straight line

$4x + 12 = 180$ ← Simplifying the expression by collecting like terms then setting it equal to 180 as the angles around a point on a straight line add up to 180°

$4x = 168$ ← Subtracting 12 from both sides eliminates the 12 on the left and gets the x term on its own

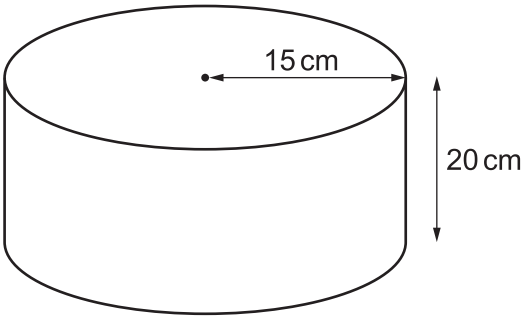
$168 \div 4 = 042$ ← Dividing both sides by 4 eliminates the 4 on the left and finds that x = 42

$42 \times 3 = 126$, $126 - 23 = 103$ ← Finding the value of the 3x - 23 angle by substituting 42 for x

The 3x - 23 angle is 103° and this is corresponding to the angle y. So they are equal

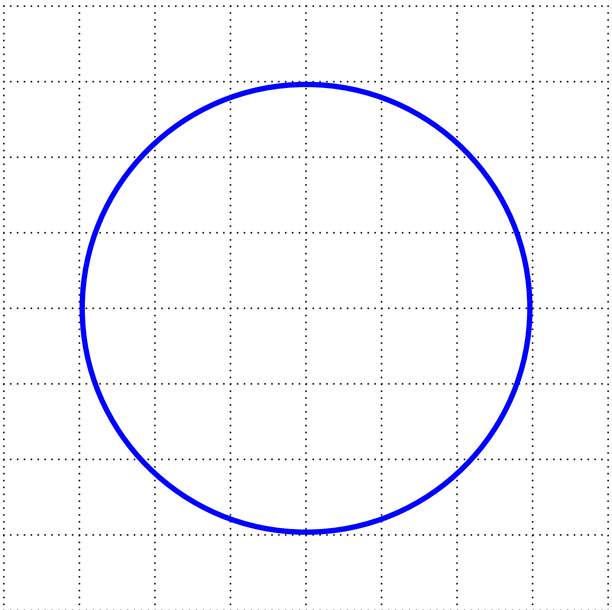
y = ...103........................ **[5]**

8 The diagram shows a cylinder with radius 15 cm and height 20 cm.



Not to scale

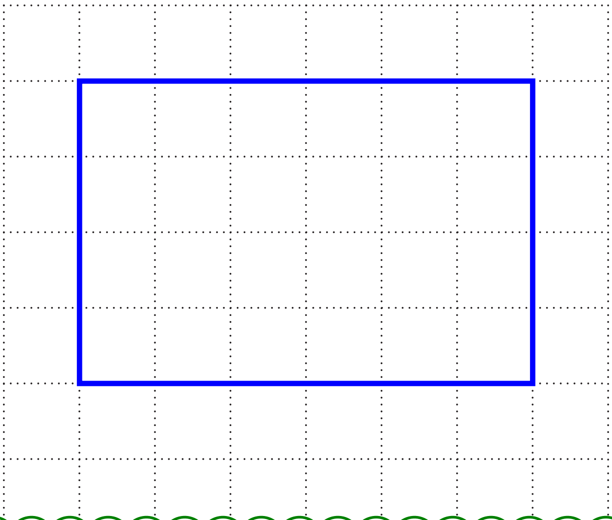
(a) On the grid below, draw the plan view of the cylinder.
Use the scale 1 cm represents 5 cm.



The plan is a 2D representation of what is seen from above.
15 ÷ 5 = 3 so the radius of 15 cm is represented by 3 cm

[2]

(b) On the grid below, draw the front elevation of the cylinder.
Use the scale 1 cm represents 5 cm.



The front elevation is a 2D representation of what is seen from the front. The diameter is 30 cm and is represented by 6 cm. The height is 20 cm and is represented by 4 cm

[2]

9 A student says that they have placed the following values in order starting with the smallest.

$\left(\frac{1}{10}\right)^2$ $\sqrt{0.25}$ 4^{-1}

Has the student done this correctly?
Show how you decide.

$\left(\frac{1}{10}\right)^2 = \frac{1}{100}$ ← 1/10 is squared by squaring the numerator and squaring the denominator. $1^2 = 1$ and $10^2 = 100$

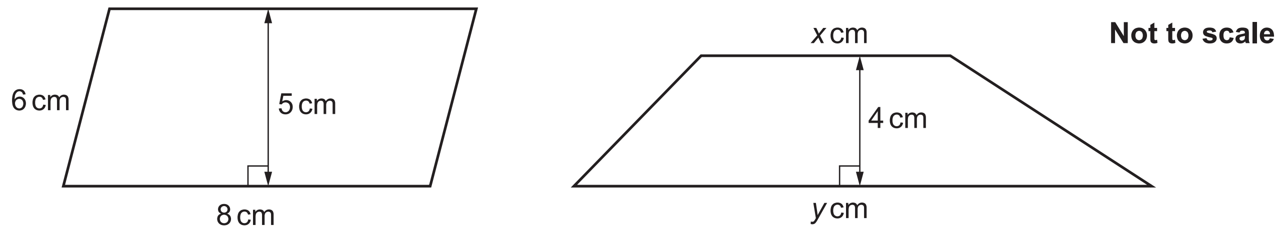
$\sqrt{0.25} = \sqrt{\frac{1}{4}} = \frac{1}{2}$ ← 0.25 is 1/4 as a fraction, which is square rooted by square rooting the numerator and square rooting the denominator. $\sqrt{1} = 1$ and $\sqrt{4} = 2$

$4^{-1} = \frac{1}{4}$ ← A negative power means to do the reciprocal. This can be done by doing 1 over

No because 4^{-1} is smaller than $\sqrt{0.25}$

[4]

10 The parallelogram and the trapezium have the same area.



The ratio of $x : y$ is 3 : 5.

Find the value of x and the value of y.
You must show your working.

8×5 ← Area of parallelogram = base x height. The base is 8 cm and the height is 5 cm. Multiplying these works out that the area of the parallelogram is 40 cm²

$\frac{1}{2}(x + \frac{5}{3}x) \times 4 = 40$ ← Area of trapezium = 1/2 (a + b)h, where a and b are the parallel sides and h is the distance between them. a is x and b is y, which can be replaced with 5/3 x as from the ratio y is 5/3 of x. h is 4. The area of the trapezium is the same as the area of the parallelogram so the expression of the area of the trapezium in terms of x can be set equal to the 40 cm²

$\frac{1}{2} \times \frac{8}{3}x = 10$ ← Dividing both sides by 4 eliminates the 4 on the left

$\frac{8}{3}x = 20$ ← Multiplying both sides by 2 eliminates the 1/2 on the left

$8x = 60$ ← Multiplying both sides by 3 eliminates the 3 as the denominator on the left

$x = \frac{60}{8}$ ← Dividing both sides by 8 eliminates the 8 on the left and gets x on its own

$y = \frac{5}{3} \times \frac{60}{8}$ ← From the ratio, y is 5/3 of x

There is no need to simplify the fractions as they cannot be simplified to integers

x = $\frac{60}{8}$

y = $\frac{300}{24}$ **[6]**

11 Write $0.\dot{2}\dot{7}$ as a fraction in its simplest form.

$x = 0.\dot{2}\dot{7}$ ← Let x be the recurring decimal

$100x = 27.\dot{2}\dot{7}$ ← Multiplying by 100 gives a different decimal with the recurring digits in the same decimal places

$99x = 27$ ← Subtracting x from 100x cancels out the recurring digits

$x = \frac{27}{99}$ ← Dividing both sides by 99 expresses x as a fraction

$= \frac{9}{33}$ ← Simplifying the fraction by dividing both the numerator and denominator by 3

Simplifying the fraction by dividing both the numerator and denominator by 3. It does not go any simpler as the numerator and denominator cannot be divided by the same amount to get smaller whole numbers

$\frac{3}{11}$ .... **[3]**

12 The time, *t* seconds, taken by each of 60 students to complete a puzzle is recorded.

The table shows information about these times.

Time (t seconds)	$20 < t \leqslant 30$	$30 < t \leqslant 40$	$40 < t \leqslant 50$	$50 < t \leqslant 70$	$70 < t \leqslant 90$
Frequency	8	0	12	30	10

(a) Two students are picked at random.
Reece works out the probability that they both took longer than 50 seconds to complete the puzzle.
Reece's working is shown below.

> The number of students who took longer than 50 seconds is 30 + 10 = 40
>
> The probability that one student took longer than 50 seconds is $\frac{40}{60} = \frac{2}{3}$
>
> The probability they both took longer than 50 seconds is $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

Explain the error in their method and write the correct calculation that Reece needs to do.
You do not need to work out the answer to the calculation.

The error is the probability for the second student is not 2/3

As there is one fewer person after the first student is picked

The correct calculation is $\frac{40}{60} \times \frac{39}{59}$ **[2]**

There is one fewer student who took longer than 50 seconds and one fewer student in total after the first student is picked

(b) Two students are picked at random from those who took 50 seconds or less.

Find the probability that one of them took 30 seconds or less and the other took more than 40 seconds.
You must show your working.

$8+0+12$ ← Adding the 8 in the interval $20 < t \le 30$, the 0 in the interval $30 < t \le 40$ and the 12 in the interval $40 < t \le 50$ works out that there were 20 students who took 50 seconds or less

$\frac{8}{20} \times \frac{12}{19} + \frac{12}{20} \times \frac{8}{19}$ ← 30 seconds or less AND more than 40 seconds OR more than 40 seconds AND 30 seconds or less. AND means to multiply. OR means to add. 8 out of the 20 students took 30 seconds or less. There is one fewer student in total for the second pick so there are then 12 out of 19 students who took more than 40 seconds. 12 out of the 20 students took more than 40 seconds. There is one fewer student in total for the second pick so there are then 8 out of 19 students who took 30 seconds or less

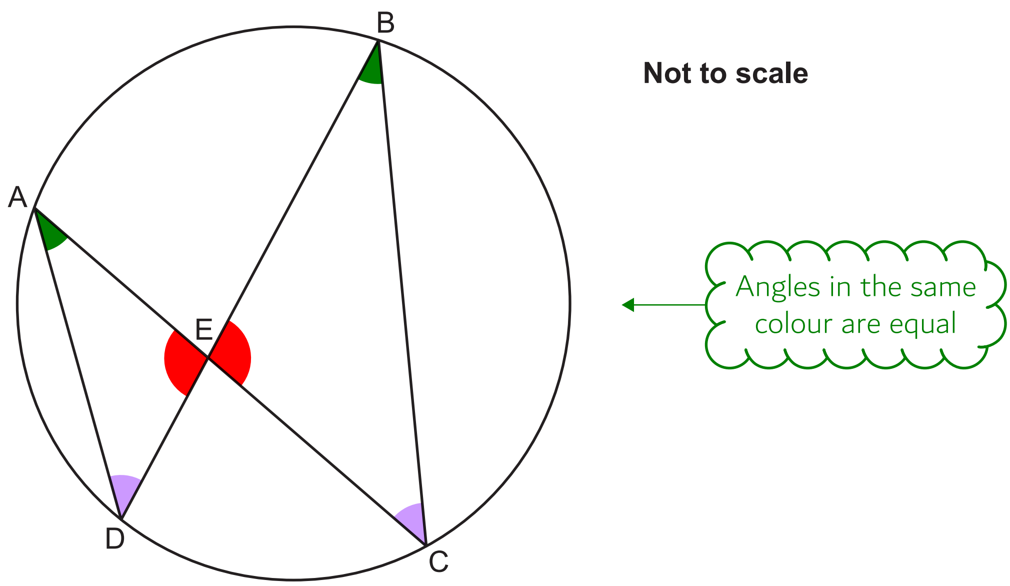
$\frac{4}{10} \times \frac{12}{19} + \frac{6}{10} \times \frac{8}{19}$ ← Simplifying the 8/20 and the 12/20 to make the multiplication easier

$\frac{48}{190} + \frac{48}{190}$ ← Multiplying the fractions by multiplying the numerators and multiplying the denominators

$48 + 48 = 96$ ← Adding the numerators of both fractions

(b) .... $\frac{96}{190}$ .... [5]

13 Points A, B, C and D lie on the circumference of a circle.
Line AC intersects line BD at point E.



Prove that triangle AED is similar to triangle BEC.

Angles AED = BEC as they are vertically opposite.

Angles DAE = EBC and ADE = ECB as they are angles in the same segment.

Therefore the triangles are similar as all their angles are the same.

....

....

....

.... **[3]**

14 The number of bees, P, in a colony is given by the formula

$P = ab^x$

where x is the number of months after the start of July.

At the start of July, there were 25 000 bees in the colony.
After one month, there were 23 500 bees in the colony.

Find the value of a and the value of b.
Give the value of b as a decimal.

$25000 = ab^0$ ← At the start of July, P is 25000 and x is 0. Substituting the known values into the equation

$= a$ ← Anything to the power of 0 is 1 so $b^0 = 1$. a x 1 = a so the right just becomes a. 25000 is a

$23500 = 25000b^1$ ← After one month, P is 23500 and x is 1. Substituting the known values into the equation

$\frac{23500}{25000} = b$ ← $b^1 = b$. Rearranging to make b the subject by dividing both sides by 25000

a = 25000

b = 0.94 **[4]**

15 **(a)** Simplify.

$\sqrt{3} \times \sqrt{15}$

$\sqrt{45}$ ← $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{9} \times \sqrt{5}$ ← $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$. Choosing a value of a which is a square number so that it can be square rooted

The square root of 9 is 3

(a) $3\sqrt{5}$ **[2]**

(b) Rationalise the denominator and simplify.

$\frac{40}{\sqrt{15}}$

$\frac{40\sqrt{15}}{15}$ ← Rationalising the denominator by multiplying both the numerator and denominator by $\sqrt{15}$

Simplifying the fraction by dividing both the numerator and denominator by 5

(b) $\frac{8\sqrt{15}}{3}$ **[3]**

(c) Work out.

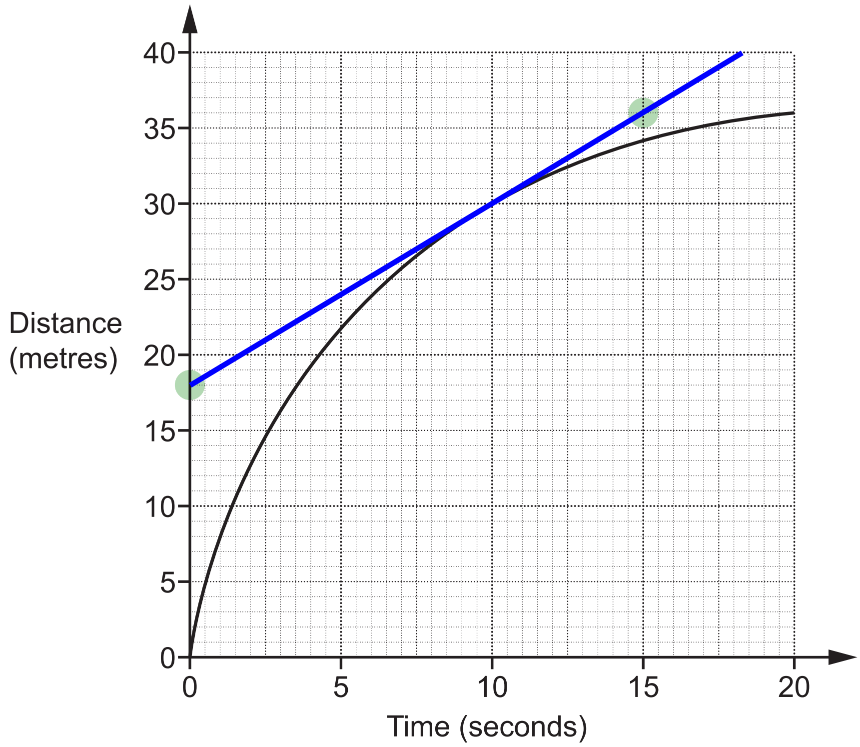
$27^{\frac{4}{3}}$

3^4 ← First dealing with the denominator of the power. The over 3 means to cube root. $3^3 = 27$ so $\sqrt[3]{27} = 3$

Raising to the power of 4 by squaring twice. $3^2 = 9$ then $9^2 = 81$

(c) 81 **[2]**

16 The graph shows the distance travelled by a particle over the first 20 seconds of its motion.



(a) Show that the average speed of the particle over the first 20 seconds of its motion is 1.8 m/s. **[1]**

$36 \div 20 = 1.8$

The unit of m/s tells us to divide the distance travelled in metres by the time taken in seconds. The total distance is 36 m and the total time is 20 seconds so dividing these gives the speed in m/s

(b) Estimate the speed of the particle at 10 seconds.
You must show working to support your estimate.

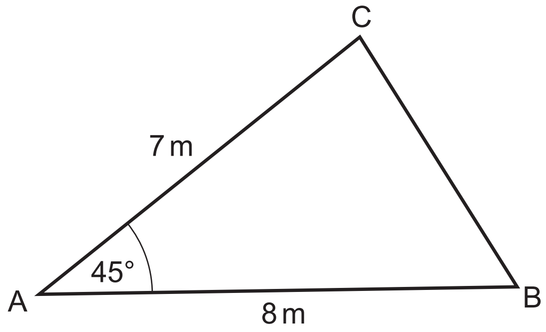
$\frac{36-18}{15-0}$

The gradient on a distance-time graph is the speed. Drawing a tangent to the curve where the time is 10 seconds then working out its gradient works out an estimate of the speed. Gradient = (change in y)/(change in x). Using the points highlighted in green, y changes from 18 to 36 and x changes from 0 to 15

The gradient can be left as an unsimplified fraction as it will not simplify to an integer

(b) $\frac{18}{15}$ .... m/s **[3]**

17 The diagram shows triangle ABC.



Not to scale

Find the area of the triangle.
Give your answer in the form $a\sqrt{b}$ where a and b are integers.

$\frac{1}{2} \times 8 \times 7 \times \sin 45$ ← Area of triangle = 1/2 absinC, where a and b are two of the sides and C is the angle between them

0	30	45	60	90
0	1	2	3	4

← Working out sin45 by listing the angles 0, 30, 45, 60, 90 then 0, 1, 2, 3, 4 under these. Square rooting the numbers underneath and putting them over 2 gives the sin values. So sin45 = $\sqrt{2}/2$

$28 \times \frac{\sqrt{2}}{2}$ ← 1/2 x 8 = 4 then 4 x 7 = 28. Multiplying this by the value of sin45

Dividing the 28 by the 2 gives 14. The $\sqrt{2}$ stays →

$14\sqrt{2}$ .... m^2 **[3]**

18 **(a)** By factorising, find the roots of $y = x^2 + 18x + 77$.

1, 77
7, 11

Factorising by finding two whole numbers which multiply to the 77 and add to the 18. Listing out the factor pairs of 77 until a pair add to 18

$(x+7)(x+11)=0$

7 and 11 are the two whole numbers which multiply to the 77 and add to the 18. Putting these two numbers in brackets with x. The roots are where the graph would cross the x-axis so are when y = 0

Either x + 7 = 0 or x + 11 = 0. Subtracting 7 from both sides of the first equation finds that x = -7 and subtracting 11 from both sides of the second equation finds that x = -11

(a) $x =$ ....-7.... and $x =$ ....-11.... **[3]**

(b) **(i)** Write $y = x^2 + 18x + 77$ in the form $y = (x + a)^2 - b$.

$y = (x+9)^2 + 77 - 81$

This is completing the square. Halving the coefficient of x (which is 18) to get 9, putting this in a bracket with x and squaring the bracket. Leaving the +77 on the outside and subtracting 9^2

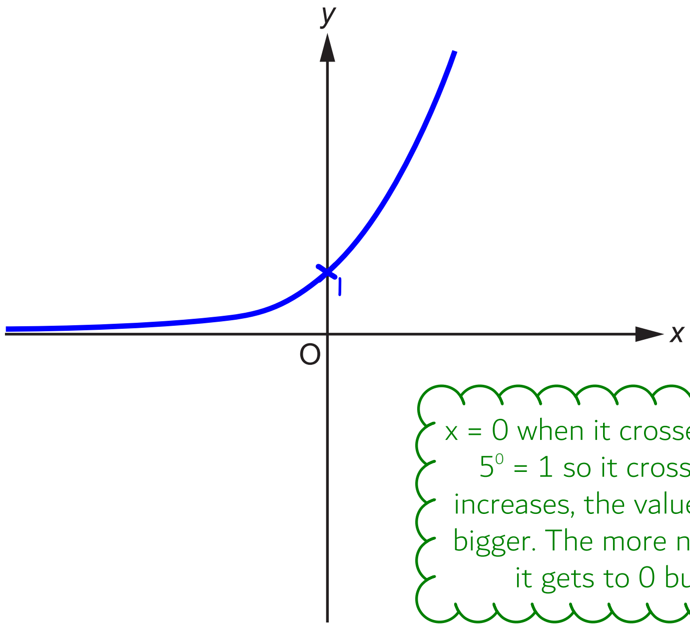
(b)(i) $y =$ ....$(x+9)^2 - 4$.... **[3]**

(ii) Write down the coordinates of the turning point of the graph of $y = x^2 + 18x + 77$.

Using the completed the square form above, the turning point is when the square bracket is equal to 0. x = -9 for this to happen and when the square bracket is 0, y = -4

(ii) (....-9.... , ...-4....) **[2]**

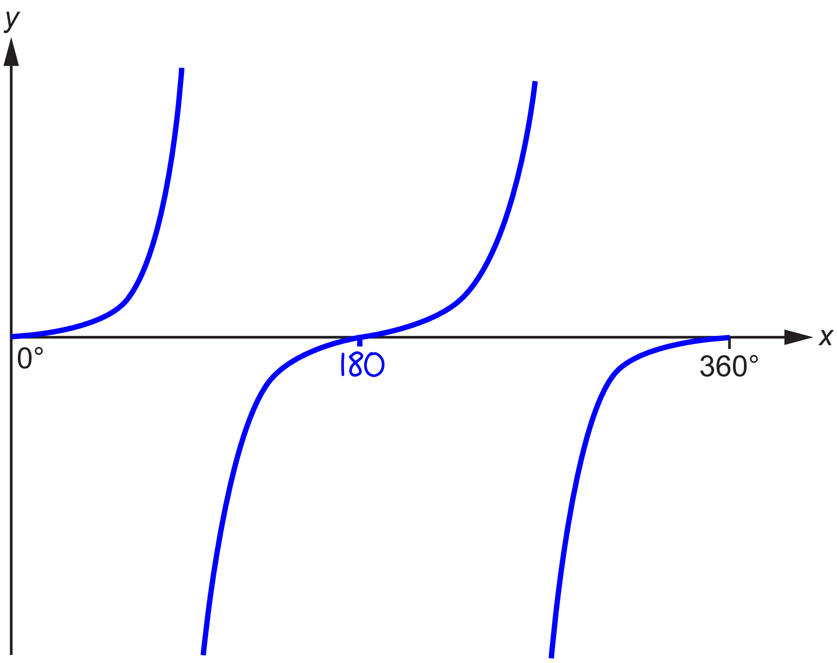
19 **(a)** Sketch the graph of $y = 5^x$ indicating any values where the graph crosses the axes.



x = 0 when it crosses the y-axis. When x = 0, $5^0 = 1$ so it crosses the y-axis at 1. As x increases, the value of 5^x gets exponentially bigger. The more negative x gets, the closer it gets to 0 but it never reaches 0

[2]

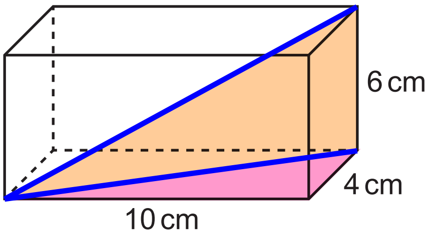
(b) Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$ indicating any values where the graph crosses the axes.



[2]

A tan graph has this curved shape which repeats every 180 degrees

20 Kai has a box in the shape of a cuboid.
The internal dimensions of the box are 10 cm by 4 cm by 6 cm.



Drawing the longest length in the cuboid. Then drawing a line on the base of the cuboid which forms a right-angled triangle so that the longest length can be worked out

Kai is given a pencil of length 13 cm.

Show that the pencil does not fit completely inside the box. **[4]**

$a^2 + b^2 = c^2$ ← Pythagoras' Theorem can be used to work out the missing length in a right-angled triangle. a and b are the two shorter sides and c is the longest side

$\sqrt{10^2 + 4^2}$ ← First using the pink right-angled triangle. Substituting 10 for a and 4 for b

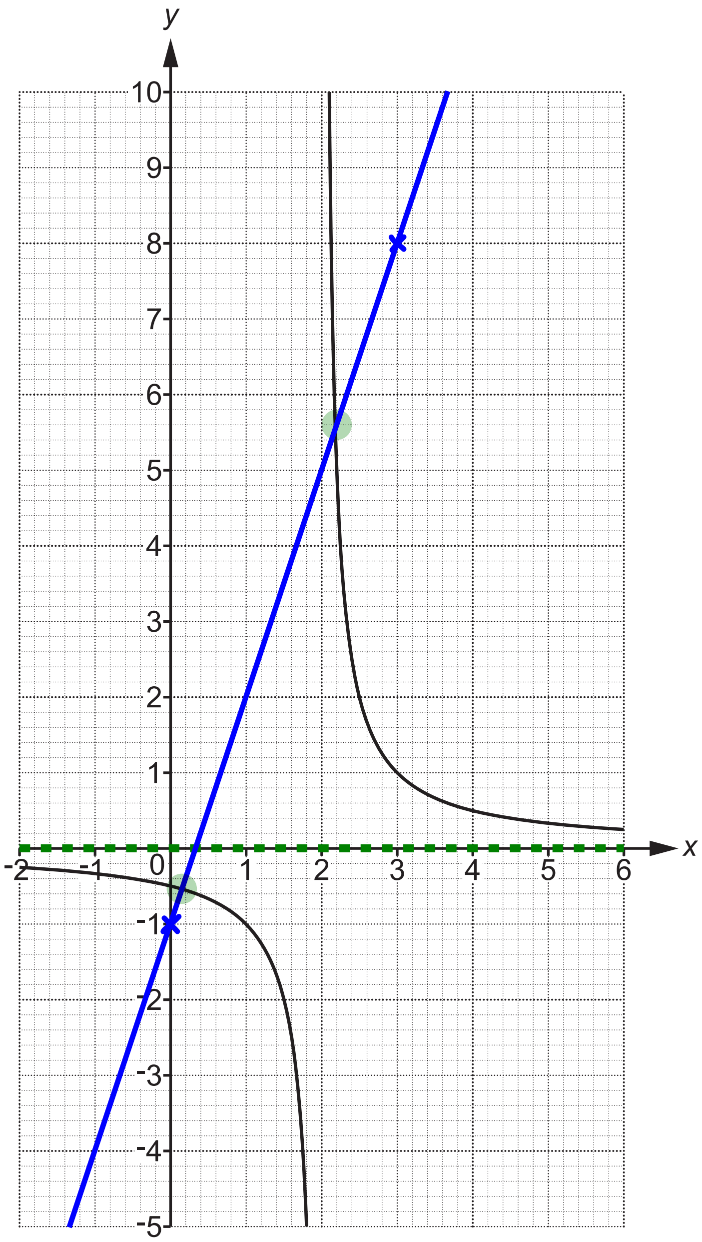
$\sqrt{116}$ ← $10^2 = 100$ and $4^2 = 16$. Then $100 + 16 = 116$

$\sqrt{116 + 6^2}$ ← Next using the orange right-angled triangle. Substituting the longest side from the pink triangle as a (which has its square root cancelled out when it is squared) and 6 as b

$$\begin{array}{r} 116 \\ +\ 36 \\ \hline 152 \end{array}$$ ← $6^2 = 36$

$\sqrt{152} < 13$ ← 13 is the square root of 169, so the longest length of $\sqrt{152}$ must be less than 13

21 The graph of $y = \frac{1}{x-2}$ is drawn on the grid for $-2 \leqslant x \leqslant 6$.



(a) There are no values of x for which $\frac{1}{x-2} = k$.

Find the value of k.

It is impossible to divide 1 to get 0. Also from the graph it can be seen that it is heading toward 0 in both directions and it will get closer and closer to the dashed line but never quite reach it

(a) $k =$ ....0...................... **[1]**

(b) **(i)** **Use the graph** to find approximate solutions to the equation $\frac{1}{x-2} = 3x - 1$.
Give your answers to **1** decimal place.
Show your working on the graph.

$3(0)-1=-1$
$3(3)-1=8$

Drawing the graph of y = 3x - 1 and seeing where the two graphs cross. When x = 0, y = -1 so plotting the coordinate of (0 , -1). When x = 3, y = 8 so plotting the coordinate of (3 , 8). Drawing a straight line through both of these points as it is in the form y = mx + c it must be a straight line graph. The two graphs cross at the points highlighted in green

(b)(i) $x =$ ......0.2...... or $x =$ ......2.2...... **[4]**

(ii) Show algebraically that $\frac{1}{x-2} = 3x - 1$ has the same solutions as $3x^2 - 7x + 1 = 0$. **[4]**

$1=(3x-1)(x-2)$

Multiplying both sides of the equation by (x - 2) to get rid of any x as a denominator

$=3x^2-6x-x+2$

Expanding the brackets on the right side

$3x^2-7x+1=0$

Simplifying by collecting like terms and subtracting 1 from both sides to show that the equations can be rearranged to get each other

END OF QUESTION PAPER