Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

GCSE
MATHEMATICS

Higher Tier Paper 1 Non-Calculator

H

Tuesday 5 November 2019 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- mathematical instruments

You must **not** use a calculator.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
TOTAL	

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided

1 Circle the calculation that decreases 250 by 15%

[1 mark]

250 ÷ 1.15 250 × 0.15 [✓ 250 × 0.85] 250 ÷ 0.85

Subtracting the 15% from 100% finds that it decreases to 85%. Dividing this by 100 converts it into the decimal 0.85, which when multiplied by decreases by 15%

2 Solve $3x = 2x$

Circle your answer.

[1 mark]

$x = -1$ [✓ $x = 0$] $x = \frac{2}{3}$ $x = \frac{3}{2}$

3 x 0 = 2 x 0. The other answers will not work when substituted into the equation

3 *A* is (2, 13) and *B* is (10, 1)

Circle the midpoint of *AB*.

[1 mark]

(4, 6)   (5, 6.5)   [✓ (6, 7)]   (8, 12)

Doing the mean of the x-coordinates of point A and B works out what is halfway between them. 2 + 10 = 12 then 12 ÷ 2 = 6, so the x-coordinate of the midpoint must be 6

4 Circle the expression equivalent to $(2x)^4$

[1 mark]

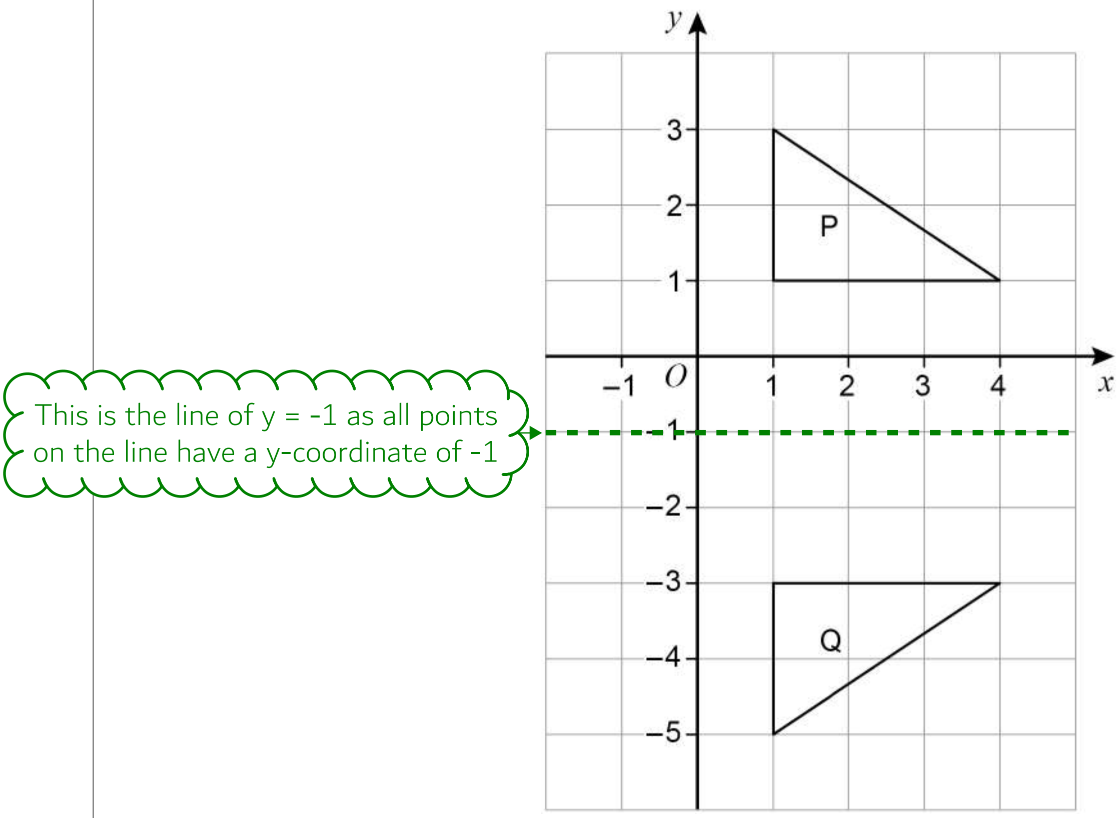
$2x^4$ $6x^4$ $8x^4$ [✓ $16x^4$]

Both the 2 and the x need to be raised to the power of 4. 2^4 = 2 x 2 x 2 x 2 = 16

Turn over for the next question

4

5 (a) Here are two triangles, P and Q.



Here is a statement.

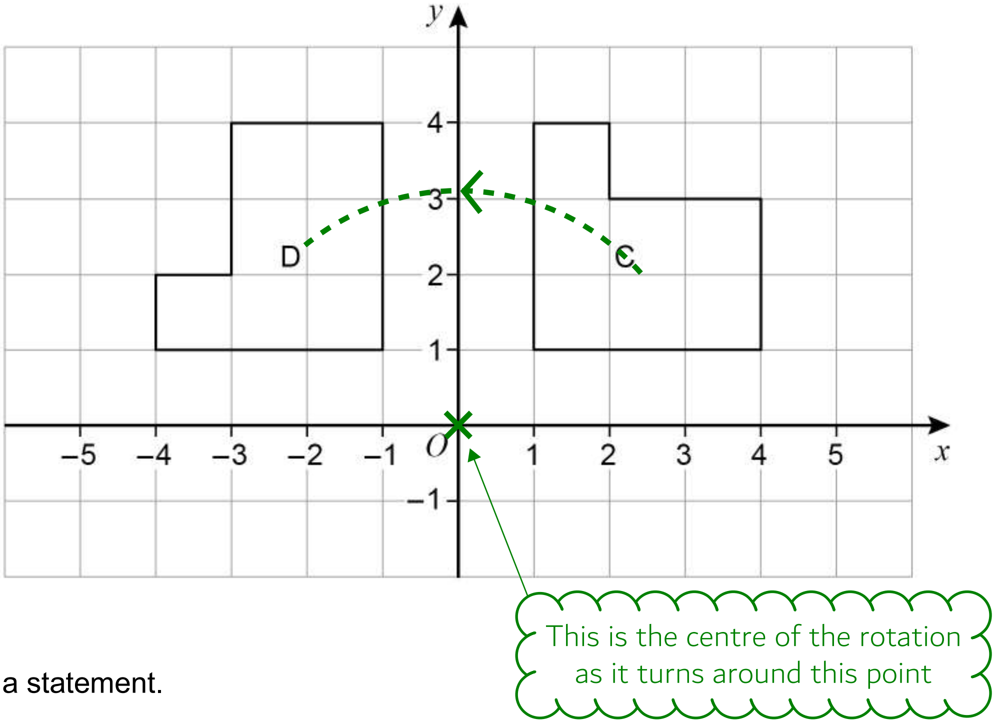
A transformation that maps P to Q is a reflection in the line $x = -1$

Make **one** criticism of the statement.

[1 mark]

The line is not x = -1

5 (b) Here are two shapes, C and D.



Here is a statement.

A transformation that maps C to D is a rotation through 90° anticlockwise.

Make **one** criticism of the statement.

[1 mark]

Centre of rotation is not given

Turn over for the next question

2

6 (a) A geometric progression starts 4 16

Work out the next term.

[1 mark]

$$\begin{array}{r} 16 \\ \times\, 4 \\ \hline 64 \end{array}$$

Geometric sequences multiply by the same amount between each term. 4 has been multiplied by 4 to get 16 so the 16 must be multiplied by 4 to get the next term

Answer 64

6 (b) A Fibonacci-type sequence starts 3 –8

The sequence is continued by adding the previous two terms.

Work out the next **two** terms.

[2 marks]

Answer -5 and -13

3 + -8 = -5
-8 + -5 = -13

7 Given that $a \times 60 = b$ work out the value of $\frac{4b}{a}$

[2 marks]

$4 \times a \times 60$

Substituting a x 60 for b in the numerator. This gives 240a

$\frac{240a}{a}$

Writing the fraction in terms of a

Answer 240

Both the numerator and denominator can be divided by a to give 240/1, which is 240

8 Write $27 \times \left(3^2\right)^7$ as a single power of 3

[3 marks]

$3^3 \times 3^{14}$

27 = 3^3. $(a^x)^y = a^{xy}$, so the 2 and 7 should be multiplied to give 14

Answer 3^{17}

$a^x \times a^y = a^{x+y}$, so the 3 and the 14 should be added to give 17

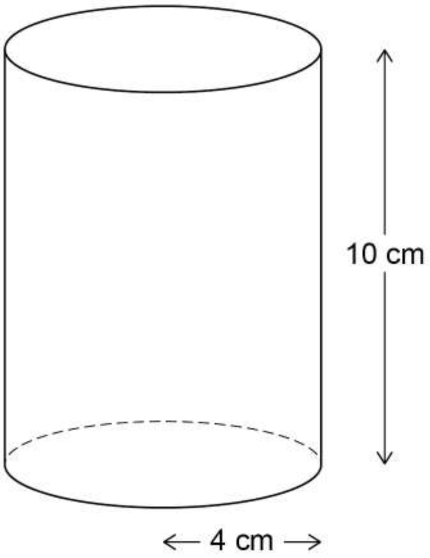
Turn over for the next question

8

9 Here are two solids.

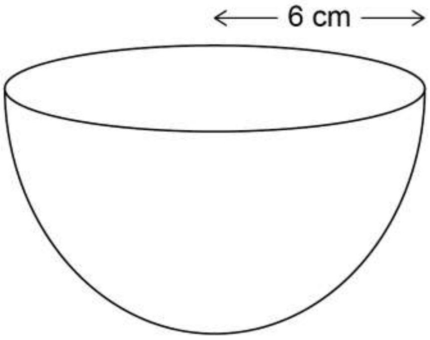
Cylinder

radius 4 cm height 10 cm



Hemisphere

radius 6 cm



volume of a hemisphere $= \frac{2}{3}\pi r^3$ where r is the radius

Which solid has the greater volume?

You **must** show your working.

[4 marks]

$4^2 \times 10$

Volume of cylinder = area of circle x height. Area of circle = π x radius^2.
Ignoring π and doing radius^2 x height

160

4^2 = 16, then 16 x 10 = 160. So the volume of the cylinder must be 160π cm^3(when the π is put back in)

$$\begin{array}{r} 36 \\ \times\ 6 \\ \hline 2_{3}16 \end{array}$$

Working out 6^3 as radius^3 is part of the formula for the hemisphere. 6^3 = 6 x 6 x 6 = 36 x 6

$$\begin{array}{r} 072 \\ 3\overline{)2^{2}1 6} \end{array}$$

$$\begin{array}{r} 72 \\ \times\ 2 \\ \hline 144 \end{array}$$

Next doing the 2/3 of 216 by dividing by the denominator then multiplying the result by the numerator

So the volume of the cylinder must be 144π cm^3

Answer Cylinder

160π cm^3 is greater than 144π cm^3

Turn over for the next question

4

10 Saj makes Rose Pink paint and Cherry Pink paint.

He mixes red paint with white paint as shown.

Rose Pink	**Cherry Pink**
red : white = 1 : 2	red : white = 4 : 3

He makes 60 litres of Rose Pink paint.

To this Rose Pink paint he adds

80 litres of red paint and 28 litres of white paint.

Has he now made Cherry Pink paint?

You **must** show your working.

[4 marks]

$60 \div 3$

1 + 2 = 3, so there are 3 parts in total in the ratio for Rose Pink. These 3 parts represent the total of 60 litres so dividing the 60 by 3 works out that 1 part of the ratio is worth 20 litres. There is therefore 20 litres of red paint

$20 \times 2 = 40$

2 parts of the ratio represent the white paint so multiplying the value of 1 part of the ratio by 2 works out that 2 parts is worth 40 litres. There is therefore 40 litres of white paint

$$\begin{array}{r} 20 \\ +80 \\ \hline 100 \end{array}$$

80 litres of red paint is added to the 20 litres of red paint which was in the Rose Pink. So he now has 100 litres of red paint

$$\begin{array}{r} 40 \\ +28 \\ \hline 68 \end{array}$$

28 litres of white paint is added to the 40 litres of white paint which was in the Rose Pink. So he now has 68 litres of white paint

$$\begin{array}{r} 0\,2\,5 \\ 4\overline{)1\,{}^{1}0\,{}^{2}0} \end{array}$$

Assuming that he has now made Cherry Pink, 4 parts of the Cherry Pink ratio represent the 100 litres of red paint. So dividing the 100 by 4 works out that 1 part of the Cherry Pink ratio would be worth 25 litres

$25 \times 3 \neq 68$

Multiplying the value of 1 part of the Cherry Pink ratio by 3 will not give the 68 litres of white paint

No

It cannot be in the ratio of 4 : 3 as it does not work when assuming that the 4 parts represents the 100 litres of red paint. Therefore it cannot be Cherry Pink

11 (a) Work out $\frac{2 \times 10^{14}}{8 \times 10^{9}}$

Give your answer in standard form.

[2 marks]

0.25×10^5

$8\overline{)2.^{2}0^{4}0}$

Dividing the 2 by the 8 gives 0.25. Dividing the 10^{14} by the 10^9 gives 10^5 as $a^x \div a^y = a^{x-y}$. So it can be written as 0.25×10^5

Answer 2.5×10^4

The 0.25 must be multiplied by 10 once to get 2.5, which is at least 1 but less than 10. Taking 1 off the power of 10 divides it by 10 to keep it the same

11 (b) $6200.07 = 6.2 \times 10^c + 7 \times 10^d$

Work out the values of c and d.

[2 marks]

$c =$ 3 $d =$ -2

$6.2 \times 10^3 = 6200$ and $7 \times 10^{-2} = 0.07$. Adding these together gives 6200.07. $\times 10^3$ basically means to multiply by ten 3 times. $\times 10^{-2}$ basically means to divide by ten 2 times

Turn over for the next question

8

12 $V = \frac{k}{H}$ where k is a constant.

Which **two** statements are correct?

Tick **two** boxes.

[1 mark]

☐ V is directly proportional to H

☑ V is inversely proportional to H ← This must be correct as increasing H decreases V

☑ V is directly proportional to $\frac{1}{H}$ ← This must be correct as it means the same as V is inversely proportional to H

☐ V is inversely proportional to $\frac{1}{H}$

13 The nth term of a sequence is $\frac{n(n-4)}{\sqrt{n+3}}$

Work out the sum of the 1st and 6th terms.

[3 marks]

$\frac{1(1-4)}{\sqrt{1+3}} = -\frac{3}{2}$ ← Substituting 1 for n in the nth term works out that the 1st term is -3/2

$\frac{6(6-4)}{\sqrt{6+3}} = \frac{12}{3} = 4$ ← Substituting 6 for n in the nth term works out that the 6th term is 4

$\frac{8}{2} - \frac{3}{2}$ ← Adding the 1st term to the 6th term. 4 can be thought of as 4/1 then multiplying both the numerator and denominator by 2 converts it into 8/2. This makes the denominators the same so that the two fractions can be subtracted

Answer $\frac{5}{2}$

14 $8300 = 100 \times 83$

Circle the number that is closest in value to $\sqrt{8300}$

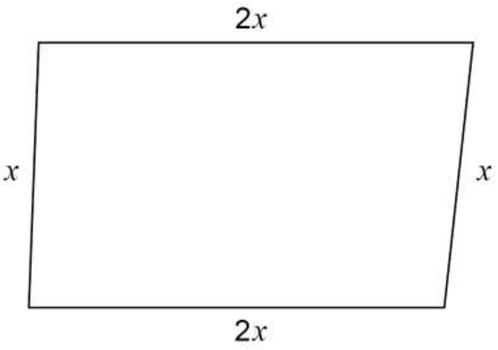
[1 mark]

19 [✓ 90] 830 900

$\sqrt{100} \times \sqrt{81} = 10 \times 9 = 90$

15 Here is a **sketch** of a quadrilateral.

All lengths are in centimetres.



Not drawn accurately

Tick **one** box for each statement.

[3 marks]

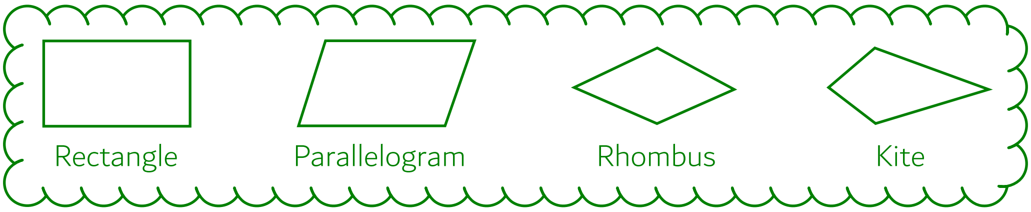
	True	**May be true**	**Not true**
The quadrilateral is a rectangle		✓	
The quadrilateral is a parallelogram	✓		
The quadrilateral is a rhombus			✓
The quadrilateral is a kite			✓

It may be a rectangle as the opposite sides are equal however the angles may not be right-angles.

It is a parallelogram as the opposite sides are equal.

It is not a rhombus as all four sides cannot be equal.

It is not a kite as there are two pairs of equal sides however the equal sides are not joined together



16 In a box there are some buttons.

45 are large and the rest are small.

Some are yellow and the rest are green.

The number of small is $\frac{5}{3}$ of the number of large.

The number of green is 300% of the number of yellow.

There are 12 small yellow buttons.

How many large green buttons are there?

You may use the two-way table to help you.

[4 marks]

	Large	**Small**	
Yellow	18	12	30
Green			
	45	75	120

$3\overline{)4^{1}5}$ = 15

This works out that 1/3 of the number of large is 15

15
× 5
75

This works out that 5/3 of the number of large is 75, which is the number of small

+45
120

Adding the number of large and number of small works out that there are 120 in total

$4\overline{)1^{1}20}$ = 030

The ratio of yellow : green is 300 : 100, which simplifies to 3 : 1. There are 4 parts in total which represent the total of 120. Dividing the 120 by 4 works out that 1 part of the ratio is worth 30. So there are 30 yellow buttons

Answer 27

$^{2}\cancel{3}{}^{1}0$
−12
18

Subtracting the 12 small yellow from the 30 yellow works out that there are 18 large yellow

$^{3}\cancel{4}{}^{1}5$
−18
27

Subtracting the 18 large yellow from the 45 large works out that there are 27 large green

7

Turn over ▶

17 $\mathbf{a} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

x-component

y-component

Work out $\mathbf{a} - 3\mathbf{b}$

Circle your answer.

[1 mark]

[✓ $\begin{pmatrix} -6 \\ 17 \end{pmatrix}$] $\begin{pmatrix} -6 \\ -13 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 17 \end{pmatrix}$ $\begin{pmatrix} 0 \\ -13 \end{pmatrix}$

$-3-3(1)=-6$

$2-3(-5)=17$

Working with the x-components gives -6 then working with the y-components gives 17

18 Solve $\dfrac{x+15}{3} = 2(x+10)$

[4 marks]

$2x+20$ Expanding the bracket on the right

$x+15=6x+60$ Multiplying both sides by 3 to eliminate the 3 as the denominator on the left

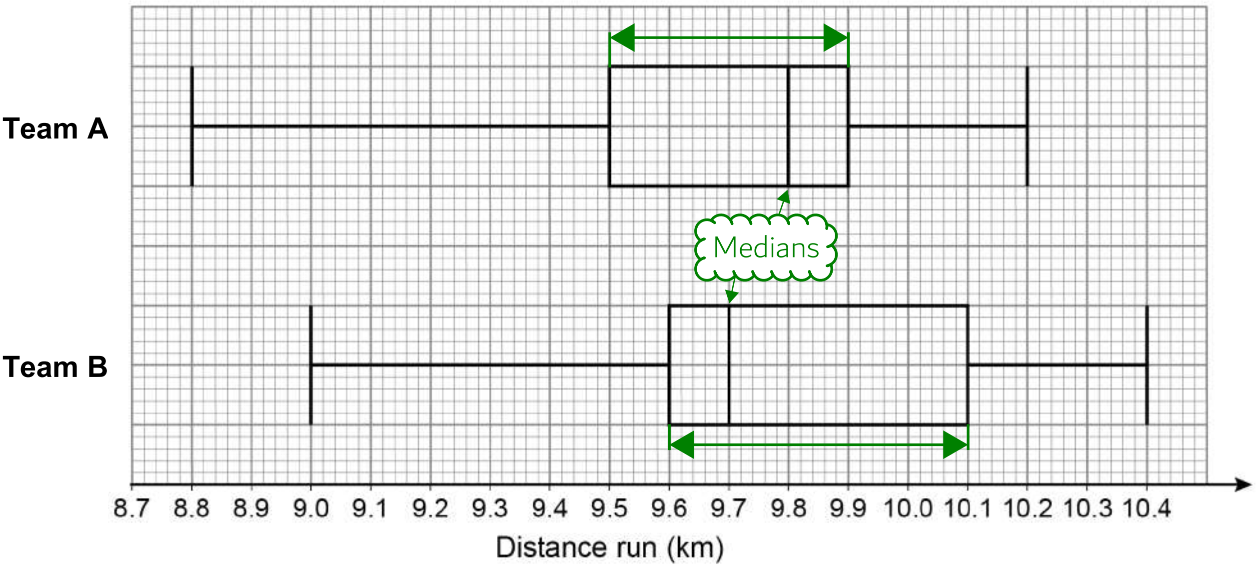
$15=5x+60$ Subtracting x from both sides to get all the x on the same side

$-45=5x$ Subtracting 60 from both sides to get the x term on its own

$x =$ -9

Dividing both sides by 5 gets x on its own

19 The box plots represent the distances run by the players in a football match.



19 (a) On average, which team's players ran further?

Tick a box.

☑ Team A ☐ Team B

Give a reason for your answer.

[1 mark]

Median is greater

19 (b) The players in Team A ran more consistent distances.

How do the box plots show this?

[1 mark]

Interquartile range is less

The interquartile ranges are shown by the green arrows. They are narrower for Team A so therefore it is more consistent

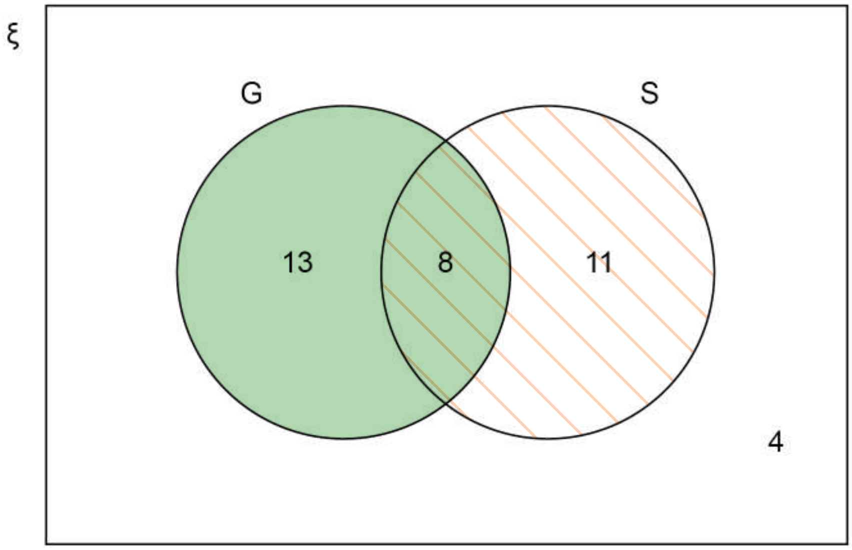
7

Turn over ▶

20 The Venn diagram shows information about some houses.

G = houses with a garage

S = houses with a shed



A house is chosen at random.

20 (a) The house has a garage.

What is the probability that it has a shed?

[1 mark]

Answer $\frac{8}{21}$

13 + 8 = 21 so 21 houses had a garage. Out of these, 8 also have a shed

20 (b) The house does **not** have a garage.

What is the probability that it does **not** have a shed?

[1 mark]

Answer $\frac{4}{15}$

11 + 4 = 15 so 15 did not have a garage. Out of these, 4 do not have a shed

20 **(c)** Show that $P(G \cap S)' > P(G \cup S')$

[2 marks]

$\frac{28}{36} > \frac{25}{36}$

There are 36 houses in total (13 + 8 + 11 + 4). G ∩ S is where it is both highlighted in green and lined in orange. The dash means everything apart from this. So 28 (13 + 11 + 4) out of the 36 houses are in (G ∩ S)'. G U S' is anything highlighted in green or not lined in orange or both green and not lined in orange. So 25 (13 + 8 + 4) out of the 26 houses are in G U S'

21 Work out $0.70\dot{4}\dot{8} - 0.001$

Circle your answer.

[1 mark]

$0.70\dot{3}\dot{8}$ $0.703\dot{8}$ $0.703\dot{8}\dot{3}$ [✓ $0.703\dot{8}\dot{4}$]

$$\begin{array}{r} 0.704\dot{8}\dot{4} \\ -0.001 \\ \hline 0.703\dot{8}\dot{4} \end{array}$$

Rewriting the recurring decimal so that the recurring digits do not have anything subtracted from them then subtracting the 0.001

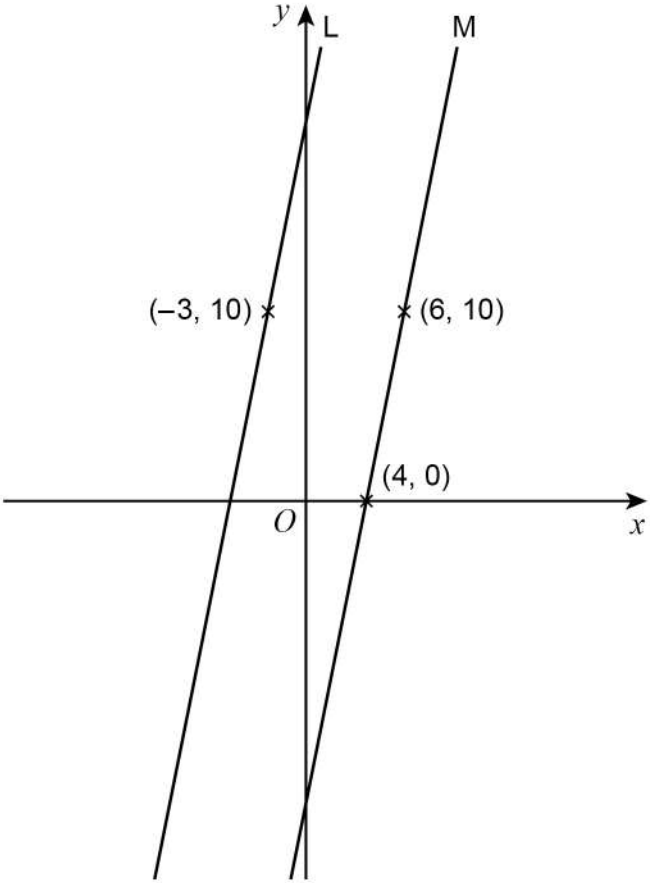
Turn over for the next question

5

22 (–3, 10) is a point on line L.

(4, 0) and (6, 10) are points on line M.

L and M are parallel.



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Work out the equation of line L.

Give your answer in the form $y = mx + c$

[3 marks]

$\frac{10-0}{6-4} = \frac{10}{2} = 5$

As the lines are parallel, they have the same gradient. Working out that the gradient of line M is 5. Gradient = (change in y)/(change in x). y changes from 0 to 10 so the change in y is 10 - 0. x changes from 4 to 6 so the change in x is 6 - 4

$10+5\times3=25$

The gradient basically means how much it goes up by for every 1 it goes to the right. From -3 it is 3 to the right to get to the y-axis so it will go up 3 lots of 5 from the y-coordinate 10 and meet the y-axis at 25

Answer $y=5x+25$

m is the gradient and c is the y-intercept

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23 (a) Factorise $5x^2 + 6x - 8$

It is in the form $ax^2 + bx + c$

[2 marks]

1,40
2,20
4,10

Multiplying a by c (the 5 by the -8) gives -40. Listing out the factor pairs of 40 until they will add to b (the 6) when one of the pair is negative

$5x^2+10x-4x-8$

-4 and 10 multiply to the -40 and add to the 6. Splitting the middle x term into these numbers of x

$5x(x+2)-4(x+2)$

Answer $(5x-4)(x+2)$

Factorising the left two terms and factorising the right two terms

Bringing the 5x and -4 into a bracket and writing the repeated (x + 2) once

23 (b) Simplify fully $\dfrac{x^2+9x+14}{x^2-4}$

[3 marks]

$$\frac{(x+7)(x+2)}{(x+2)(x-2)}$$

Factorising the numerator by putting 7 and 2 in brackets with x as these two numbers multiply to the 14 and add to the 9. Factorising the denominator by using difference of two squares: $A^2 - B^2 = (A + B)(A - B)$

Answer $\dfrac{x+7}{x-2}$

(x + 2) is a common factor to the numerator and denominator so it can be cancelled out. There are no other common factors

Turn over for the next question

8

24 Work out $\sqrt{18} - \dfrac{28}{\sqrt{50}}$

Give your answer in the form $\dfrac{\sqrt{a}}{b}$ where a and b are integers.

[4 marks]

$\sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

Simplifying $\sqrt{18}$ by using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ to split it into $\sqrt{9} \times \sqrt{2}$. Then square rooting the 9 gives 3

$\sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

Simplifying $\sqrt{50}$ by using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ to split it into $\sqrt{25} \times \sqrt{2}$. Then square rooting the 25 gives 5

$\dfrac{28}{5\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

Replacing $\sqrt{50}$ with $5\sqrt{2}$ then rationalising the denominator by multiplying both the numerator and denominator by $\sqrt{2}$

$\dfrac{28\sqrt{2}}{10}$

$\sqrt{2} \times \sqrt{2} = 2$. Then $5 \times 2 = 10$

$\dfrac{30\sqrt{2}}{10} - \dfrac{28\sqrt{2}}{10}$

Replacing $\sqrt{18}$ with $3\sqrt{2}$. Multiplying by 10 and putting it over 10 so that both fractions have the same denominator and can be subtracted

$\dfrac{2\sqrt{2}}{10}$

The numerators can be subtracted and the denominators stay the same

Answer $\dfrac{\sqrt{2}}{5}$

Simplifying by dividing both the numerator and denominator by 2 puts it into the desired form

25 A bag contains 8 balls.

3 are red and 5 are blue.

2 balls are taken from the bag at random without replacement.

25 (a) Write down the probability that there is **at least** 1 red ball still in the bag.

[1 mark]

Answer 1

Picking both red balls will still leave 1 red ball in the bag. Therefore it is certain that there is at least 1 red ball still in the bag. The probability of something certain is 1

25 (b) Work out the probability that there are **at least** 2 red balls still in the bag.

[3 marks]

$\frac{3}{8} \times \frac{2}{7}$

This works out that the probability of there not being at least 2 red balls still in the bag is 6/56. The only way of there not being at least 2 red balls still in the bag is if both balls picked are red. Red AND red. AND means to multiply the probabilities. 3 out of the 8 balls are red so the probability of getting red on the first pick is 3/8. On the second pick there is 1 fewer red and 1 fewer ball in total so it is 2/7

$\frac{56}{56} - \frac{6}{56}$

It is certain that there will either be at least 2 red balls still in the bag or that there will not be at least 2 red balls still in the bag. So the probabilities of both must add up to 1 and subtracting the probability of there not being at least 2 red balls still in the bag from 1 leaves the probability of there being at least 2 red balls still in the bag. 1 is written as 56/56 so that the denominators of both fractions are the same and they can be subtracted

Answer $\frac{50}{56}$

8

Turn over ▶

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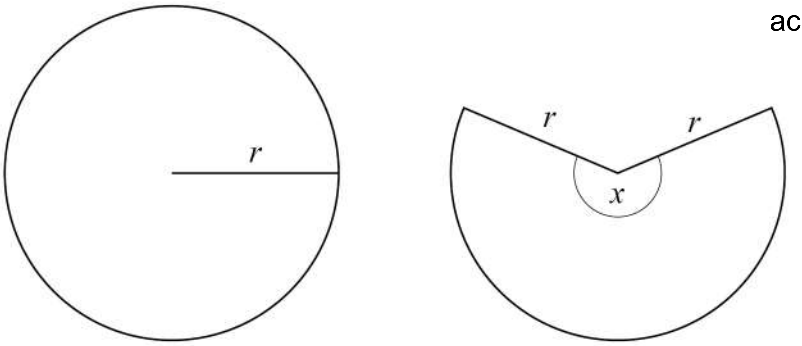
26 Here are a circle and a sector of the circle.

They each have radius r.

Not drawn accurately



circumference of circle = perimeter of sector

Work out the size of angle x.

Give your answer in terms of π

[4 marks]

$2\pi r = \frac{x}{360} \times 2\pi r + 2r$

Expressing the circumference of the circle and setting this equal to an expression of the perimeter of the sector. Circumference of circle = 2πr. x/360 is the fraction of the circumference the arc of the sector is so doing this fraction of the circumference. Adding 2 lots of r as these are on the perimeter

$\frac{2\pi r - 2r}{2\pi r} = \frac{x}{360}$

Subtracting 2r from both sides then dividing both sides by 2πr

$\frac{\pi - 1}{\pi} \times 360 = x$

Simplifying the fraction on the left by dividing all terms on the numerator and denominator by 2 and r. Then multiplying both sides by 360 to get x on its own

Answer $\frac{360\pi - 360}{\pi}$ degrees

Multiplying the numerator of the fraction by the 360

27 A curve has the equation $y = x^2 - 6x + 17$

The turning point of the curve is at $(a, 8)$

27 (a) By completing the square, or otherwise, work out the value of a.

[2 marks]

$y=(x-3)^2+d$

Completing the square by halving the coefficient of x, putting this in a bracket with x and squaring the bracket. Leaving the outside as d as something needs to be subtracted from the 17 but we are only concerned with the x-coordinate

Answer 3

The turning point occurs when the square bracket is equal to 0 as this is the smallest a squared number can be. x = 3 for this to happen

27 (b) The turning point of the curve $y = x^2 + 4x + b$ also has y-coordinate 8

Work out the value of b.

[2 marks]

$y=(x+2)^2+b-4$

Completing the square by halving the coefficient of x, putting this in a bracket with x and squaring the bracket. 2^2 must be subtracted from the outside of the bracket to keep it equal to the original

$8=b-4$

Substituting in the y-coordinate of the turning point. At the turning point, the square bracket is equal to 0 as this is the smallest a squared number can be so this can be ignored

Answer 12

Adding 4 to both sides gets b on its own

8

28 Work out the value of $100^{-\frac{1}{2}}$

[2 marks]

Dealing with the 1/2 first. This as a power means to do the positive square root. The positive square root of 100 is 10. Then the negative power means to do the reciprocal, which means to do 1 over the 10

Answer $\frac{1}{10}$

29 Show that the value of $5\sin 30^\circ \times \cos 30^\circ \times 8\tan 30^\circ$ is an integer.

[4 marks]

0	30	45	60	90
0	1	2	3	4
4	3	2	1	0

Writing out the angles we need to know the trig values for. Under these writing 0, 1, 2, 3, 4 for the sin values and 4, 3, 2, 1, 0 for the cos values. Square rooting the 1 under the 30 gives 1 then putting this over 2 finds that sin 30 is 1/2. Square rooting the 3 under the 30 gives $\sqrt{3}$ then putting this over 2 finds that cos30 is $\sqrt{3}/2$

$\frac{1}{2} \div \frac{\sqrt{3}}{2}$

Dividing sin30 by cos30 finds tan30

$\frac{1}{2} \times \frac{2}{\sqrt{3}}$

To divide by a fraction: keep the first part, change the division to a multiply, flip the second fraction. To multiply fractions, the numerators can be multiplied and the denominators can be multiplied. The 2 on the numerator and denominator cancel out so tan30 is $1/\sqrt{3}$

$5\left(\frac{1}{2}\right) \times \frac{\sqrt{3}}{2} \times 8\left(\frac{1}{\sqrt{3}}\right)$

Substituting the values into the expression

$\frac{5}{2} \times \frac{\sqrt{3}}{2} \times \frac{8}{\sqrt{3}}$

Expanding the brackets by multiplying the numerators by what is outside of them

$\frac{40\sqrt{3}}{4\sqrt{3}}$

Multiplying the fractions by multiplying all the numerators and multiplying all the denominators

10

The 3 cancel out and dividing the 40 by the 4 gives 10, which is an integer

END OF QUESTIONS

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