

Please write clearly in block capitals.

Centre number

Candidate number

Surname _____

Forename(s) _____

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GCSE MATHEMATICS

H

Higher Tier Paper 2 Calculator

Thursday 7 June 2018

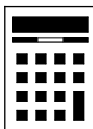
Morning

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
28–29	
TOTAL	



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

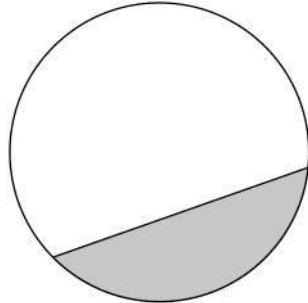
Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided

1 Here is a circle.



Circle the word that describes the shaded part.

[1 mark]

segment

chord

sector

arc

The area enclosed between a chord and the circumference

2 Circle the number that is in standard form.

[1 mark]

0.25×10^4

6×10^7

38×10^{-3}

$4 \times 10^{\frac{1}{2}}$

Standard form is $a \times 10^n$, where $1 \leq a < 10$ and n is an integer

3 y is $1\frac{1}{2}$ times x .

Circle the ratio that is equivalent to $y : x$

[1 mark]

2 : 5

5 : 2

3 : 2

2 : 3

y could be 3 and x could be 2. 3 is $1\frac{1}{2} \times 2$

4 Work out 40 as a percentage of 10
Circle your answer.

[1 mark]

4%

25%

300%

400%

$$40/10 \times 100 = 400$$

40/10 works out 40 as a fraction of 10. Multiplying a fraction by 100 converts it into a percentage

Turn over for the next question



- 5 Match each sequence to its description.
One has been done for you.

[4 marks]

1 1 2 3 5 8	Arithmetic progression
1 2 4 8 16 32	Geometric progression
1 2 3 4 5 6	Fibonacci sequence
1 3 6 10 15 21	Triangular numbers
1 4 9 16 25 36	Cube numbers
1 8 27 64 125 216	Square numbers

1, 2, 4, 8, 16, 32 is a geometric progression as it is multiplying by the same amount (2) between each term. 1, 2, 3, 4, 5, 6 is an arithmetic progression as the same amount (1) is being added between each term. 1, 3, 6, 10, 15, 21 are triangular numbers as it starts with 1, then adds 2, then adds 3, then adds 4, then adds 5, then adds 6. 1, 4, 9, 16, 25, 36 are square numbers as they are $1^2, 2^2, 3^2, 4^2, 5^2, 6^2$. 1, 8, 27, 64, 125, 216 are cube numbers as they are $1^3, 2^3, 3^3, 4^3, 5^3, 6^3$.



6 The table shows information about the population of a city.

Population in 2001	Population in 2011
420 000	480 000

Liam claims,

“From 2011 to 2021 the population of the city will increase by the same percentage as from 2001 to 2011”

He works out,

$$\begin{aligned} \text{population increase from 2001 to 2011} &= 480\,000 - 420\,000 \\ &= 60\,000 \end{aligned}$$

$$\begin{aligned} \text{population in 2021} &= 480\,000 + 60\,000 \\ &= 540\,000 \end{aligned}$$

Does the population of 540 000 match his claim?

You **must** show your working.

[3 marks]

$$\frac{60000}{420000} \times 100 = 14.2... \%$$

60000/420000 expresses the increase as a fraction of the population in 2001. Multiplying this by 100 converts it into a percentage

$$\frac{60000}{480000} \times 100 = 12.5 \%$$

60000/480000 expresses the same increase as a fraction of the population in 2011. Multiplying this by 100 converts it into a percentage

Answer **No**

Liam increased by 60000 for both 2001 to 2011 and 2011 to 2021.
The percentage increase is different for both of these though

Turn over for the next question



- 7 On three days, Ali throws darts at a target.
Here are his results.

	Number of throws	Number of hits	Number of misses
Monday	20	15	5
Tuesday	30	22	8
Wednesday	40	17	23
Total	90	54	36

- 7 (a) Work out **two** different estimates for the probability of Ali hitting the target.

[2 marks]

Answer $\frac{54}{90}$ and $\frac{17}{40}$

54 out of the total number of throws were hits.
17 out of the 40 throws on Wednesday were hits

- 7 (b) Which of your two answers is the better estimate for the probability of Ali hitting the target?

Give a reason for your answer.

[1 mark]

Answer $\frac{54}{90}$

Reason It was based on more throws



- 8 Theo starts with savings of £18
James starts with no savings.

Each week from now,

Theo will save £4.50 and James will save £4

In how many weeks will Theo and James have savings in the ratio 15 : 8 ?

[3 marks]

45:24

Using table mode by pressing MENU then 3. $f(x) = 18 + 4.50x$. $g(x) = 4x$. Start: 1. End: 30. Step: 1

This lists out the amount of money each person has each week. The x column is the number of weeks. The f(x) column is the amount of money Theo has. The g(x) column is the amount of money James has. Scrolling down until the amount Theo has to the amount James has simplifies to 15 : 8. 45 : 24 can be simplified to 15 : 8 by dividing both sides by 3

Answer _____

6



9 The length of each side of a regular pentagon is 8.4 cm to 1 decimal place.

9 (a) Complete the error interval for the length of one side.

[2 marks]

$$8.4 \pm \frac{0.1}{2}$$

$$\underline{8.35} \text{ cm} \leq \text{length} < \underline{8.45} \text{ cm}$$

Adding and subtracting half of the resolution works out the upper and lower bound. The resolution is 0.1 as this is the place value of the first decimal place

9 (b) Complete the error interval for the perimeter.

[1 mark]

$$\underline{8.35} \times 5$$

$$\underline{8.45} \times 5$$

Pentagons have 5 sides. The perimeter is found by multiplying the side length by 5

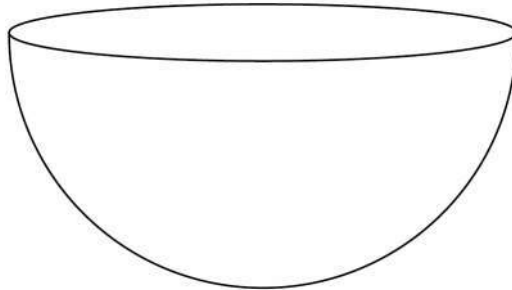
$$\underline{41.75} \text{ cm} \leq \text{perimeter} < \underline{42.25} \text{ cm}$$



10

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

A container is a hemisphere of radius 30 cm



Sand fills the container at a rate of 4000 cm^3 per minute.

Does it take **less than** a quarter of an hour to fill the container?

You **must** show your working.

[3 marks]

$s^d t$

This is basically a speed, distance, time problem. The speed is the rate the sand fills the container and the distance is the volume of the container

$$\frac{\left(\frac{4}{3}\pi \times 30^3\right)}{2} \div 4000 = 14.1$$

From the formula triangle, time = distance/speed. The volume of the container is found by substituting in the radius into the volume of a sphere formula then dividing by 2 as it is half of a sphere. The speed is 4000 cm^3 per minute. As the speed is in terms of minutes, the time calculated is in minutes

Quarter of an hour is 15 minutes
as $60/4 = 15$. 14.1 is less than this

Answer _____

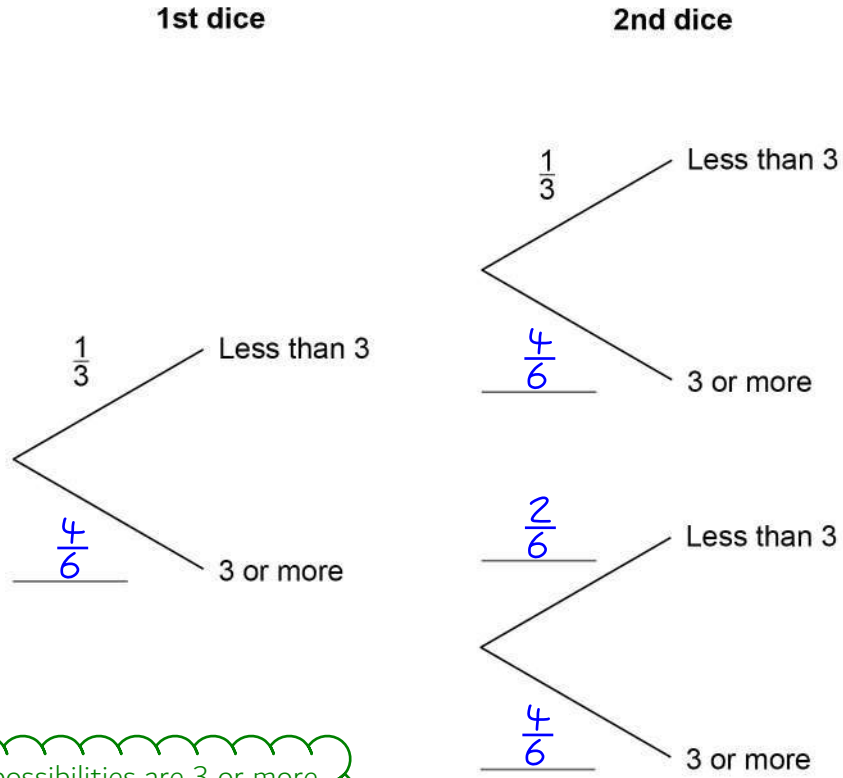
Yes



11 Two ordinary fair dice are rolled.

11 (a) Complete the tree diagram.

[1 mark]



4 out of the 6 possibilities are 3 or more.
2 out of the 6 possibilities are less than 3

11 (b) Work out the probability that **both** dice land on a number less than 3

[1 mark]

$\frac{1}{3} \times \frac{1}{3}$

Less than 3 AND less than 3. AND means to multiply the probabilities

Answer $\frac{1}{9}$



- 11 (c) Work out the probability that **exactly one** of the dice lands on a number less than 3

[2 marks]

$$\frac{1}{3} \times \frac{4}{6} + \frac{4}{6} \times \frac{2}{6}$$

Less than 3 AND 3 or more OR 3 or more AND less than 3. AND means to multiply the probabilities. OR means to add the probabilities

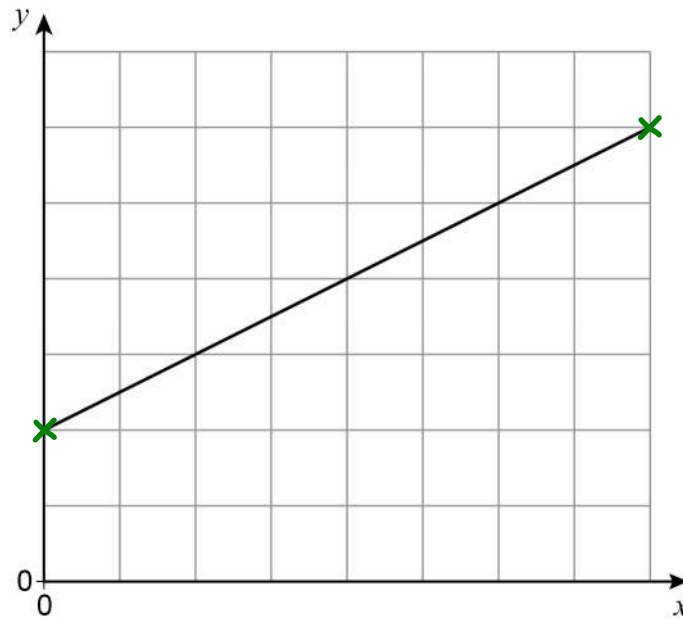
Answer $\frac{4}{9}$

Turn over for the next question

Turn over ►



- 12 A straight line is drawn on the centimetre grid.



Fay assumes that the scale is 1 cm represents 1 unit.

- 12 (a) Use her assumption to work out the gradient of the line.

[1 mark]

Gradient = (change in y)/(change in x). y changed by 4 between the two crosses. x changed by 8 between the two crosses

Answer $\frac{4}{8}$



12 (b) In fact, the scale is 1 cm represents 2 units.

Which statement is correct?

Tick **one** box.

[1 mark]

The answer to part (a) is too big

The answer to part (a) stays the same

The answer to part (a) is too small

As the change in y doubles and the change in x doubles,
the gradient will be an equivalent fraction. $8/16 = 4/8$

Turn over for the next question



13

Show that, for $x \neq -1$

$$\frac{8x^2 - 8}{4x + 4}$$
 simplifies to the form $ax + b$ where a and b are integers.
[3 marks]

$$8(x^2 - 1)$$

Bringing 8 out as a factor on the numerator

$$\frac{8(x+1)(x-1)}{4(x+1)}$$

Factorising $x^2 - 1$ using difference of two squares.
Bringing 4 out as a factor on the denominator

$$2(x-1)$$

$8/4 = 2$. Cancelling out the $(x + 1)$ as it is a
common factor to the numerator and denominator

$$2x - 2$$

Expanding the bracket to get into the desired form



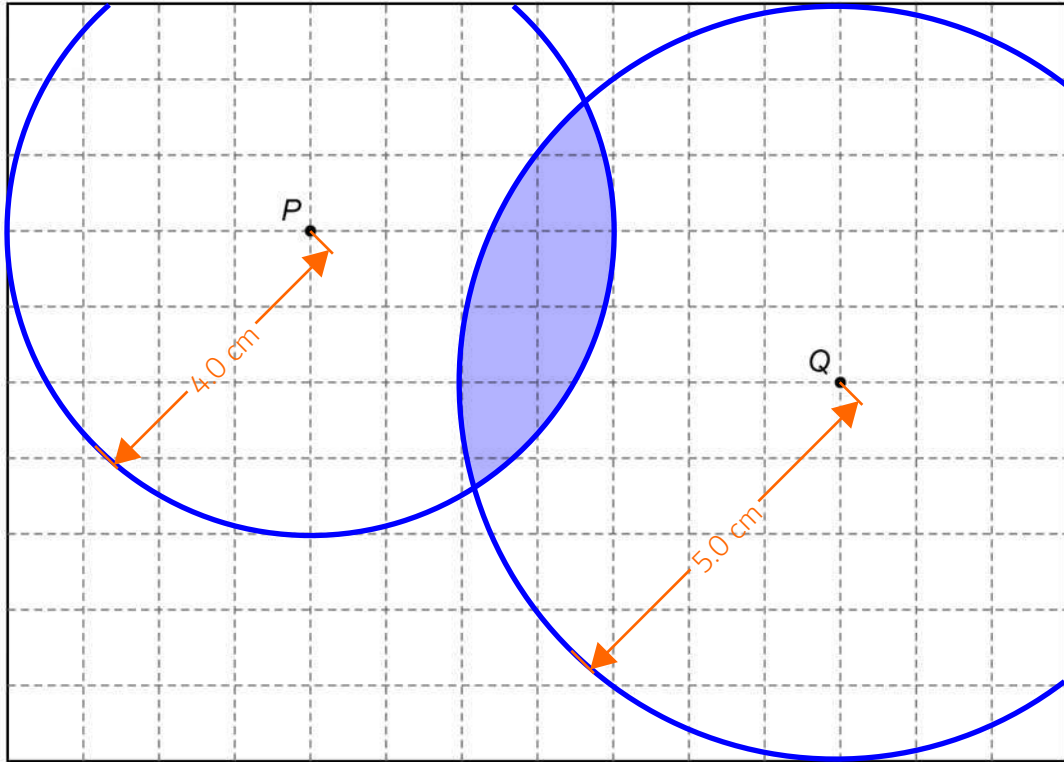
14

The scale drawing represents a garden.

Water from a sprinkler at P reaches up to 20 metres from P .

Water from a sprinkler at Q reaches up to 25 metres from Q .

Scale: 1 cm represents 5 m



Using a pair of compasses,

show the region that water from **both** sprinklers reaches.

[2 marks]

Turn over for the next question

$20/5 = 4$ so the water from P reaches 4cm on the scale drawing. $25/5 = 5$ so the water from Q reaches 5cm on the scale drawing. Drawing circles around both points using these radii shows the regions which the water reaches. The area within both circles is the area which can be reached by both



15 100 men and 100 women took a test.

Scores

	Median	Interquartile range	Range
Men	28	7.5	31
Women	30	9	37

Using this data, which statement **must** be true?

Tick **one** box.

[1 mark]

Men had a higher average score than women

Men had more consistent scores than women

A woman had the highest score

A man had the lowest score

Men had a lower interquartile range (which is a measure of consistency) so the second statement must be true



16 Some concrete has volume 3.8 m^3

16 (a) The density of the concrete is 2400 kg/m^3

Work out the mass of the concrete.

[2 marks]

$d^m v$

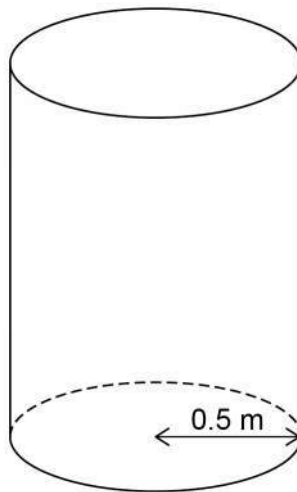
This is the formula triangle for density, mass and volume

$$2400 \times 3.8$$

From the formula triangle, mass = density \times volume

Answer 9120 kg

16 (b) The 3.8 m^3 of concrete is made into the shape of a cylinder.
The base has radius 0.5 metres.



Work out the height of the cylinder.

[2 marks]

$$\pi \times 0.5^2 \times h = 3.8$$

A cylinder is similar to a prism so its volume = cross sectional area \times length. The cross section is a circle and its area is found using $\pi \times \text{radius}^2$. The length is the height

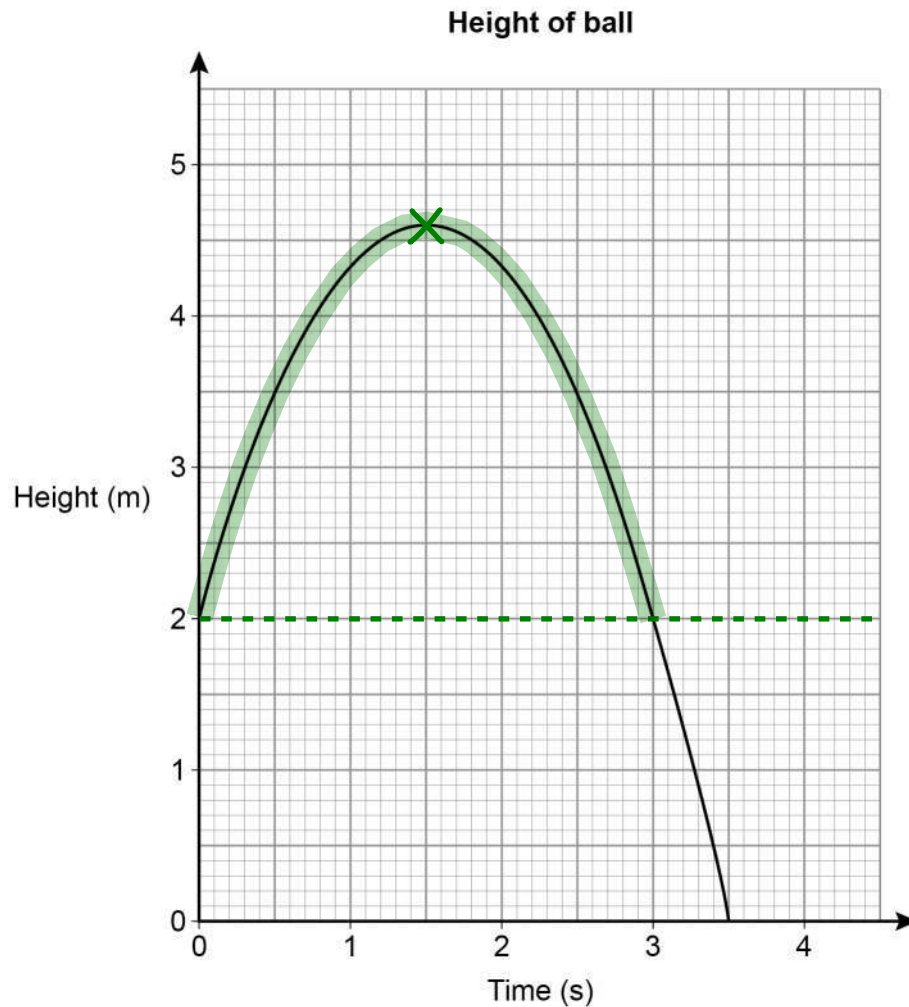
$$h = \frac{3.8}{\pi \times 0.5^2}$$

Rearranged to make the height the subject

Answer 4.8 m



- 17 A ball is thrown vertically upwards.
The graph shows the height of the ball above the ground after it is thrown.



- 17 (a) For how many seconds is the ball at a height of **more than 2 metres**?

The highlighted part of the line is when it had a height of more than 2 metres

[1 mark]

Answer 3 s

- 17 (b) After how many seconds is the ball at instantaneous rest when it is in the air?

The gradient was 0 at this point so it had no speed and therefore wasn't moving

[1 mark]

Answer 1.5 s



- 17 (c) Work out the average speed of the ball when it is moving downwards.

[2 marks]

$$\frac{4.6}{2}$$

The units of m/s means to divide the distance in metres by the time in seconds. The distance travelled while moving downward was 4.6m and it took 2 seconds

Answer 2.3 m/s

- 18 The solution of $3^x = 300$ lies between two consecutive integers. Work out the two integers.

[1 mark]

Using table mode by pressing MENU then 3. $f(x) = 3^x$. Ignore $g(x)$. Start: 1. End: 30. Step: 1

This lists out the powers of 3 from 3^1 to 3^{30} . $3^5 = 243$ and $3^6 = 729$ so as it is a continuous increasing function, x must be between 5 and 6

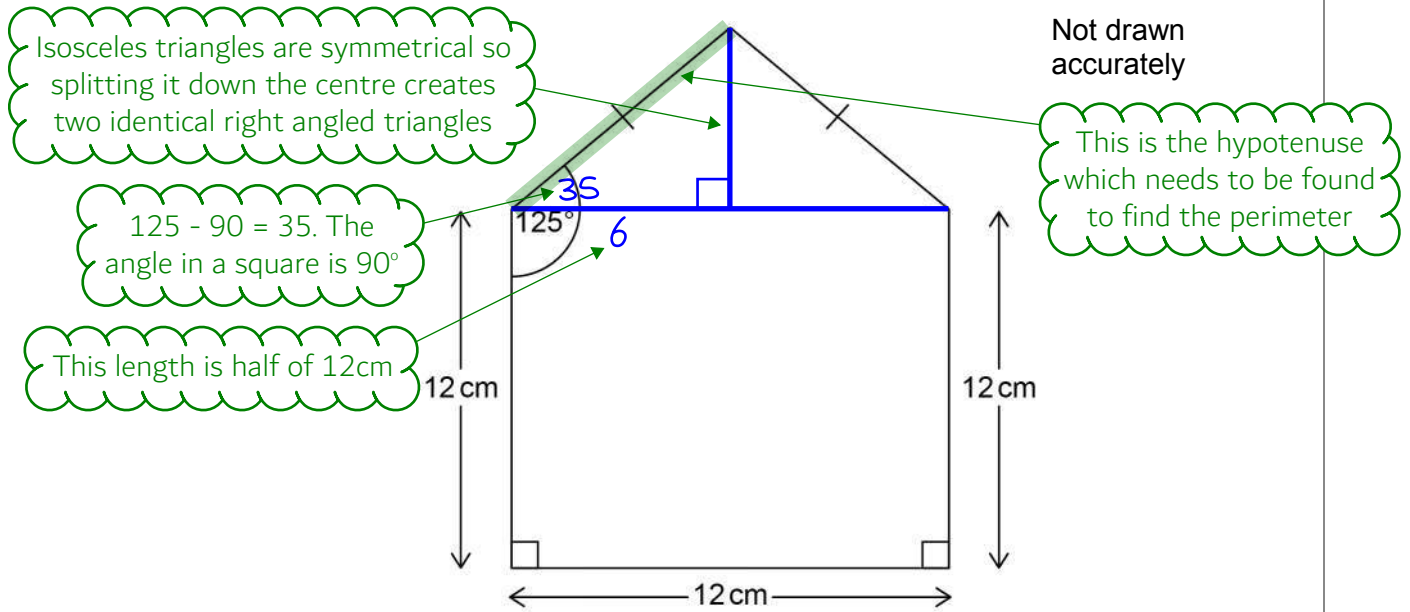
Answer 5 and 6

Turn over for the next question

Turn over ►



19 A pentagon is made from a square and an isosceles triangle.



Work out the perimeter of the pentagon.

[4 marks]

SOH CAH TOA

Writing out SOH CAH TOA as formula triangles. 6cm is the adjacent and we are looking for the hypotenuse so both of these are ticked. There are two ticks on CAH so this formula can be used

$12 \times 3 + 2 \left(\frac{6}{\cos 35} \right)$

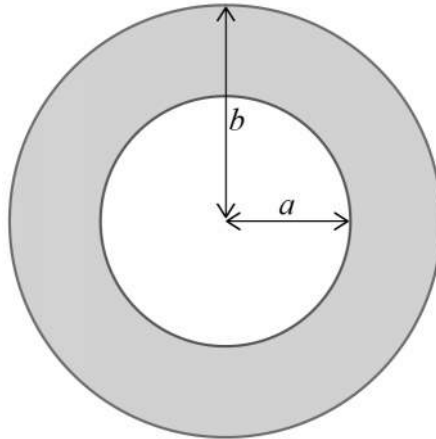
From the formula triangle, hypotenuse = adjacent / (cos of the angle). The adjacent is 6 and the angle is 35. Multiplying $6 / \cos 35$ by 2 as there are two of these lengths on the perimeter. Adding this to 3 lots of the 12cm lengths on the square works out the perimeter

Answer 50.6 cm



20

Here is an inflated swimming ring with dimensions in centimetres.



The volume of the ring, $V \text{ cm}^3$, is given by

$$V = 0.25\pi^2(b - a)^2(b + a)$$

Work out the volume when $a = 20$ and $b = 30$

Give your answer to 3 significant figures.

[3 marks]

$$0.25\pi^2(30-20)^2(30+20)$$

V is the subject so no rearranging is needed. Substituting 20 for a and 30 for b in the formula gives the volume

The answer of 12337.0055 is rounded to 3 significant figures

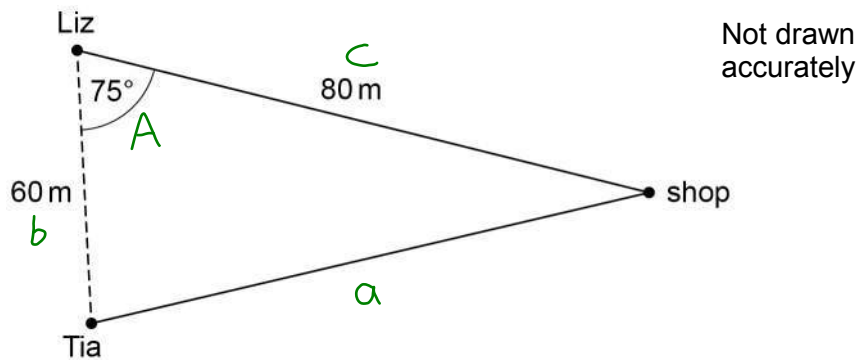
Answer 12300 cm^3

Turn over for the next question

Turn over ►



- 21 Liz and Tia are walking towards a shop along different straight paths.
The diagram shows their positions at 2 pm



- 21 (a) Assume they walk at the same speed.

Who will arrive at the shop first?

You **must** show your working.

[3 marks]

$$a^2 = b^2 + c^2 - 2bc \cos A$$

The distance between Tia and the shop needs to be found. There isn't a right angle in the triangle so right angled trigonometry can't be used. There aren't opposite pairs of sides and angles so the sine rule can't be used. Therefore the cosine rule must be used

$$a = \sqrt{60^2 + 80^2 - 2 \times 60 \times 80 \times \cos 75}$$

$$= 86.7$$

Rearranging to make a the subject by square rooting both sides and substituting in the values works out the distance between Tia and the shop

Answer _____ Liz

Liz is closer so as they both go at the same speed she must get there first

- 21 (b) In fact, Liz walks at a faster speed than Tia.

How does this affect the answer to part (a)?

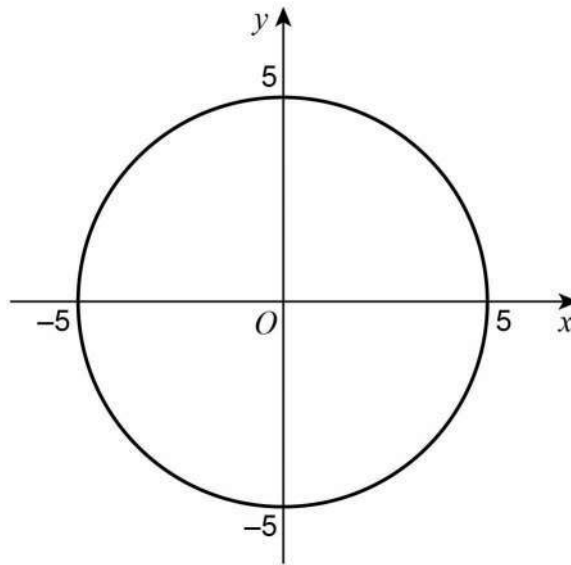
[1 mark]

It doesn't

Liz will still get there first as she will get there even sooner



- 22 A circle, centre O , passes through $(5, 0)$.



What is the equation of the circle?

Circle your answer.

[1 mark]

$x^2 + y^2 = 25$

$x^2 + y^2 = 5$

$x^2 + y^2 = 10$

$x^2 + y^2 = 100$

Turn over for the next question

The general equation of a circle with its centre at the origin is $x^2 + y^2 = r^2$, where r is the radius. The radius is 5 so r^2 must be 25. Alternatively, substituting in the coordinates of any of the points where the circle crosses the axis finds that only the first equation is satisfied

Turn over ►



23

Solids X and Y are similar.

X has volume 64 cm^3 Y has volume 343 cm^3 The surface area of X is 176 cm^2

Work out the surface area of Y.

[3 marks]

$$176 \times \left(\sqrt[3]{\frac{343}{64}} \right)^2$$

343/64 expresses the volume scale factor. Cube rooting this expresses the length scale factor. Squaring this expresses the area scale factor. Multiplying the surface area of X by this works out the surface area of Y

Answer 539 cm^2



24

A tank is a cuboid measuring 50 cm by 35 cm by 20 cm

All lengths are to the **nearest centimetre**.

A container has a capacity of **exactly** 34 litres.

1 litre = 1000 cm³

Which has the greater capacity?

Tick **one** box.

Tank

Container

Cannot tell

Show working to support your answer.

[4 marks]

$$\frac{(50 + \frac{1}{2})(35 + \frac{1}{2})(20 + \frac{1}{2})}{1000} = 36.8$$

The upper bound of the volume of the tank in litres. Adding half of the resolution to each measurement works out the upper bound of each of them. Volume of cuboid = length x width x height. Dividing the volume in cubic centimetres by 1000 converts it into litres

$$\frac{(50 - \frac{1}{2})(35 - \frac{1}{2})(20 - \frac{1}{2})}{1000} = 33.3$$

The lower bound of the volume of the tank in litres. Subtracting half of the resolution from each measurement works out the lower bound of each of them. Volume of cuboid = length x width x height. Dividing the volume in cubic centimetres by 1000 converts it into litres

Turn over for the next question

The upper bound is greater than 34 litres and the lower bound is less than 34 litres. Therefore it is impossible to tell which has the greater capacity



25

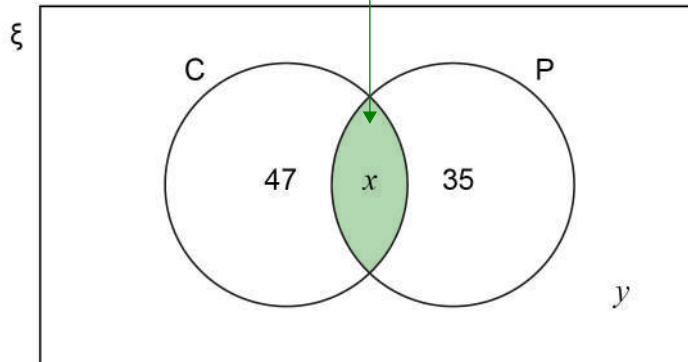
The Venn diagram shows some information about 150 students.

$\xi = 150$ students

C = students who study Chemistry

P = students who study Physics

The intersection of C and P is the number of students who studies Physics and also Chemistry



The probability that a Physics student, chosen at random, also studies Chemistry is $\frac{5}{12}$

One of the 150 students is chosen at random.

Work out the probability that the student does **not** study either Chemistry or Physics.

[4 marks]

$$\frac{x}{x+35} = \frac{5}{12}$$

Expressing the fraction of the Physics students who also study Chemistry. This is the probability that a Physics student, chosen at random, also studies Chemistry. So the expression must equal to $\frac{5}{12}$

$$12x = 5x + 175$$

Multiplying both sides of the equation by the denominators eliminates them

$$7x = 175$$

Subtracting $5x$ from both sides

$$x = 25$$

Dividing both sides by 7 finds x

$$\frac{150 - 47 - 25 - 35}{150}$$

Answer

$$\frac{43}{150}$$

Subtracting the numbers of students who study Physics, Chemistry or both from the 150 students finds y . Expressing this as a fraction of the 150 students finds the probability that the student does not study either Chemistry or Physics



26

A curve has equation $y = 4x^2 + 5x + 3$ A line has equation $y = x + 2$ Show that the curve and the line have **exactly** one point of intersection.Do **not** use a graphical method.**[4 marks]**

$$0 = 4x^2 + 4x + 1$$

Subtracting the equations from each other eliminates the y terms and leaves an equation just in terms of x

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 4 \times 1}}{2 \times 4}$$

Solving using the quadratic formula

$$= -\frac{1}{2}$$

There is only one solution of x so as one of the equations is linear, there will also only be one solution of y. Therefore there is exactly one point of intersection

Turn over for the next question



27

Prove algebraically that $2.7\dot{5}$ converts to the fraction $\frac{124}{45}$

[3 marks]

$$x = 2.7\dot{5}$$

Setting x equal to the recurring decimal

$$10x = 27.5\dot{5}$$

As there is 1 recurring digit, multiplying by 10 once lines up the recurring digit in the same decimal place

$$9x = 24.8$$

Subtracting x from $10x$ eliminates the recurring digit

$$x = \frac{24.8}{9} = \frac{124}{45}$$

Rearranging to express x as a fraction by dividing both sides by 9. Entering the fraction into the calculator simplifies it to the desired fraction



28 $f(x) = 5 - x$ and $g(x) = 3x + 7$

28 (a) Simplify $f(2x) + g(x - 1)$

[3 marks]

$$5 - 2x + 3(x - 1) + 7$$

$f(2x)$. $2x$ is substituted for x in $f(x)$

$g(x - 1)$. $(x - 1)$ is substituted for x in $g(x)$

Expanding the bracket and collecting like terms

Answer

$$x + 9$$

28 (b) Solve $g^{-1}(x) = 2x$

[3 marks]

$$x = 3y + 7$$

Replacing $g(x)$ with x and x with y . Inverse functions are basically when the x and y swap

$$\frac{x-7}{3} = 2x$$

Made y the subject by subtracting 7 from both sides then dividing by 3. This is $g^{-1}(x)$. Setting this equal to $2x$

$$x - 7 = 6x$$

Multiplying both sides by 3 to eliminate the denominator

$$-7 = 5x$$

Subtracting x from both sides to get the x terms on their own

Dividing both sides by 5 finds x

$$x = \frac{-7}{5}$$

END OF QUESTIONS

