

Please write clearly in block capitals.

Centre number       Candidate number

Surname \_\_\_\_\_

Forename(s) \_\_\_\_\_

Candidate signature \_\_\_\_\_

**GCSE  
MATHEMATICS**

**H**

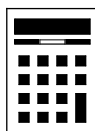
Higher Tier Paper 3 Calculator

Monday 11 November 2019 Afternoon Time allowed: 1 hour 30 minutes

**Materials**

For this paper you must have:

- a calculator
- mathematical instruments.



**Instructions**

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

**Information**

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
<b>TOTAL</b>	

**Advice**

In all calculations, show clearly how you work out your answer.



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to [curtis@cgmaths.co.uk](mailto:curtis@cgmaths.co.uk)

Answer **all** questions in the spaces provided

- 1 Circle the relative frequency that represents 13 successes out of 50 trials.

[1 mark]

0.13

26

13 : 50

0.26

13/50 is the relative frequency as a fraction. Converting this to a decimal using the calculator

- 2 The equation of a straight line is  $2y = 3x + 5$

Circle the gradient of the line.

[1 mark]

 $\frac{2}{3}$  $\frac{3}{2}$ 

3

5

The general equation of a straight line is  $y = mx + c$ , where  $m$  is the gradient.  
Dividing both sides of  $2y = 3x + 5$  by 2 gives  $y = \frac{3}{2}x + \frac{5}{2}$ . So the gradient is  $\frac{3}{2}$

- 3  $(2x - 4)(3x + 5)$  is expanded and simplified.

Circle the term which is part of the answer.

[1 mark]

 $2x$  $-2x$  $22x$  $-22x$ 

$6x^2 + 10x - 12x - 20$  ← Expanding the brackets

Simplifying by collecting like terms will give  $-2x$  as one of the terms as  $10x - 12x = -2x$



4 When rounded to 3 significant figures,  $x = 6.37$

Circle the correct error interval.

[1 mark]

$$6.365 \leq x < 6.375$$

$$6.36 \leq x < 6.38$$

$$6.369 \leq x < 6.379$$

$$6.365 \leq x < 6.3749$$

Adding and subtracting half of the resolution works out the upper and lower bound. The resolution is 0.01 as this is the place value of the 3rd significant figure.  $6.37 + 0.01/2 = 6.375$  and  $6.37 - 0.01/2 = 6.365$

5 Solve the simultaneous equations

$$7x + 2y = 36 \quad \leftarrow \text{First equation}$$

$$3x + 2y = 16 \quad \leftarrow \text{Second equation}$$

[3 marks]

$$4x = 20$$

Subtracting the second equation from the first equation cancels out the y terms leaving an equation just in terms of x.  $7x - 3x = 4x$ .  $2y - 2y = 0$ .  $36 - 16 = 20$

$$x = 5$$

Dividing both sides by 4 eliminates the 4 on the left and gets x on its own

$$35 + 2y = 36$$

Substituting 5 for x in the first equation.  $7 \times 5 = 35$

$$2y = 1$$

Subtracting 35 from both sides eliminates the 35 on the left and gets the y term on its own

$$x = \underline{\quad 5 \quad} \quad y = \underline{\quad 0.5 \quad}$$

Dividing both sides by 2 eliminates the 2 on the left and gets y on its own



6 (a) Tom is tiling a wall.

He needs to buy at least 100 tiles.

The tiles are sold in large packs and small packs.

Large pack 40 tiles £18

Small pack 28 tiles £14

*Special offer*

25% reduction when you buy 3 or more **large** packs

Work out the cheapest cost for Tom to buy the packs of tiles he needs.

[3 marks]

$$18 \times \frac{100-25}{100}$$

Reducing the price of a large pack by 25% works out that each large pack would cost £13.50 using the special offer. This is cheaper than the small pack so the cheapest cost must be by using the special offer. Subtracting the 25% from 100% expresses the percentage it decreases to. Putting this over 100 converts it into a fraction, which when the £18 is multiplied by it is reduced by 25%

$$13.50 \times 3$$

Buying 3 large packs gets 120 tiles as  $40 \times 3 = 120$ . This is at least 100 tiles. Multiplying the cost of each large pack in the special offer by 3 works out the cost of the tiles

Answer £ 40.50



6 (b) Tom is also tiling a floor.

The floor is a rectangle with length 600 cm and width 240 cm

Each tile is a square with side 40 cm

Tom uses this method to work out the number of tiles he needs.

$$\begin{aligned} \text{Number of tiles that will fit along the length} &= 600 \div 40 \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{Number of tiles that will fit along the width} &= 240 \div 40 \\ &= 6 \end{aligned}$$

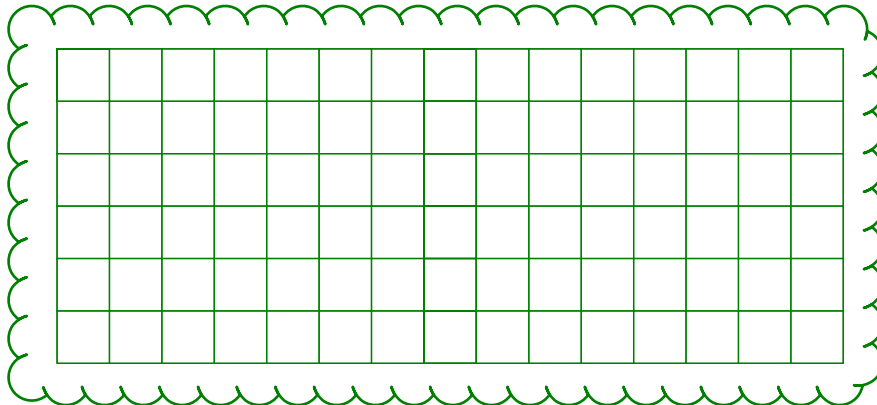
$$\begin{aligned} \text{Total number of tiles needed} &= 15 + 6 \\ &= 21 \end{aligned}$$

Give a reason why Tom's method is wrong.

[1 mark]

Should be  $15 \times 6$

The first two parts are correct. 15 tiles fit along the length and 6 tiles fit along the width. However adding them does not work out the total number of tiles needed. There are 6 rows of 15 so they should be multiplied



Turn over for the next question



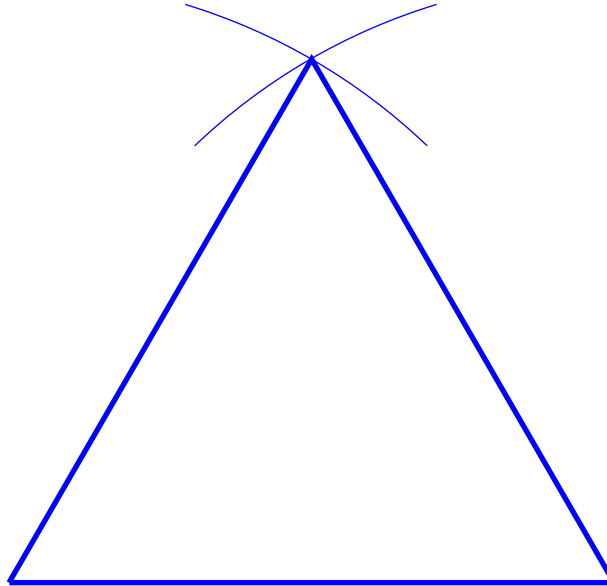
7 An equilateral triangle has side length 16 metres.

Using ruler and compasses only, construct a scale drawing of the triangle.

Use the scale 1 centimetre represents 2 metres.

**[3 marks]**

**Scale:** 1 cm represents 2 m



Every 2 metres is represented by 1 centimetre. So dividing the 16 metres by 2 works out that each side should be represented by 8 centimetres. First drawing a horizontal line which is 8 cm long using a ruler. Then scribing an arc with radius 8 cm from each end of the line. Drawing two lines from the ends of the first line to where the two arcs meet ensures that both of the other sides are exactly 8 cm



8 In a choir there are 35 men and 48 women.

The probability that a man chosen at random wears glasses is  $\frac{2}{5}$

The probability that a woman chosen at random wears glasses is  $\frac{3}{8}$

8 (a) Work out the number of people in the choir who wear glasses.

[3 marks]

$$\frac{2}{5} \times 35 = 14$$

This works out that  $\frac{2}{5}$  of the men is 14. 'of' means to multiply

$$\frac{3}{8} \times 48 = 18$$

This works out that  $\frac{3}{8}$  of the women is 18. 'of' means to multiply

$$14 + 18$$

Adding the 14 men wearing glasses and the 18 women wearing glasses works out that there are 32 people wearing glasses

Answer 32

8 (b) A person is chosen at random from the choir.

Work out the probability that the person does **not** wear glasses.

[2 marks]

$$35 + 48$$

Adding the 35 men and 48 women works out that there are 83 people in total

$$83 - 32$$

Subtracting the 32 who wear glasses from the 83 people works out that there are 51 who do not wear glasses

Answer  $\frac{51}{83}$

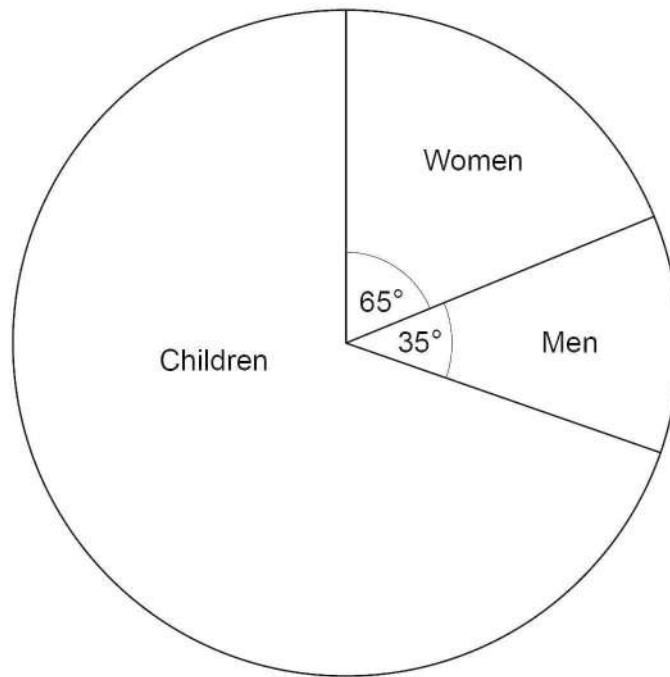
51 out of the 83 people do not wear glasses





9

The pie chart shows information about people at a theme park.



There were 450 **more** women than men.

Work out the number of children.

[3 marks]

$$360 - 65 - 35 = 260$$

There are  $360^\circ$  in total in a pie chart. Subtracting the angles for women and men works out that the angle for children is  $260^\circ$

$$65 - 35$$

Working out that there are  $30^\circ$  more for women than men. This represents the 450 people

$$450 \div 30$$

Dividing the 450 people by the  $30^\circ$  representing it works out that  $1^\circ$  represents 15 people

$$15 \times 260$$

Multiplying what  $1^\circ$  represents by the  $260^\circ$  for children works out that there were 3900 children

Answer 3900



10 Density =  $\frac{\text{mass}}{\text{volume}}$

The mass is divided by 2 and the volume is multiplied by 4

What happens to the density?

Circle your answer.

[1 mark]

$\times 2$

$\div 2$

$\times 8$

$\div 8$

Dividing the mass by 2 divides the density by 2. As it is the denominator, multiplying the volume by 4 divides the density by 4. Dividing by 2 then dividing by 4 is dividing by 8

11 Work out

cube root of 512 : reciprocal of 0.4

Give your answer in the form  $n : 1$

[3 marks]

$8:2.5$  ←  $\sqrt[3]{512} = 8$  and  $1/0.4 = 2.5$ . Reciprocal means 1 divided by

$8 \div 2.5$  ← The 2.5 has been divided by 2.5 to get 1. So the 8 must also be divided by 2.5

Answer 3.2 : 1

Turn over for the next question

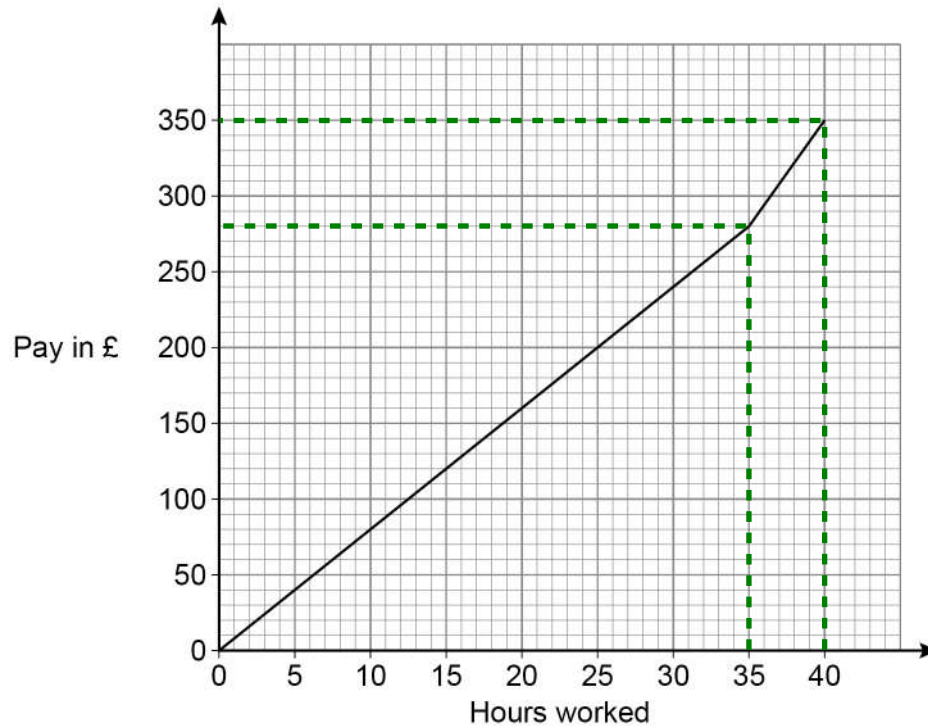


12 The graph shows how much Molly is paid for working for up to 40 hours.

She receives

a basic rate of pay for the first 35 hours worked

a higher rate of pay for the next 5 hours worked.



Work out the difference between the higher rate of pay and the basic rate of pay.

Give your answer in £ per hour.

[3 marks]

$$280 \div 35 = 8$$

£280 was earned in the first 35 hours. £ per hour means to divide the pay in £ by the hours worked. So the basic rate of pay is £8 per hour

$$350 - 280$$

This works out that £70 was earned in the next 5 hours

$$70 \div 5$$

£ per hour means to divide the pay in £ per hour by the hours worked. So the higher rate of pay is £14 per hour

Answer £ 6 per hour

$$14 - 8$$

Difference = largest - smallest



13 Naga states a hypothesis.

“Most people read more than 100 books a year.”

She asks a sample of five people in a book club how many books they read last month. The table shows the results.

	Lynn	Ali	Paul	Chen	Ruth
Number of books	10	11	8	10	13

13 (a) Show how Naga could use the data to support her hypothesis.

[2 marks]

$$100 \div 12 = 8.\dot{3}$$

There are 12 months in a year so dividing the 100 books by 12 months works out how many books they would need to read per month

4 out of the 5 would read more than 100 books a year

Assuming that they read the same number each month, Lynn, Ali, Chen and Ruth would read more than 100 books a year as they are reading more than 8.3 a month

13 (b) Give two reasons why this sample should **not** be used to support her hypothesis.

[2 marks]

Reason 1 The sample is too small

Only 5 people were sampled and it was only done during one month. Using a larger sample of people and over a longer time would give more reliable results

Reason 2 The sample is biased

The people asked go to a book club so may be more likely to read more books



14 A graph has equation  $y = x^3 + a$  where  $a$  is an integer.

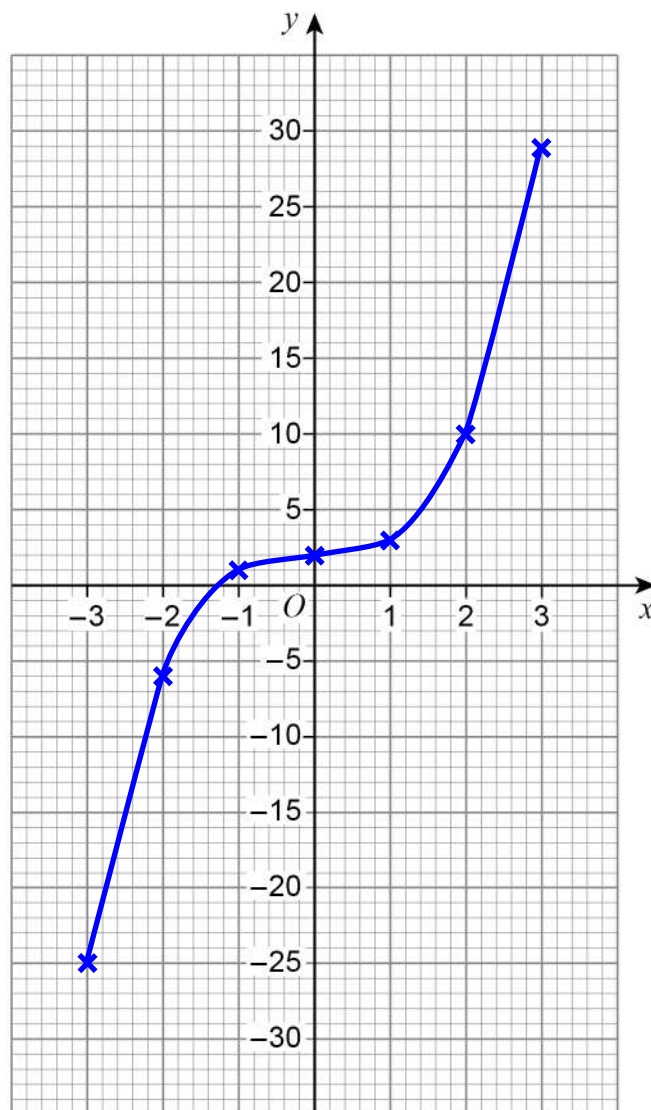
The graph passes through the point (3, 29)

Draw the graph for values of  $x$  from  $-3$  to  $3$

[3 marks]

$$a = 29 - 3^3 = 2$$

Rearranging to find  $a$  by subtracting  $x^3$  from both sides to give  $a = y - x^3$ .  
Then substituting in 3 for  $x$  and 29 for  $y$  as these are the  $x$ -coordinate and  $y$ -coordinate of the point (3, 29) which the graph goes through. So  $a$  is 2



Using table mode on the calculator. Setting  $f(x) = x^3 + 2$ . Start:  $-3$ .  
End:  $3$ . Step:  $1$ . This gives a table of values for the equation. The points can be plotted then they can be joined up with a curve



15

When you earn money you pay income tax.

The amount you pay depends on how much you earn that year.

You pay

0% on the first £12 500 you earn

20% on the next £37 500 you earn

40% on the next £112 500 you earn.

0% bracket

20% bracket

40% bracket

One year, Kim paid £9260 income tax.

Work out how much she earned that year.

[4 marks]

$$\frac{20}{100} \times 37500$$

This works out that 20% of the £37500 is £7500. Putting 20% over 100 converts it into a fraction, which when multiplied by the £37500 finds 20% of it

$$9260 - 7500$$

More than £7500 tax has been paid so some of the tax must be in the 40% bracket. Subtracting the £7500 from the £9260 works out that £1760 tax was paid in the 40% bracket

$$\frac{1760}{40} \times 100$$

The £1760 tax is 40% of the earning in the 40% bracket. So dividing it by 40 finds 1% then multiplying it finds 100%, which is the amount earned in the 40% bracket

$$4400 + 37500 + 12500$$

£4400 was earned in the 40% bracket. All of the £37500 must have been earned in the 20% bracket. All of the £12500 must have been earned in the 0% bracket. Adding these together works out the total amount earned

Answer £ 54400



- 16** A building company employs  
2 labourers  
14 joiners  
9 electricians  
8 plumbers.

For a job, the company needs one of each type of worker.

- 16 (a)** In how many ways can the company choose the four workers?

[2 marks]

$$2 \times 14 \times 9 \times 8$$

Using the product rule for counting

Answer \_\_\_\_\_ 2016 \_\_\_\_\_

- 16 (b)** One labourer and two plumbers are on holiday.

In how many ways can the company now choose the four workers?

[2 marks]

$$1 \times 14 \times 9 \times 6$$

Using the product rule for counting. This time there is 1 fewer labourer so there is only 1 of them and there is 2 fewer plumbers so there are 6 of them

Answer \_\_\_\_\_ 756 \_\_\_\_\_



17  $f(x) = 3x^2 - 4x + 8$  for all values of  $x$

Jenny says,

“ $f(10)$  must equal  $2 \times f(5)$ , because 10 is  $2 \times 5$ ”

Is Jenny correct?

Show working to support your answer.

[2 marks]

$$f(10) = 3 \times 10^2 - 4 \times 10 + 8 = 268$$

Substituting 10 for  $x$  in  $f(x)$  works out that  $f(10) = 268$

$$f(5) = 3 \times 5^2 - 4 \times 5 + 8 = 63$$

Substituting 5 for  $x$  in  $f(x)$  works out that  $f(5) = 63$

$$2 \times 63 \neq 268$$

$2 \times 63 = 126$ , which does not equal to 268

No

$2 \times f(5)$  does not equal to  $f(10)$

18 Work out the **two** roots of  $(7x + 1)(2x - 3) = 0$

Circle **both** roots.

[1 mark]

$$\left(-\frac{1}{7}\right)$$

$$\frac{1}{7}$$

$$-\frac{3}{2}$$

$$\left(\frac{3}{2}\right)$$

Two brackets are multiplied to give 0, so one of the two brackets must equal to 0. If  $7x + 1 = 0$ ,  $7x = -1$ ,  $x = -1/7$ . If  $2x - 3 = 0$ ,  $2x = 3$ .  $x = 3/2$

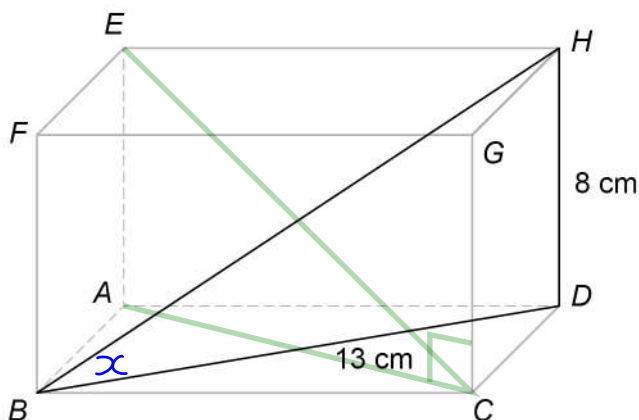




19 Here is a cuboid.

$DH = 8 \text{ cm}$

$DB = 13 \text{ cm}$



19 (a) Work out the size of angle  $DBH$ .

[2 marks]

SOHCAHTOA

Using right-angled trigonometry so writing SOH CAH TOA as formula triangles. Ticking O as the 8 cm is the opposite and ticking A as the 13 cm is the adjacent. There are two ticks on the TOA formula triangle so this one can be used

$\tan x = \frac{8}{13}$

From the formula triangle, tan of the angle = opposite/adjacent

$x = \tan^{-1}\left(\frac{8}{13}\right)$

Doing the inverse tan of both sides gets x on its own

Answer 31.6 degrees

19 (b) Using your answer to part (a), work out the size of angle  $ECG$ .

[1 mark]

$90 - 31.6$

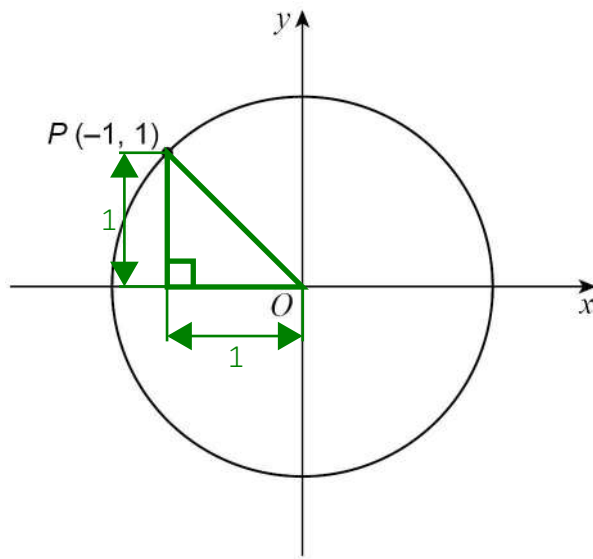
Angle ECA is the same as angle DBH and angle ACG is a right-angle. So subtracting the 31.6° from 90° finds angle ECG

Answer 58.4 degrees



20

$P(-1, 1)$  is a point on the circle, centre  $O$ , radius  $r$ .



Not drawn  
accurately

Work out the value of  $r$ .

Circle your answer.

[1 mark]

1

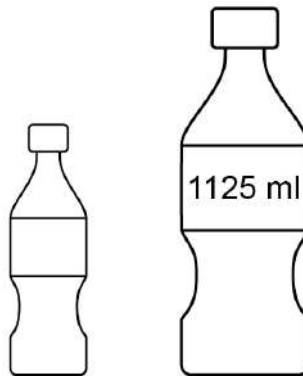
2

 $\sqrt{2}$  $2\sqrt{2}$ 

A right-angled triangle can be formed with the radius. Both of the shorter sides are 1 and the longer side is the radius, which can be found using Pythagoras' Theorem.  $a^2 + b^2 = c^2$ , where  $c$  is the longest side. So  $c = \sqrt{1^2 + 1^2} = \sqrt{2}$



- 21 Juice is sold in small bottles and large bottles.  
The volume of the large bottle is 1125 ml.



volume of small bottle : volume of large bottle = 2 : 5

A café has small glasses and large glasses.

volume of small glass : volume of large glass = 4 : 7

A small bottle fills 6 small glasses with no juice left over.

How many large glasses can be filled by a large bottle?

You **must** show your working.

[4 marks]

$$\frac{1125}{5} \times 2$$

5 parts of the ratio of the bottles represents the volume of the large bottle. So dividing the 1125 ml by 5 works out the value of 1 part of the ratio. Multiplying this by 2 works out that the value of the 2 parts which represent the volume of the small bottle is worth 450 ml

$$450 \div 6$$

Dividing the volume of the small bottle by the 6 small glasses works out that the volume of a small glass is 75 ml

$$\frac{75}{4} \times 7$$

4 parts of the ratio of the glasses represent the volume of the small glass. So dividing the 75 ml by 4 works out the value of 1 part of the ratio. Multiplying this by 7 works out that the value of the 7 parts which represent the volume of the large glass is worth 131.25 ml

$$1125 \div 131.25$$

Dividing the volume of the large bottle by the volume of the large glass works out how many large glasses can be filled by a large bottle

8.57... can be rounded down to 8 as there is not enough to fill 9

Answer 8



22 The **only** solution to  $x^2 + bx + c = 0$  is  $x = 5$

Work out the values of  $b$  and  $c$ .

[2 marks]

$$(x-5)(x-5)$$

This must be the factorised form of the left side of the equation in order for  $x = 5$  to be the only solution

$$x^2 - 5x - 5x + 25$$

Expanding the brackets

$$b = \underline{\quad -10 \quad} \quad c = \underline{\quad 25 \quad}$$

$-5x - 5x = -10x$  so  $b$  is  $-10$ .  $c$  is the constant not involving  $x$  so must be  $25$

23  $x : y = \frac{1}{4} : \frac{2}{3}$

What is  $x$  as a fraction of  $y$ ?

Circle your answer.

[1 mark]

$$\frac{8}{3}$$

$$\frac{1}{6}$$

$$\frac{3}{7}$$

$$\frac{3}{8}$$

$$\frac{1}{4} \div \frac{2}{3}$$

$x$  could be  $1/4$  when  $y$  is  $2/3$ . Expressing  $x$  as a fraction of  $y$

$$\frac{1}{4} \times \frac{3}{2}$$

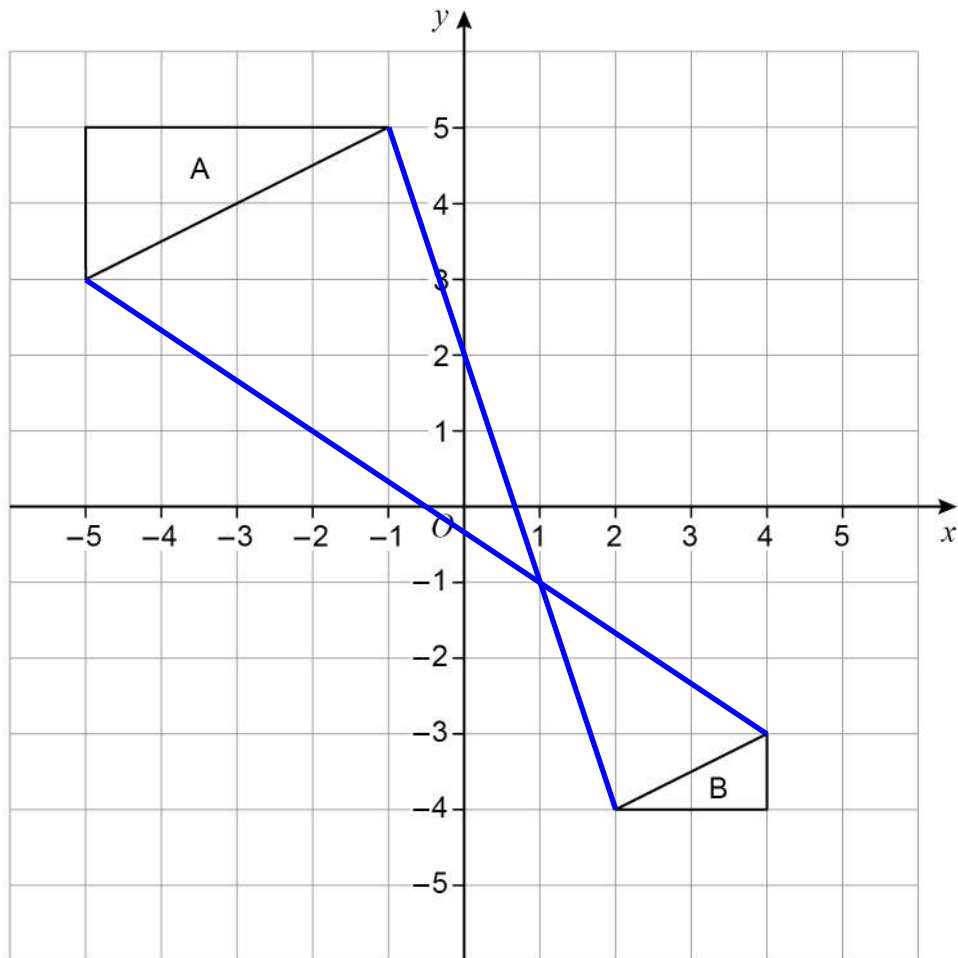
To divide by a fraction: keep the first part, change the division to a multiplication, flip the second fraction

To multiply fractions: the numerators can be multiplied and the denominators can be multiplied.  $1 \times 3 = 3$  and  $4 \times 2 = 8$



24

Shape A and shape B are shown on the grid.

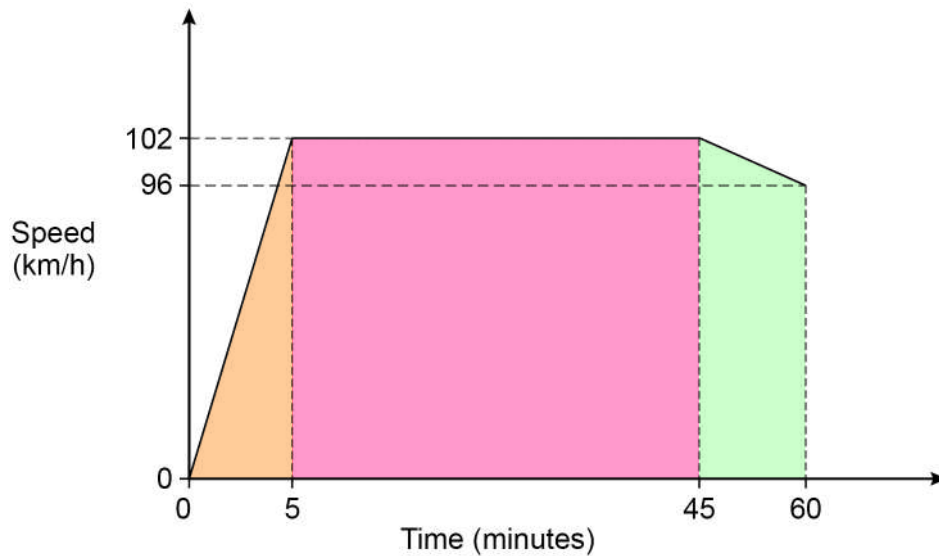
Describe the **single** transformation that maps shape A to shape B.**[3 marks]**Enlargement, scale factor  $-1/2$ , centre  $(1, -1)$ 

It must be an enlargement as it has changed size. The scale factor is  $-1/2$  as it is half the size and has flipped. The centre is  $(1, -1)$  as this is where the straight lines going through the corners meet



25

Here is a sketch of a speed-time graph for the first part of a journey.



The total distance for the journey is 130 kilometres.

How far is left to travel?

**[4 marks]**

On a speed-time graph, the distance is the area under the line. All the times need to be in hours so that it is compatible with the unit of speed, which involves hours. There are 60 minutes in an hour so putting all the times over 60 converts them into hours

$$\frac{1}{2} \times \frac{5}{60} \times 102 = 4.25$$

Area of the orange triangle. Area of triangle =  $\frac{1}{2} \times \text{base} \times \text{height}$ .  
The base is  $\frac{5}{60}$  and the height is 102

$$\frac{40}{60} \times 102 = 68$$

Area of the pink rectangle. Area of rectangle = length  $\times$  width.  
The length is  $\frac{40}{60}$  and the width is 102

$$\frac{1}{2} (102 + 96) \times \frac{15}{60} = 24.75$$

Area of the green trapezium. Area of trapezium =  $\frac{1}{2} (a + b) \times h$ ,  
where  $a$  and  $b$  are the parallel sides and  $h$  is the distance between  
them.  $a$  is 102,  $b$  is 96 and  $h$  is  $\frac{15}{60}$

$$130 - 4.25 - 68 - 24.75$$

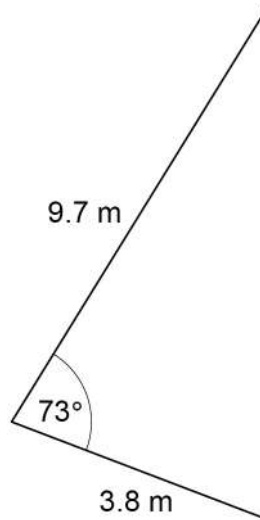
Subtracting the distances (which are the same as the areas) from  
the total distance for the journey leaves how far is left to travel

Answer 33 km

Turn over ►



26 Here is a triangular sail.



Not drawn  
accurately

26 (a) Vicky needs to buy waterproofing liquid for the sail.

She will put **3 coats** of liquid on **each** side of the sail.

A litre of liquid covers 8.5 square metres of sail.

How many 1-litre bottles of liquid does Vicky need?

[3 marks]

$$\frac{1}{2} \times 9.7 \times 3.8 \times \sin 73$$

Area of triangle =  $\frac{1}{2} ab \sin C$ , where a and b are two sides and C is the angle between them.  $\frac{1}{2} bh$  can't be used so easily as the height is not given

$$17.6... \times 3 \times 2$$

Multiplying the area of the triangle by 3 as 3 coats of liquid are needed. Multiplying this by 2 as there are 2 sides of the sail

$$105.7... \div 8.5$$

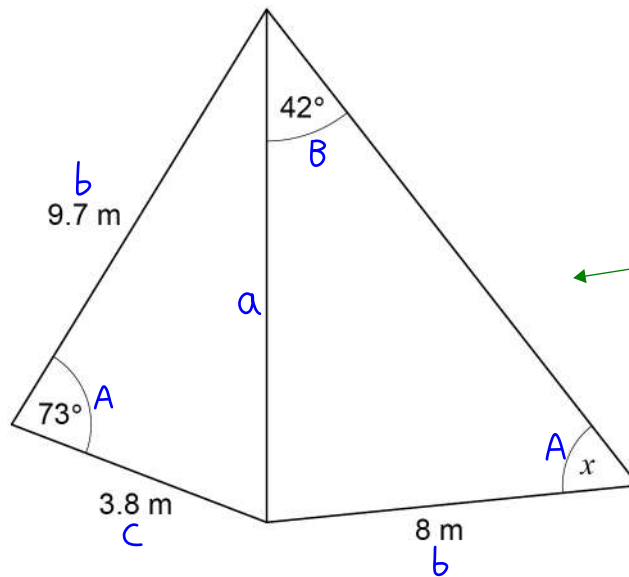
Dividing the result by 8.5 works out how many lots of 8.5 square metres it is and therefore how many litres are needed

Answer \_\_\_\_\_ 13

12.4... litres are needed. 13 1-litre bottles of liquid are needed as 12 is not enough and it needs to be a whole number of bottles



26 (b) Another sail is joined to the first sail as shown.



Not drawn  
accurately

Labelling the triangles.  
Side a is opposite angle A.  
Side b is opposite angle B

$x$  is an acute angle.

Work out the size of angle  $x$ .

[5 marks]

$$a^2 = b^2 + c^2 - 2bc \cos A$$

The sine rule cannot be used in the left triangle as there are not two pairs of opposite sides and angles. Therefore the cosine rule should be used

$$a = \sqrt{9.7^2 + 3.8^2 - 2 \times 9.7 \times 3.8 \times \cos 73}$$

Square rooting both sides and substituting in the values finds the joining side. Storing the exact value of 9.32... on the calculator

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

The sine rule can now be used in the right triangle as there are two pairs of opposite sides and angles

$$A = \sin^{-1}\left(\frac{9.32 \dots \sin 42}{8}\right)$$

Rearranging to find  $A$  by multiplying both sides by  $a$  then doing the inverse sin of both sides. Substituting in the values. Using the exact value stored on the calculator for 9.32...

Answer                     51.3                     degrees

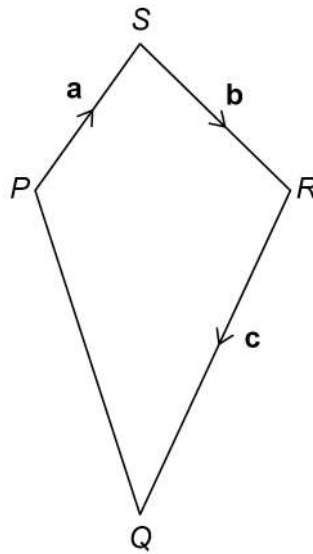




27

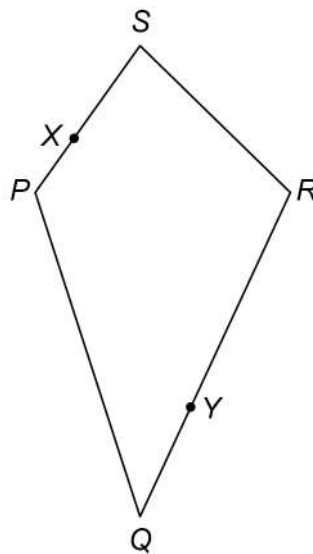
Here is quadrilateral  $PQRS$ .

$$\overrightarrow{PS} = \mathbf{a} \quad \overrightarrow{SR} = \mathbf{b} \quad \overrightarrow{RQ} = \mathbf{c}$$

Not drawn  
accurately

$X$  is a point on  $PS$  where  $PX : XS = 1 : 2$

$Y$  is a point on  $RQ$  where  $RY : YQ = 2 : 1$

Not drawn  
accurately

Is  $XY$  parallel to  $PQ$ ?

Show working to support your answer.

[3 marks]

$$\vec{PQ} = a + b + c$$

$$\vec{PQ} = \vec{PS} + \vec{SR} + \vec{RQ}$$

$$\vec{XY} = \frac{2}{3}a + b + \frac{2}{3}c$$

$\vec{XY} = \vec{XS} + \vec{SR} + \vec{RY}$ .  $\vec{XS} = \frac{2}{3}\vec{PS}$  as it is represented by 2 parts in the ratio  $PX : XS$  and there are 3 parts in total.  $\vec{RY} = \frac{2}{3}\vec{RQ}$  as it is represented by 2 parts in the ratio  $RY : YQ$  and there are 3 parts in total

No, as  $XY$  is not a multiple of  $PQ$

For vectors to be parallel it must be possible to multiply one of the vectors to get the other.  $a$  and  $c$  have been multiplied by  $\frac{2}{3}$  however the  $b$  has not been multiplied by  $\frac{2}{3}$  to get from  $\vec{PQ}$  to  $\vec{XY}$

Turn over for the next question



28  $f(x) = 2x - 3$  and  $g(x) = x^2$

Show that  $f^{-1}(55) = fg(4)$

[4 marks]

$$x = 2y - 3$$

Switching  $f(x)$  for  $x$  and  $x$  for  $y$  in  $f(x)$  then rearranging to find  $y$  finds the inverse function  $f^{-1}(x)$

$$\frac{x+3}{2} = y$$

Adding 3 to both sides then dividing both sides by 2 makes  $y$  the subject. So the inverse function  $f^{-1}(x) = (x + 3)/2$

$$f^{-1}(55) = \frac{55+3}{2} = 29$$

Substituting 55 for  $x$  in  $f^{-1}(x)$  finds that  $f^{-1}(55) = 29$

$$g(4) = 4^2 = 16$$

Substituting 4 for  $x$  in  $g(x)$  finds that  $g(4) = 16$

$$f(16) = 2 \times 16 - 3 = 29$$

Substituting 16 for  $x$  in  $f(x)$  finds that the composite function  $fg(4) = 29$

Both  $f^{-1}(x)$  and  $fg(x)$  are equal to 29 so they must be equal to each other

END OF QUESTIONS

