

Please write clearly in block capitals.

Centre number Candidate number

Surname _____

Forename(s) _____

Candidate signature _____

GCSE MATHEMATICS

H

Higher Tier Paper 1 Non-Calculator

Tuesday 21 May 2019

Morning

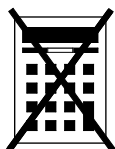
Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- mathematical instruments

You must **not** use a calculator.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
TOTAL	

Advice

In all calculations, show clearly how you work out your answer.



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

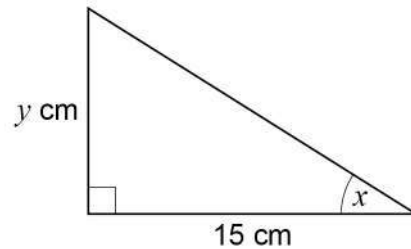
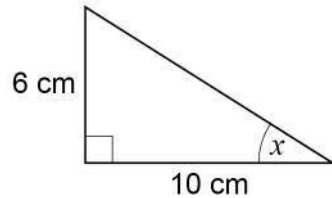
Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided

- 1 Here are two right-angled triangles.

Not drawn
accurately



Circle the value of y .

[1 mark]

11

7.5

9

4

As they both have a right angle and one of the other angles is x , the missing angle must also be the same. Therefore the triangles are similar. Each of the sides has been scaled by the same amount

- 2 Work out the value of $\left(1\frac{2}{3}\right)^2$

Circle your answer.

[1 mark]

$1\frac{4}{9}$

$3\frac{1}{3}$

$2\frac{4}{9}$

$2\frac{7}{9}$

Convert into an improper fraction then multiply by itself. To multiply fractions, multiply the numerators and denominators. Divide the numerator by the denominator to work out the whole number and leave the remainder as a fraction to convert the answer to a mixed number

- 3 Work out the arc length, in metres, of a semicircle of radius 6 metres.

Circle your answer.

[1 mark]

3π

6π

12π

18π

The arc on a semicircle is half of the circumference of the full circle.
Circumference = $2\pi r$ where r is the radius



4 Circle the fraction that is equivalent to 4.625

[1 mark]

$$\frac{39}{8}$$

$$\frac{37}{8}$$

$$\frac{185}{4}$$

$$\frac{17}{4}$$

Convert the fractions into decimals by dividing the numerators by the denominators

5 (a) Write 0.00097 in standard form.

[1 mark]

Answer _____

Standard form: $a \times 10^n$ where $1 \leq a < 10$ and n is a whole number. Count the number of times the decimal point needs to be moved to get a number between 1 and 10. n is the number of times it has been moved to the left (as it moves to the right in this case n will be negative)

5 (b) Work out $\frac{3 \times 10^5}{4 \times 10^3}$

Give your answer as an ordinary number.

[2 marks]

First divide 3 by 4 then multiply the answer by $10^5/10^3$
 $a^x/a^y = a^{x-y}$

This will give an answer in standard form which needs to be converted to an ordinary number

Answer _____



6 Anna plays a game with an ordinary, fair dice.

If she rolls 1 she wins.

If she rolls 2 or 3 she loses.

If she rolls 4, 5 or 6 she rolls again.

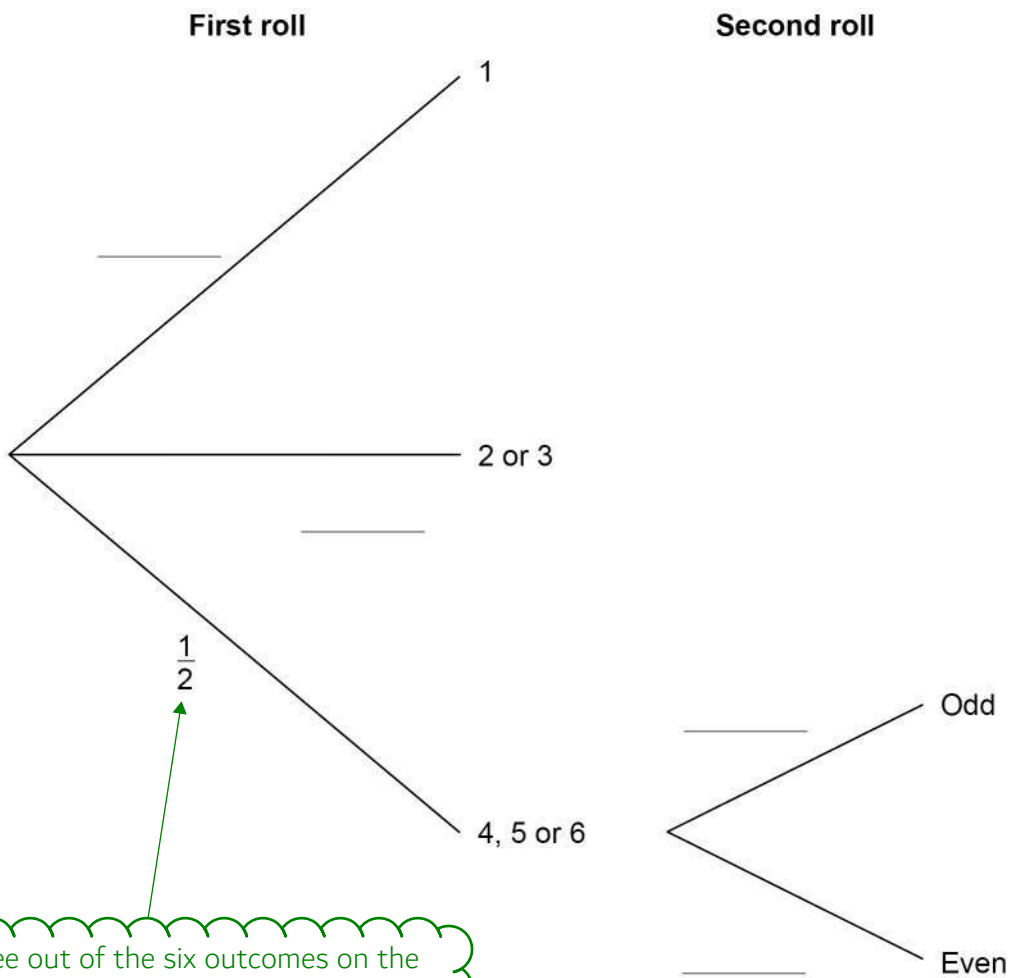
When she has to roll again,

if she rolls an odd number she wins

if she rolls an even number she loses.

6 (a) Complete the tree diagram with the four missing probabilities.

[2 marks]



$\frac{1}{2}$

Three out of the six outcomes on the ordinary dice are 4, 5 or 6. $\frac{3}{6}$ simplifies to $\frac{1}{2}$. Express the other probabilities as fractions in a similar way. There is no need to simplify the fractions



6 (b) Is Anna more likely to win or to lose?

You **must** work out the probability that she wins.

[4 marks]

AND means to multiply, OR means to add. To win, roll a 1 OR roll 4, 5, 6 AND odd. Substitute in the probabilities from the tree diagram then work with the fractions to get the probability of winning. If it is more than half, she is more likely to win

Turn over for the next question

Turn over ►



- 7 Three friends arrive at a party.
Their arrival increases the number of people at the party by 20%
In total, how many people are now at the party?

[2 marks]

20% of the people is 3. As it has increased by 20%, the number of people is now at 120%. Multiply 20% to get 120%

Answer _____

- 8 Work out the value of $(3^{12} \div 3^5) \div (3^2 \times 3)$

[3 marks]

Do not work out the value of any of the powers of 3 until they have been fully simplified to a single power of 3

$$a^x / a^y = a^{x-y}$$

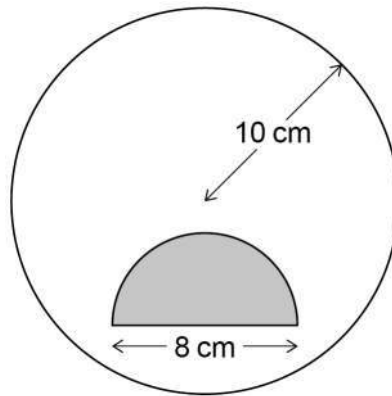
$$a^x \times a^y = a^{x+y}$$

Answer _____



- 9 A shaded semicircle is inside a circle as shown.

Not drawn
accurately



The **radius** of the circle is 10 cm

The **diameter** of the semicircle is 8 cm

How many times bigger is the unshaded area than the shaded area?

[4 marks]

Dividing the unshaded area by the shaded area works out how many times larger it is. Subtract the shaded area from the area of the circle to get the unshaded area.
Area of circle = πr^2 , where r is the radius.

Answer _____

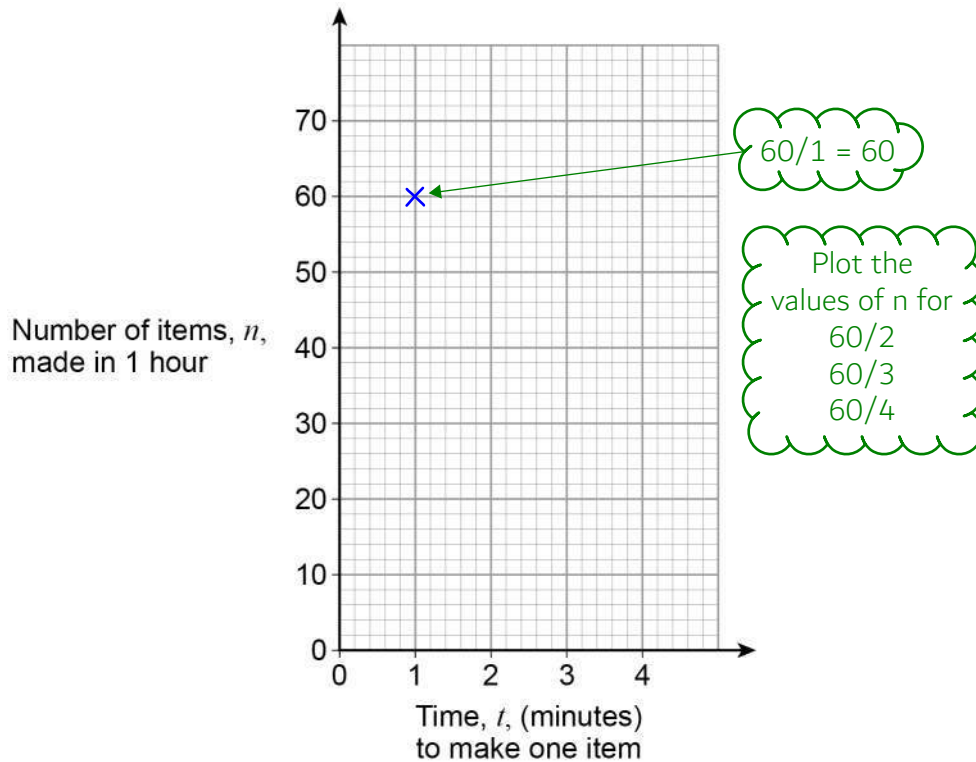
Turn over for the next question



- 10** The number of items, n , made in 1 hour by a machine is given by $n = \frac{60}{t}$
- t is the time in minutes the machine takes to make one item.
- The value of t changes for different types of item.

- 10 (a)** On the grid below, draw the graph of $n = \frac{60}{t}$ for values of t from 1 to 4

[2 marks]



- 10 (b)** The machine takes 3 minutes 30 seconds to make one item.
- Use your graph to estimate the value of n .

[2 marks]

Answer _____

30 seconds is half a minute so draw a line
up from 3.5 minutes to the line drawn then
across to the number of items on the y-axis



- 11 Ed and Fay shared £330 in the ratio 7 : 4
Ed gives Fay some of his money.
Fay now has the same amount as Ed.

How much does Ed give Fay?

[3 marks]

Work out how much Ed and Fay had by dividing £330 into the ratio. To do this, add up the number of parts in the ratio then divide 330 by this amount to get what 1 part is worth. Multiply the worth of 1 part by 7 and 4 to get 7 and 4 parts. Half of difference of the amounts of money they had works out how much Ed gives Fay

Answer £ _____

- 12 The next term of a sequence is made by adding the previous two terms.
Which of these sequences follows this rule?
Circle your answer.

[1 mark]

-9 2 -7 -5 -12

-3 5 -2 3 1

~~0 -3 -3 0 -3~~

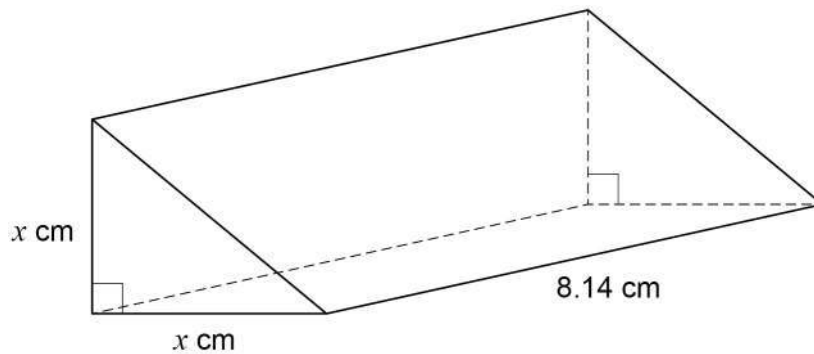
-1 -1 -2 -3 1

Check each sequence to see if it follows the rule. It isn't this one as
 $0 + -3 = -3$
But $-3 + -3 = -6$, not 0



13

The triangular cross section of a prism is an isosceles right-angled triangle.



The volume of the prism is 102 cm^3

Use approximations to estimate the value of x .

You **must** show your working.

[3 marks]

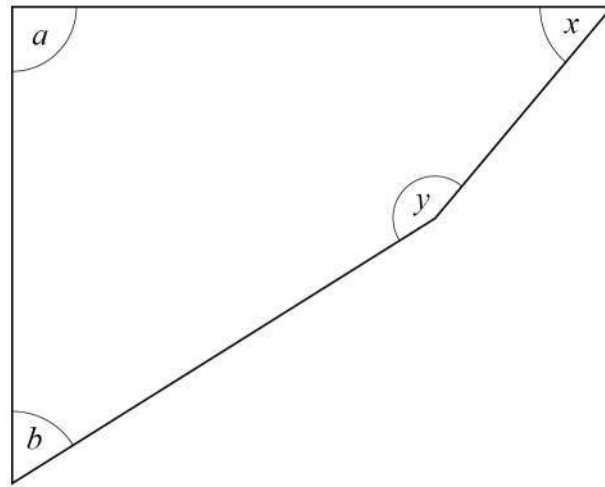
Volume of prism = cross sectional area \times length
 The cross section is a triangle.
 Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
 The length is approximately 8cm.
 Express the volume of the prism in terms of x ,
 simplify the expression fully then set it equal to
 the approximate volume. Rearrange to find x

Answer _____



14

Here is a quadrilateral.

Not drawn
accurately

$$a = 90^\circ \quad \text{and} \quad a : b = 5 : 3$$

$$x : y = 1 : 3$$

Show that $b = x$ **[3 marks]**

First find b by using the ratio $a : b$. 5 parts represents a so dividing 90 by 5 works out the value of 1 part. Multiplying by 3 works out the value of the 3 parts which represent b .

There are 360 degrees in total in a quadrilateral. Subtracting the angles a and b from 360 leaves the total of angles x and y .

Divide the total of x and y into the ratio of $x : y$ to find x .

The value of b should be the same as the value of x

Turn over ►



15 Here is some information about the test marks of 120 students.

Mark, m	$0 < m \leq 10$	$10 < m \leq 20$	$20 < m \leq 30$	$30 < m \leq 40$	$40 < m \leq 50$
Frequency	20	28	40	20	12

15 (a) Complete the cumulative frequency table.

[1 mark]

Mark, m	$m \leq 10$	$m \leq 20$	$m \leq 30$	$m \leq 40$	$m \leq 50$
Cumulative frequency	20	48			

Cumulative frequency means to add up the frequencies as they go. $20 + 28 = 48$. $48 + 40 = \dots$

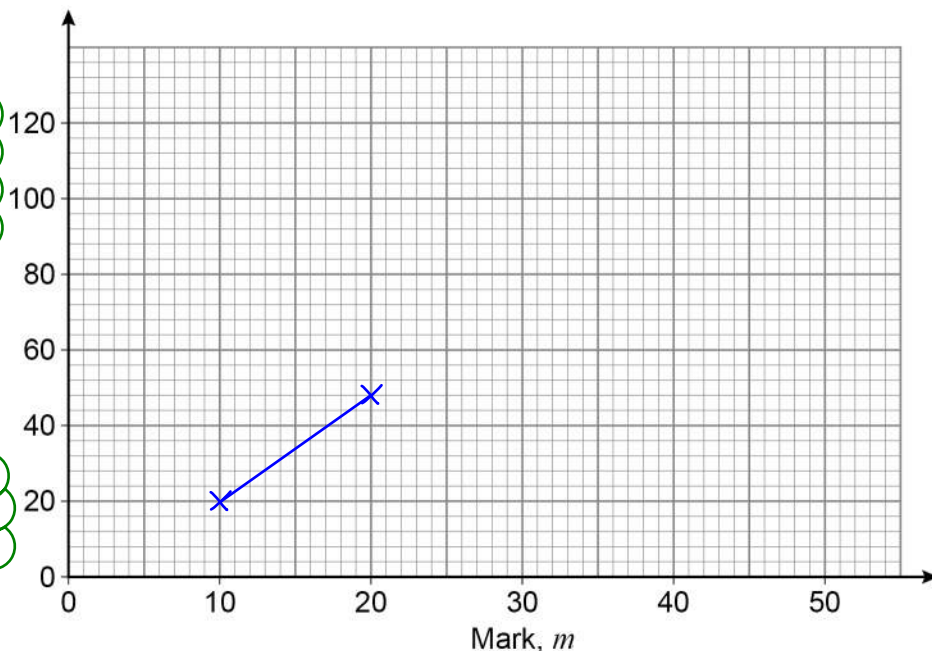
15 (b) Draw a cumulative frequency graph.

[2 marks]

The scale goes up 20 every 5 boxes. $20/5 = 4$ so the scale is going up in 4 every small box

Cumulative frequency

There were no students with a mark less than 0



The cumulative frequencies are plotted at the end points of each category as it is reached by the end but not before. Each of the points can be joined up using a series of straight lines



15 (c) Students who scored 15 marks or fewer take another test.

Use your graph to estimate how many students take another test.

[2 marks]

Read up from 15 to the line
then across to the frequency

Answer _____

16 Simplify fully

$$\frac{4x - 8x^2}{12x - 6}$$

[3 marks]

Factorise the numerator and denominator then cancel out
any common factors from the numerator and denominator.
The highest common factor brought out for the numerator
should be negative as the x^2 term is negative

Answer _____

Turn over for the next question



17 Toby is forming and solving equations.

17 (a)

The product of half of a number and three more than the number
is the same as
the square of the number

Toby uses y to represent the number.

Write an equation that Toby could form.

[2 marks]

Product means multiplied together.
'The same as' means equals

Answer _____

17 (b) Toby forms another equation.

$$x = \frac{9}{8x}$$

He wants to work out the values of x .

Here is his working.

There is a
mistake here

$$x = \frac{9}{8x}$$

$$8x^2 = 9$$

$$8x = 3 \text{ or } 8x = -3$$

$$x = \frac{3}{8} \text{ or } x = -\frac{3}{8}$$

What error has he made in his working?

[1 mark]



18 Here is an identity.

$$x^2 - y^2 \equiv (x + y)(x - y)$$

18 (a) Use the identity to work out the value of $193^2 - 7^2$
You **must** show your working.

[2 marks]

x is 193 and y is 7. Substitute these values into the right side of the identity

Answer _____

18 (b) Factorise $100a^2 - 81b^2$

$$\begin{aligned} x^2 &= 100a^2 \\ x &= 10a \\ y^2 &= 81b^2 \\ y &= \dots \end{aligned}$$

Use the identity

[1 mark]

Answer _____

19 Circle the fraction that is equivalent to $0.\dot{1}$

The dot above the 1 means it is recurring (goes on forever)

[1 mark]

$$\frac{1}{9}$$

$$\frac{1}{99}$$

$$\frac{1}{10}$$

$$\frac{11}{100}$$

1/9 is close to 1/10

1/99 is close to 1/100

It can't be this one as $1/10 = 0.1$

It can't be this one as $11/100 = 0.11$

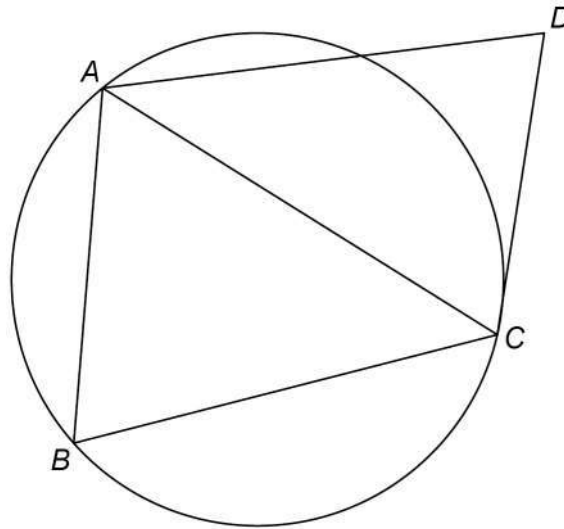


20

A , B and C are points on a circle.

CD is a tangent.

Not drawn
accurately



20 (a) Assume that triangle ABC is isosceles with $AC = BC$

Prove that AB is parallel to DC .

[4 marks]

The base angles of isosceles triangles are equal.
The alternate segment theorem can be used.
Lines are parallel if there are alternate angles



20 (b) In fact, triangle ABC is equilateral.

Tick the **two** boxes for the statements that **must** be correct.

[1 mark]

AB is parallel to DC

This was proven in (a) as the angles in the triangle will still be equal and the alternate segment theorem will still apply

AC bisects angle BCD

Both halves of BCD (BCA and ACD) are both equal as all the angles in an equilateral triangle are equal and the alternate segment theorem

AC bisects angle BAD

CD could have any length so angle CAD could be anything. It doesn't have to be the same as BAC

21 Solve the simultaneous equations

$$2x + 3y = 5p$$

Equation 1

$$y = 2x + p$$

Equation 2

where p is a constant.

Give your answers in terms of p in their simplest form.

[4 marks]

1. Substitute the right side of Equation 2 for y in Equation 1.
2. Rearrange to make x the subject and simplify.
3. Substitute the value of x (in terms of p) for x in Equation 2.
4. Simplify to find y in terms of p

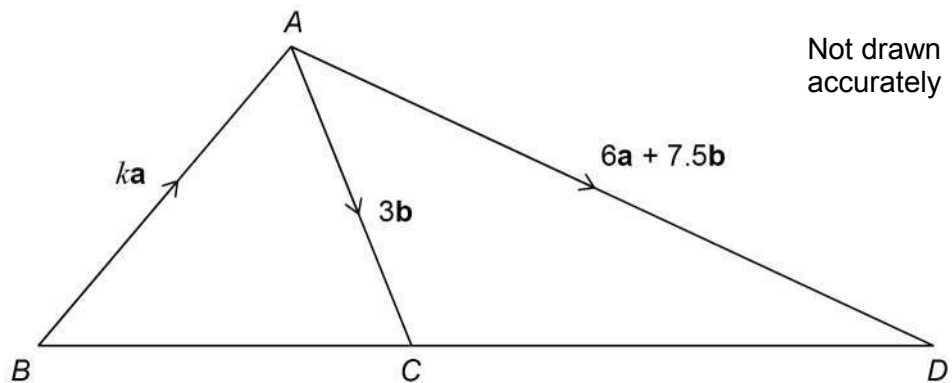
$x =$ _____ $y =$ _____

9

Turn over ►



- 22 ABC and ACD are triangles.
 k is a constant.



- 22 (a) Show that $\vec{CD} = 6\mathbf{a} + 4.5\mathbf{b}$

[1 mark]

$$\vec{CD} = \vec{CA} + \vec{AD}$$

\vec{CA} is in the opposite direction to \vec{AC}

- 22 (b) BCD is a straight line.

Work out the value of k .

You **must** show your working.

[3 marks]

$$\vec{BC} = \vec{BA} + \vec{AC}$$

As BCD is a straight line, \vec{BC} must be in the same direction as \vec{CD} . So \vec{BC} is a scaled down version of \vec{CD}

Answer _____



23 Simplify $8^4 \div 32^{\frac{2}{5}}$

Give your answer in the form 2^m where m is an integer.

[3 marks]

Express both 8 and
32 as powers of 2

$$(a^x)^y = a^{xy}$$

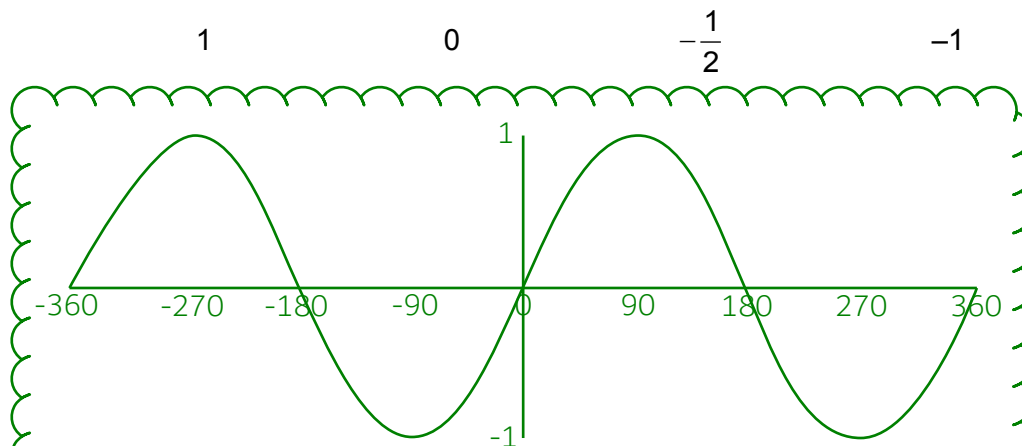
$$a^x / a^y = a^{x-y}$$

Answer _____

24 $f(x) = \sin(x - 90^\circ)$

Circle the value of $f(0^\circ)$

[1 mark]



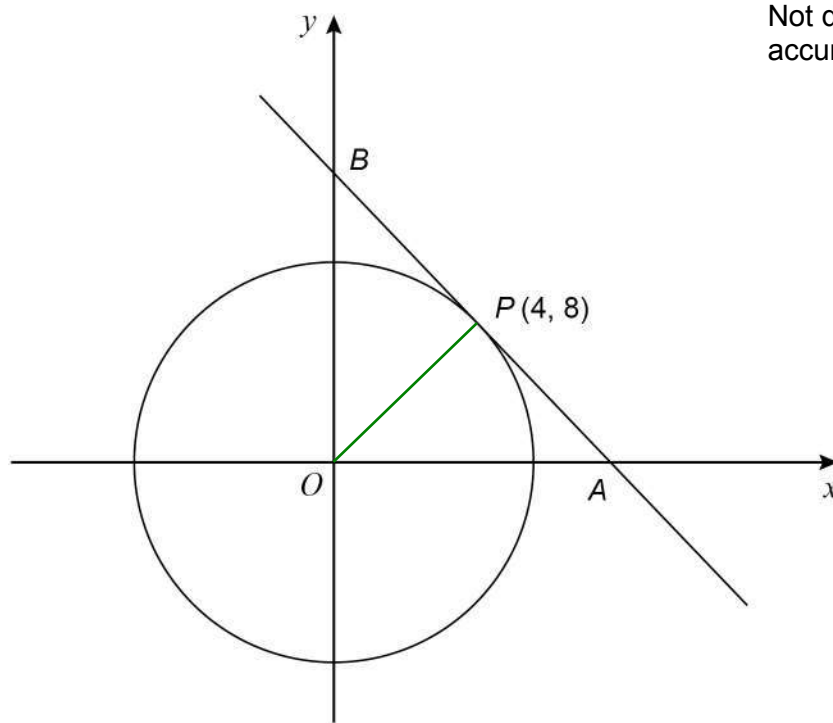
The sine curve keeps repeating every 360 degrees. Substitute 0 into $f(x)$ then read off the sin value using the graph above

Turn over for the next question

Turn over ►



- 25 $P(4, 8)$ is a point on a circle, centre O .
The tangent at P intersects the axes at points A and B .



- 25 (a) Show that the gradient of the tangent is $-\frac{1}{2}$

[2 marks]

The tangent is perpendicular to the radius so both gradients multiply together to get -1. Work out the gradient of the radius first. Gradient = (change in y)/(change in x)



25 (b) Work out the length AB .

Give your answer in the form $a\sqrt{5}$ where a is an integer.

You **must** show your working.

[4 marks]

Pythagoras' Theorem can be used to work out the length AB as OAB is a right-angled triangle. Work out where the tangent crosses the y -axis at point B and the x -axis at point A to work out the lengths of OA and OB . To do this, the equation of the tangent needs to be found. The general equation of a straight line is $y = mx + c$, where m is the gradient and c is the y -intercept. $y = 0$ at point A

Answer _____ units

Turn over for the next question



- 26 The turning point of the graph $y = (x + a)^2 + b$ has x -coordinate -2
(3, 1) is another point on the graph.

Work out the y -coordinate of the turning point.

[3 marks]

The turning point is where the square bracket has the minimum value, which is 0 (the lowest a squared number can be is 0). Find a by setting $x + a = 0$ and substituting the x -coordinate of the turning point for x .
Find b by rearranging the equation to make b the subject and substituting in the x and y values from the point (3, 1). When the square bracket is 0 (at the minimum point, which is the turning point), $y = b$

Answer _____



27

Angle x is acute.

$$\cos x = \sin 60^\circ \times \tan 30^\circ$$

Work out the size of angle x .You **must** show your working.**[3 marks]**

	0	30	45	60	90
Sin:	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos:	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

The angles we need to remember the trig values for are 0, 30, 45, 60 and 90. To find the sin values, list 0, 1, 2, 3, 4 then square root them all and put them over 2. The cos values are the same but the other way around

$$\tan 30 = \sin 30 / \cos 30$$

Answer _____ degrees

END OF QUESTIONS