.CG Maths.
Worked Solutions
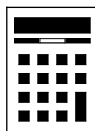
GCSE
MATHEMATICS

H

Higher Tier   Paper 2 Calculator

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24	
TOTAL	

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

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Answer **all** questions in the spaces provided.

1 Circle the fraction that is equal to 1.25%

[1 mark]

$$\frac{1}{8} \qquad \frac{1}{25} \qquad \boxed{\frac{1}{80}} \qquad \frac{1}{125}$$

1.25/100 = 1/80 ← Putting the percentage over 100 converts it into a fraction

2 Circle the expression that means   the probability of A and **not** B.

[1 mark]

P(A' ∪ B)   P(A ∪ B')   P(A' ∩ B)   P(A ∩ B')

Probability of not A or B

Probability of A or not B

Probability of not A and B

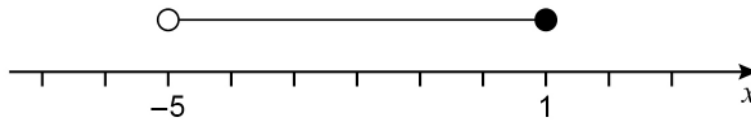**3** Circle the triangular number.

[1 mark]

9   12   15   18

Start with 1. Then add 2, then add 3, then add 4… 1, 3, 6, 10, 15…

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4   Circle the inequality represented by the diagram.

[1 mark]

$-5 < x < 1$ $-5 < x \leqslant 1$ $-5 \leqslant x < 1$ $5 \leqslant x \leqslant 1$

Where the dot is open, it cannot be equal to that value.
Where the dot is closed, it can be equal to that value

5   Solve $5(2x - 1) = 6x + 9$

[3 marks]

$10x - 5 = 6x + 9$ ← *Expanding the brackets on the left side*

$4x - 5 = 9$ ← *There is the least x on the right so subtracting the 6x from both sides*

$4x = 14$ ← *Adding 5 to both sides to eliminate the -5 and get the x term on its own*

$$x = \dfrac{14}{4}$$

Dividing both sides by 4 gets x on its own. 14 cannot be divided by 4 to get a whole number so it can be left as a fraction

7

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IB/M/Jun22/8300/2H

6 Show that 2125 can be written as

a cube number **multiplied** by a prime number between 10 and 20

[2 marks]

Using table mode, enter f(x) = 2125/x^3. Start at 1 and end at 30 with a step of 1

This divides 2125 by the first 30 cube numbers. From the table of values, 2125/5^3 = 17, which is a prime number between 10 and 20

$5^3 \times 17 = 2125$

7 Sam types a constant number of words per minute.

He takes 8 minutes to type a report of 416 words.

How long does it take him to type an essay of 1534 words?

Give your answer in minutes and seconds.

[3 marks]

$\dfrac{1534}{416} \times 8$

Expressing the 1534 words as a fraction of the 416 words then doing that fraction of the 8 minutes

29.5

Answer _____29_____ minutes _____30_____ seconds

29.5 minutes is 29$\frac{1}{2}$ minutes. Half a minute is 30 seconds as there are 60 seconds in a minute and half of 60 is 30

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IB/M/Jun22/8300/2H

8 A school play takes place each day from Monday to Friday.

Here are the attendances on four of the days.

Monday	Tuesday	Wednesday	Thursday
72	83	88	97

For all **five** days, the mean attendance is 90

Work out the attendance on Friday.

[3 marks]

m^t_n

Writing the formula triangle for mean, where m is the mean, t is the total (which is all of the values added together), and n is the number (the number of values)

90×5

From the formula triangle, covering t finds that the total = mean x number. Multiplying the mean of 90 by the 5 days finds that the total attendance of all five days is 450

450−72−83−88−97

Subtracting the attendance for the other four days from the total attendance of all five days leaves the attendance of Friday

Answer _____ 110

Turn over for the next question

Turn over ▶

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0 5

9 Rosie makes phone calls to try to sell broadband.

Today, she made 120 calls.

The table shows the results.

Result of call	Frequency
Not answered	33
Answered but sale not made	81
Answered and sale made	6

9 (a) Write down the relative frequency that a call was **not answered**.

[1 mark]

Answer $\frac{33}{120}$

33 out of the 120 calls were not answered

9 (b) During the **rest of the week**, Rosie will make 500 calls.

Using the results in the table, how many sales does she expect to make during the **rest of the week**?

[2 marks]

$\frac{6}{120} \times 500$

6 out of the 120 calls resulted in a sale made. Expressing this fraction then doing that fraction of the 500 calls as it can be expected that the fraction which resulted in a sale will be the same for all 500

Answer 25

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10 Harry and Ellie each bought a printer and a hard drive.

Here is some information about how much they paid.

	Printer	**Hard drive**
Harry	£80	£25
Ellie	10% less than Harry	20% more than Harry

Ellie says,

"In total, I paid more than Harry because 20% is greater than 10%"

Is she correct?

Tick a box.

Yes ☐   No ☑

Show calculations to support your answer.

[2 marks]

$\frac{10}{100} \times 80 = 8$ ← Converting 10% into a fraction by putting it over 100 then multiplying it by the £80 finds that 10% of £80 is £8
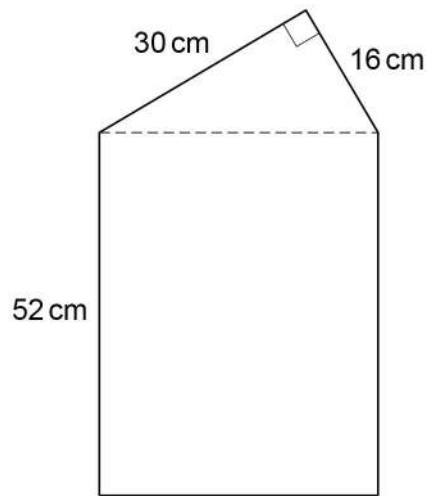
$\frac{20}{100} \times 25 = 5$ ← Converting 20% into a fraction by putting it over 100 then multiplying it by the £25 finds that 20% of £25 is £5

Ellie paid £8 less for the printer and £5 more for the hard drive. Overall this is £3 less than Harry. Therefore Ellie is not correct

5

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11   A shape is made by joining a right-angled triangle to a rectangle.

30 cm   16 cm

Not drawn
accurately

52 cm

Work out the area of the shape.

[5 marks]

$\frac{1}{2} \times 16 \times 30 = 240$

Area of triangle = 1/2 x base x height. The base
of the triangle is 16cm and the height is 30cm

$a^2 + b^2 = c^2$

There is a missing side on the right-angled triangle
so Pythagoras' Theorem can be used to find it

$\sqrt{30^2 + 16^2}$

The missing side is the longest side so is c. Rearranging to make c
the subject finds that c = $\sqrt{a^2 + b^2}$. Substituting 30 for a and 16 for b

52×34

The missing side is the width of the rectangle.
Area of rectangle = length x width. The length is
52cm and the width is 34cm

$1768 + 240$

Adding together the area of the rectangle and the
area of the triangle gives the area of the shape

Answer _____2008_____ cm^2

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12 $4y = 5x$

Which statement is correct?

4 x 5 = 5 x 4 so y could be 5 and x could be 4

Tick **one** box.

[1 mark]

☐ y is 80% of x ← y is bigger than x so it cannot be this option

☑ y is 125% of x ← 5 is 25% more than 4 so it must be this option

☐ x is 20% of y ← 4 is not 20% of 5 so it cannot be this option

☐ x is 400% of y ← y is bigger than x so it cannot be this option

Turn over for the next question

6

Turn over ▶

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13   Outside a cafe there is a large plastic ice cream cornet.

The cornet is a hemisphere on top of a cone.

The cone and the hemisphere each have radius 24 cm

The cone has perpendicular height 117 cm

Volume of a cone = $\frac{1}{3}\pi r^2 h$
r is the radius
h is the perpendicular height

Volume of a hemisphere = $\frac{2}{3}\pi r^3$
r is the radius

13 (a)   Work out the total volume of the cornet.

[4 marks]

$$\frac{1}{3}\pi \times 24^2 \times 117 + \frac{2}{3}\pi \times 24^3$$

Adding the volume of the cone and the volume of the hemisphere gives the volume of the cornet

Volume of the cone. The radius is 24cm and the perpendicular height is 117cm

Volume of the hemisphere. The radius is 24cm

Answer _____ 31680π _____ cm³

13 (b) The actual cornets that the cafe sells are **similar** to the plastic one.

For the actual cornets, the cone and the hemisphere each have radius 2 cm

How many times greater is the volume of the plastic cornet than an actual cornet?

[3 marks]

$\left(\dfrac{24}{2}\right)^3$

The radius of the plastic cornet is 24cm. The radius of the actual cornet is 2cm. These are both lengths. Dividing them works out the length scale factor. The unit of length is cm and the unit of volume is cm³. So the length scale factor needs to be cubed to work out the volume scale factor
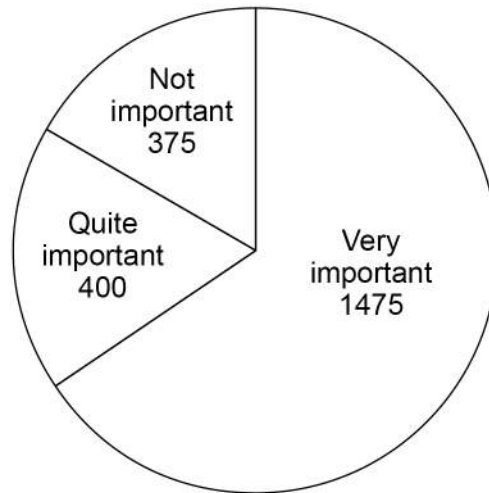
Answer _____ 1728 _____

Turn over for the next question

Turn over ▶

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14 A survey was held in a football stadium.

A sample of the crowd was asked about the importance of a family area.

The pie chart represents the answers.

14 (a) The total number of people in the crowd was 29 250

Estimate how many people in the crowd think that a family area is **very important**.

Assume that the sample is representative of the crowd.

[3 marks]

$375 + 400 + 1475$ ← This works out that there are 2250 people in total in the sample

$\frac{1475}{2250} \times 29250$ ← Expressing the number of people in the sample who think that a family area is very important as a fraction of the total number of people in the sample then doing this fraction of the total number of people in the crowd works out an estimate of how many people in the crowd think that a family area is very important

Answer _____ 19175 _____

14 (b) In fact,

50% of the **sample** were sitting in the family area

10% of the **crowd** were sitting in the family area.

What is this likely to mean about the actual number of people in the crowd who think that a family area is very important?

Tick **one** box.

[1 mark]

☐ It is larger than the answer to part (a)

☐ It is the same as the answer to part (a)

☑ It is lower than the answer to part (a)

> The sample is not representative of the crowd. People sitting in the family area can be assumed to be more likely to think that a family area is important. As a higher proportion were sitting in the family area in the sample, this makes the answer to part (a) likely to be too high

15 In the grid, the **product** of each row, column and diagonal is 1

> 2) There are now two numbers in this diagonal. 1 x 1/8 = 1/8. This must be multiplied by 8 to get 1

> 3) There are now two numbers in this row. 8 x 1/4 = 2. This must be multiplied by 1/2 to get 1

8	$\frac{1}{4}$	$\frac{1}{2}$
$\frac{1}{16}$	1	16
2	4	$\frac{1}{8}$

> 4) The others are filled in using similar methods

> 1) Starting with this column, there are two numbers so the third can be worked out. Multiplication can be done in any order so 1/4 x 4 = 1. Then 1 must be multiplied by 1 to get 1

Complete the grid.

[2 marks]

6

Turn over ▶

16 Amol owns a sandwich shop.

The shop is open from Monday to Saturday.

In June, Amol sold 3000 sandwiches.

16 (a) Amol wants to work out the mean number of sandwiches he sold per day in June.

His method is   3000 ÷ 30 = 100

Make **one** criticism of Amol's method.

[1 mark]

The shop wasn't open for 30 days

> The mean is the total number of sandwiches divided by the number of days open. There are 30 days in June but the shop is only open from Monday to Saturday and not on Sundays

16 (b) Amol received £6660 from selling the 3000 sandwiches in June.

The numbers of sandwiches sold were in the ratio

meat : cheese : vegan = 9 : 4 : 7

The price of a meat sandwich is £2.39

The price of a cheese sandwich is £1.89

Work out the price of a vegan sandwich.

[4 marks]

9+4+7
3000÷20

> There are 3000 sandwiches in total. Adding the 9, 4 and 7 works out that there are 20 parts in total in the ratio. These represent the 3000 sandwiches so dividing by 20 works out that 1 part of the ratio is 150

150×9=1350
150×4=600
150×7=1050

> Multiplying the 150 by each of the numbers of parts works out that there were 1350 meat, 600 cheese and 1050 vegan

1350×2.39=3226.50
600×1.89=1134

> Multiplying the number of meat and cheese sandwiches by the price of each works out the money received from the meat and cheese sandwiches
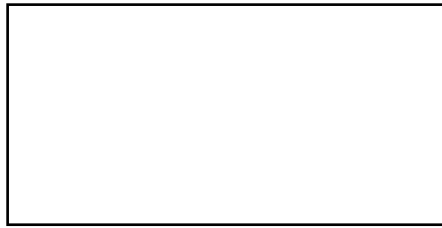
6660−3226.50−1134
2299.50÷1050

> Subtracting the money received from the meat and cheese sandwiches from the £6660 received in total leaves the money received from the vegan sandwiches. Then dividing this by the 1050 vegan sandwiches works out the price of a vegan sandwich

Answer £ _____ 2.19 _____

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17 Here is the plan of a solid.

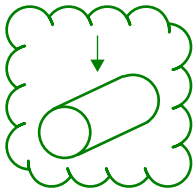Circle the solid that it could be.

[1 mark]

sphere   cone   hemisphere   (cylinder)

A cylinder viewed from above could appear as a rectangle

18 Solve $x^2 + 7x - 11 = 0$

Give your solutions as decimals.

[2 marks]

$$\frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times -11}}{2 \times 1}$$
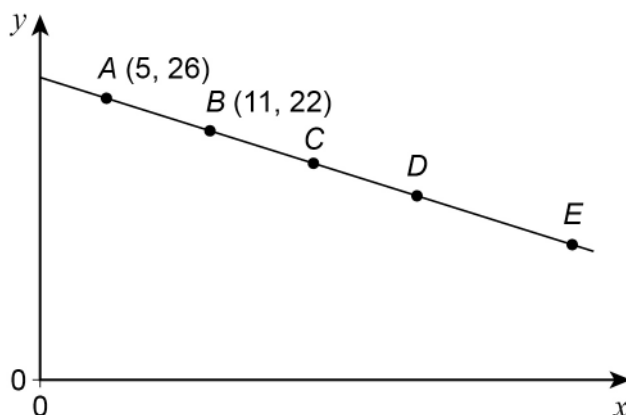
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quadratic formula can be used to solve the equation. It is in the form $ax^2 + bx + c = 0$ so a = 1, b = 7 and c = -11

Answer   1.3, -8.3

8

Turn over ▶

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19   *A*, *B*, *C*, *D* and *E* are points on a straight line.

Not drawn accurately

y↑

A (5, 26)

B (11, 22)

C

D

E

0

0   x

A, *B*, *C* and *D* are equally spaced.

$AD : DE = 2 : 1$

Work out the coordinates of *E*.

[3 marks]

$11-5=6$

This works out that there is 6 in the x-direction between points A and B

$22-26=-4$

This works out that there is -4 in the y-direction between points A and B

$6 \times 3 = 18$

There are 3 equal jumps between A and D. Multiplying the 6 in the x-direction by 3 works out that there is 18 in the x-direction between A and D

$-4 \times 3 = -12$

There are 3 equal jumps between A and D. Multiplying the -4 in the y-direction by 3 works out that there is -12 in the y-direction between A and D

$5 + \frac{18}{2} \times 3 = 32$

$26 - \frac{12}{2} \times 3 = 8$

Answer (___32___ , ___8___)

2 parts of the ratio represent the distance between A and D. Dividing this by 2 works out what 1 part of the ratio is worth. There are 3 parts in total in the ratio from A to E. So multiplying the worth of 1 part by 3 works out the distance between A and E in both the x and y-directions. Adding these distances to the x and y-coordinates of A works out the coordinates of E

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20 A company makes and sells boxes of washing powder.

Washing powder
1.68 kg
£5.60
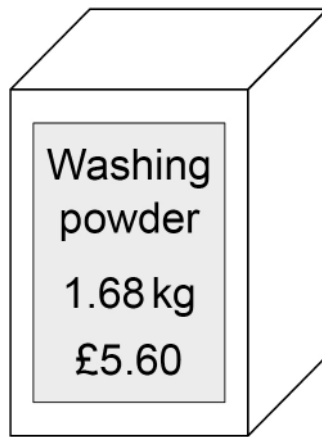
The company wants to increase the amount of money it receives **per kg** of powder.

To get the required increase it can

increase the price to £5.88

or

reduce the mass of powder in the box by x%

Work out the value of x to 2 decimal places.

[4 marks]

$$\frac{5.88}{1.68} = \frac{5.60}{m}$$

Money per kg means to divide the money by the mass. Dividing the new price of £5.68 by the mass of 1.68kg expresses the money per kg. This must be equal to the £5.60 divided by the new mass, m

$$\frac{m}{5.60} = \frac{1.68}{5.88}$$

Doing the reciprocal of both sides to make m the numerator and bringing m onto the left

$$m = \frac{1.68}{5.88} \times 5.60$$

Rearranging to find m by multiplying both sides by 5.60

$$\frac{1.6 - 1.68}{1.68} \times 100$$

1.6 - 1.68 expresses the change in mass. Putting this over the original mass expresses the change as a fraction. Multiplying this by 100 converts it into a percentage change

-4.761... becomes a decrease of 4.76% to 2 decimal places. The negative is ignored as x is a reduction

$x =$ _____ 4.76 _____

7

Turn over ▶

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21 Which of these is the equation of a circle?

Circle your answer.

[1 mark]

$$x^2 - y^2 = 6 \qquad \boxed{x^2 + y^2 = 6} \qquad y = x^2 - 6 \qquad y = x^2 + 6$$

The general equation of a circle is $x^2 + y^2 = r^2$, where r is the radius

22 Circle the reciprocal of 8^5

[1 mark]

$$\boxed{8^{-5}} \qquad\qquad 5^{-8} \qquad\qquad -8^5 \qquad\qquad 5^8$$

Negative power means reciprocal

23 Factorise $3x^2 - 16x - 12$

[2 marks]

It is in the form $ax^2 + bx + c$. First multiply a by c. Then look for two numbers which multiply to this and add to b

3 x -12 = -36. Entering table mode and setting f(x) = 36/x. Start: 1. End: 30. Step: 1

This lists out the factor pairs of 36. One of the pair must be negative to multiply to -36

$3x^2 - 18x + 2x - 12$

$3x(x-6) + 2(x-6)$

The two numbers which multiply to -36 and add to -16 are -18 and 2. Splitting the middle x term into these number of x

Answer $\qquad (3x+2)(x-6)$

Factorising both halves separately. 3x is the highest common factor of $3x^2$ and -18x. Bringing this out as a factor and leaving the remainder in the bracket. 2 is the highest common factor of 2x and -12. Bringing this out as a factor and leaving the remainder in the bracket

Bringing the 3x and +2 together into a bracket and multiplying it by the (x - 6)

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24 A straight line

is perpendicular to the straight line through (2, 8) and (6, 15)

and

passes through (0, 9) and (x, 17)

Work out the value of x.

[4 marks]

$\dfrac{15-8}{6-2} = \dfrac{7}{4}$

Expressing the gradient of the straight line through (2 , 8) and (6 , 15). Gradient = (change in y)/(change in x). 15 - 8 expresses the change in y and 6 - 2 expresses the change in x

$\dfrac{17-9}{x-0} = -\dfrac{4}{7}$

Expressing the gradient of the straight line. Gradient = (change in y)/(change in x). 17 - 9 expresses the change in y and x - 0 expresses the change in x. This is perpendicular so the gradient must be the negative reciprocal of 7/4

$\dfrac{x}{17-9} = -\dfrac{7}{4}$

Doing the reciprocal of both sides to make x the numerator. x - 0 = x

$x = -\dfrac{7}{4}(17-9)$

Rearranged to make x the subject by multiplying both sides by (17 - 9)

$x =$ _____ -14 _____

8

Turn over ▶

25   f(x) = 2x + 5

Show that   3f(x) − 12f^{-1}(x)   simplifies to an integer.

$x = 2y + 5$

Finding the inverse function by switching f(x) with x and x with y then rearranging to make y the subject

$x - 5 = 2y$

Subtracting 5 from both sides to get the y term on its own

$\frac{x-5}{2} = y$

Dividing both sides by 2 to make y the subject. So the inverse function f^{-1}(x) = (x - 5)/2

$3(2x+5) - 12\left(\frac{x-5}{2}\right)$

Expressing 3f(x) - 12f^{-1}(x)

$6x + 15 - 6x + 30$

Expanding the brackets

45

Collecting like terms and simplifying to an integer

26 Two objects, J and K, are applying pressure to areas of ground.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

For J, the force is 18.9 newtons and the area is $0.45\,\text{m}^2$

pressure for J : pressure for K = 7 : 8

area for J : area for K = 9 : 5

Work out the force for K.

[4 marks]

$\rho^F_{\ A}$ ← Writing the formula as a formula triangle

$\dfrac{18.9}{0.45}$ ← Pressure for J = (force for J)/(area for J)

$\dfrac{42}{7} \times 8 = 48$ ← From the ratio of the pressures, 7 parts represent the pressure for J. Dividing the pressure for J by 7 expresses 1 part of the ratio. Multiplying this by 8 finds the value of the 8 parts of the ratio which represent the pressure for K

$\dfrac{0.45}{9} \times 5 = 0.25$ ← From the ratio of the areas, 9 parts represent the area for J. Dividing the area for J by 9 expresses 1 part of the ratio. Multiplying this by 5 finds the value of the 5 parts of the ratio which represent the area for K

48×0.25 ← From the formula triangle, force = pressure x area

Answer _____ 12 _____ newtons

8

Turn over ▶

27 To be rented, a bedroom must have a floor area of at least 6.51 m²

A bedroom has a rectangular floor.

The floor measures 2.4 m by 2.9 m, each correct to 2 significant figures.

Show that the bedroom can be rented.

[3 marks]

$$\left(2.4 - \frac{0.1}{2}\right)\left(2.9 - \frac{0.1}{2}\right) = 6.6\ldots$$

Working out the lower bound of the area. Area of rectangle = length x width. The lower bounds of the length and width are expressed by subtracting half of the resolution of each measurement. The resolution is 0.1 for both as the 2nd significant figure is in the tenths place. 6.6... is more than 6.51 so the area is at least 6.51m²
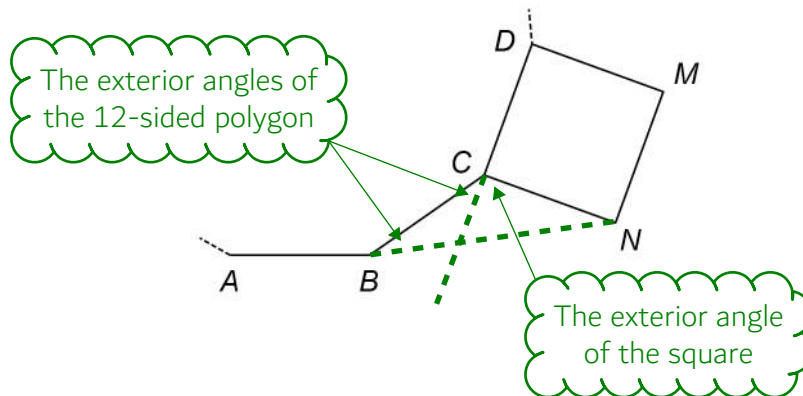
28   *AB*, *BC* and *CD* are sides of a regular 12-sided polygon.

CDMN is a square.

The exterior angles of the 12-sided polygon

The exterior angle of the square

Not drawn accurately

Prove that points *A*, *B* and *N* lie on a straight line.

[4 marks]

The exterior angles of any polygon add up to 360°. So dividing 360 by the number of exterior angles (which is the same as the number of sides) gives the exterior angle

360 ÷ 4 = 90
The exterior angle of the square is 90°

360 ÷ 12 = 30
The exterior angle of the 12-sided polygon is 30°

90 + 30 = 120
Angle BCN is 120°   Adding the two exterior angles gives angle BCN

(180 - 120)/2 = 30
Angle CBN is 30° as triangle CBN is isosceles as sides CN = CB

The base angles of an isosceles triangle are equal and the angles in any triangle add up to 180°. So subtracting the 120° from 180° degrees leaves the sum of angles CBN and CNB. Dividing this by 2 works out both equal angles

This is the exterior angle of the 12-sided polygon therefore ABN must be a straight line

The exterior angle forms a straight line with one of the sides of the polygon

7

2 3

24

29 The equation of a curve is $y = x^2 - 18x + 70$

By completing the square, work out the coordinates of the turning point.
You **must** show your working.

[3 marks]

$y = (x-9)^2 + 70 - (-9)^2$

Completed the square by halving the coefficient of x to get -9, putting this in a bracket with x and squaring the bracket. Leaving the 70 on the outside and subtracting the square of the -9

The turning point occurs when the square bracket is equal to 0 as this is the minimum value a squared number can have. x - 9 = 0, so x = 9. When the square bracket is 0, y = 70 - (-9)^2 = -11

Answer (___9___ , ___-11___)

END OF QUESTIONS

3

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