

Please write clearly in block capitals.

Centre number

Candidate number

Surname \_\_\_\_\_

Forename(s) \_\_\_\_\_

Candidate signature \_\_\_\_\_

# GCSE MATHEMATICS

# H

Higher Tier          Paper 3 Calculator

Tuesday 11 June 2019

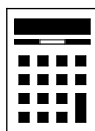
Morning

Time allowed: 1 hour 30 minutes

### Materials

For this paper you must have:

- a calculator
- mathematical instruments.



### Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
<b>TOTAL</b>	

### Advice

In all calculations, show clearly how you work out your answer.



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to [curtis@cgmaths.co.uk](mailto:curtis@cgmaths.co.uk)

Answer **all** questions in the spaces provided

- 1 Work out £1.50 as a fraction of 60p  
Circle your answer.

[1 mark]

$\frac{2}{5}$

$\frac{1}{4}$

$\frac{4}{1}$

$\frac{5}{2}$

£1.50 is 150p.  $150/60 = 5/2$

- 2 For a biased dice,  $P(6) = \frac{3}{5}$   
Circle the probability of two sixes when the dice is rolled twice.

[1 mark]

$\frac{6}{25}$

$\frac{6}{10}$

$\frac{9}{25}$

$\frac{9}{5}$

Six AND six so  $\frac{3}{5} \times \frac{3}{5}$

- 3 Circle the lowest common multiple (LCM) of 5, 15 and 25

[1 mark]

5

45

75

150

5 isn't a multiple of 15. 45 isn't a multiple of 25. 75 is the lowest of the numbers which is a multiple of 5, 15 and 25



- 4 Circle the **two** roots of  $(x - 5)(x + 3) = 0$

[1 mark]

-5

-3

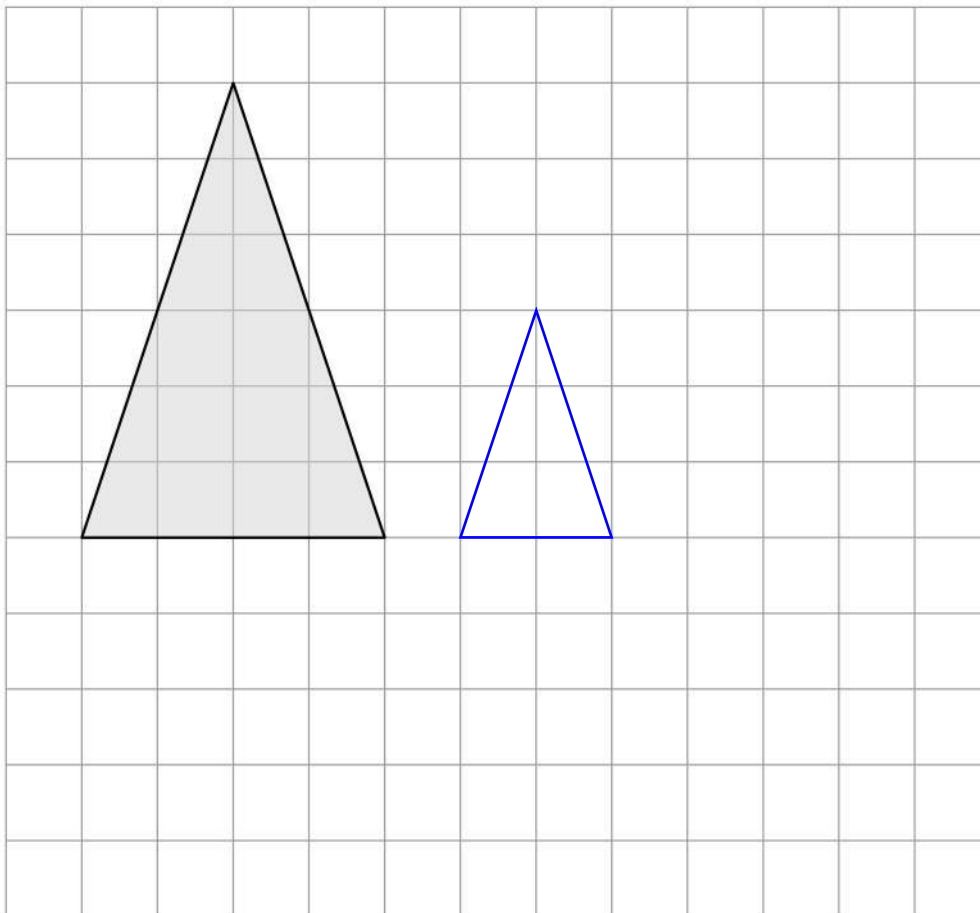
3

5

Either  $x - 5 = 0$  or  $x + 3 = 0$   
Rearranging these to make  $x$  the subject finds the roots

- 5 On the grid, draw an enlargement of the triangle with scale factor  $\frac{1}{2}$

[2 marks]



The base had a length of 4. Multiplying this by  $\frac{1}{2}$  works out that the new base should have a length of 2.

The height was 6. Multiplying this by  $\frac{1}{2}$  works out that the new height should be 3

Turn over ►



6

To the nearest pound, Jon has £9

To the nearest 50p, Ellie has £6.50

Work out the maximum possible total amount of money.

**[3 marks]**

$$(9 + \frac{1}{2} - 0.01) + (6.50 + \frac{0.50}{2} - 0.01)$$

Jon: to work out the maximum amount he has, add half of the resolution (£1 in this case as it is to the nearest pound) to £9 to get the upper bound. However, this amount isn't possible as it would round to £10 so £0.01 needs to be taken away.

Ellie: to work out the maximum amount she has, add half of the resolution (£0.50 in this case as it is to the nearest 50p) to £6.50 to get the upper bound. However, this amount isn't possible as it would round to £7 so £0.01 needs to be taken away.

Add the maximum amount for Jon and Ellie to get the maximum possible total amount of money

Answer £ 16.23



7 Two solids, J and K, have the same density.

Complete the table.

Include units in your answers.

[3 marks]

	J	K
Mass	48 g	78 g
Volume	8 cm <sup>3</sup>	13 cm <sup>3</sup>
Density	6 g/cm <sup>3</sup>	6 g/cm <sup>3</sup>

3.  $78/6 = 13$   
The unit of  
volume will be  
the same as for J

1.  $48/8 = 6$   
We have divided g by cm<sup>3</sup>  
so the unit will be g/cm<sup>3</sup>

2. J and K have  
the same density

$d = \frac{m}{v}$

From the formula triangle:  
Density = mass/volume  
Volume = mass/density

8 Rearrange  $y = 3x - 2$  to make  $x$  the subject.

Circle your answer.

[1 mark]

$$x = \frac{y}{3} - 2$$

$$x = \frac{y+2}{3}$$

$$x = \frac{y-2}{3}$$

$$x = \frac{y}{3} + 2$$

Add 2 to both sides then divide by 3  
on both sides to make x the subject



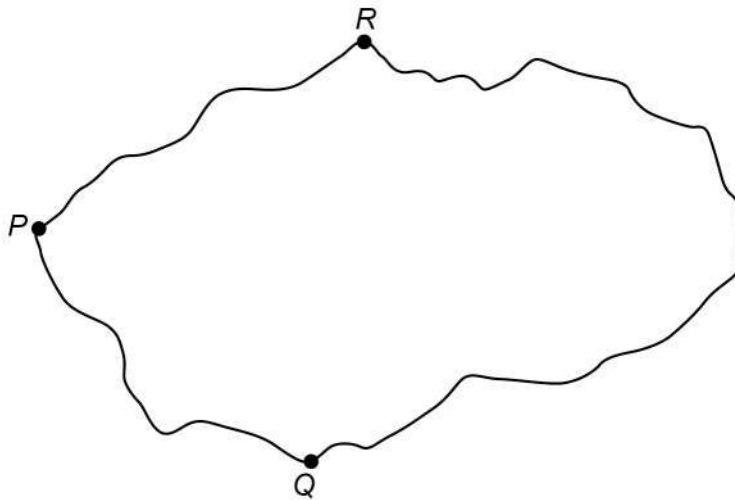
9

Towns  $P$ ,  $Q$  and  $R$  are connected by roads  $PQ$ ,  $PR$  and  $QR$ .

$PR$  is 10 km longer than  $PQ$ .

$QR$  is twice as long as  $PR$ .

The total length of the three roads is 170 km



Not drawn  
accurately

Work out the length of  $PQ$ .

[4 marks]

$$x + (x + 10) + (2(x + 10))$$

Let  $x$  be  $PQ$ .  $PR$  is  $x + 10$  and  $QR$  is  $2(x + 10)$ .  
Expressing the total length of the three roads

$$x + x + 10 + 2x + 20$$

Expanding the brackets and getting rid  
of the brackets which aren't needed

$$4x + 30 = 170$$

Collecting the like terms and simplifying. The  
expression of the total length must be equal to 170

$$x = \frac{170 - 30}{4}$$

Making  $x$ ,  $PQ$ , the subject by  
subtracting 30 then dividing by 4

Answer 35 km



10

Mia wants to borrow £6000 and repay it, with interest, after two years.  
She sees two offers for loans.

**Offer 1**  
Compound interest  
3% per year

**Offer 2**  
Compound interest  
First year 1%  
Second year 5%

Mia says,

“I will pay back the same amount because the average of 1% and 5% is 3%”

Is she correct?

You **must** show your working.

[3 marks]

$$6000 \times 1.03^2 = 6365.40$$

100% + 3% = 103%, which is 1.03 as a decimal (divide by 100 to convert a percentage to a decimal). So multiplying £6000 by 1.03 twice (or  $1.03^2$ ) increases it by 3% twice

$$6000 \times 1.01 \times 1.05 = 6363$$

100% + 1% = 101%, which is 1.01 as a decimal. 100% + 5% = 105%, which is 1.05 as a decimal. So multiplying £6000 by 1.01 then by 1.05 increases it by 1% then 5%

No

Offer 1 has a different amount to pay back than Offer 2

Turn over for the next question





11 Here are two sets of numbers, A and B.

Set A

200	160
104	100

Set B

270	400	483
300	$x$	

mean of Set A : mean of Set B = 3 : 8

Work out the value of  $x$ .

[4 marks]

$$\left( \frac{200 + 160 + 104 + 100}{4} \div 3 \right) \times 8 \times 5 - 270 - 400 - 483 - 300$$

Adding up all the numbers in Set A and dividing by 4 to work out the mean. 3 parts of the ratio represents the mean of Set A so dividing by 3 works out 1 part. Multiplying by 8 works out 8 parts, which represents the mean of Set B.

Mean = total/number, total = mean x number

So multiplying the mean of Set B by 5 (the number of numbers in Set B) works out the total of Set B. Subtracting the other numbers leaves  $x$

Answer 427



12

A straight line

has gradient 4

and

passes through the point (5, 23)

Work out the equation of the line.

Give your answer in the form  $y = mx + c$ **[3 marks]**

$$c = 23 - 4(5)$$

Rearranging  $y = mx + c$  to get  $c = y - mx$  then  
substituting 23 for  $y$ , 5 for  $x$  and 4 for  $m$  (the gradient)

$m$  is the gradient and  $c$  was found above

Answer

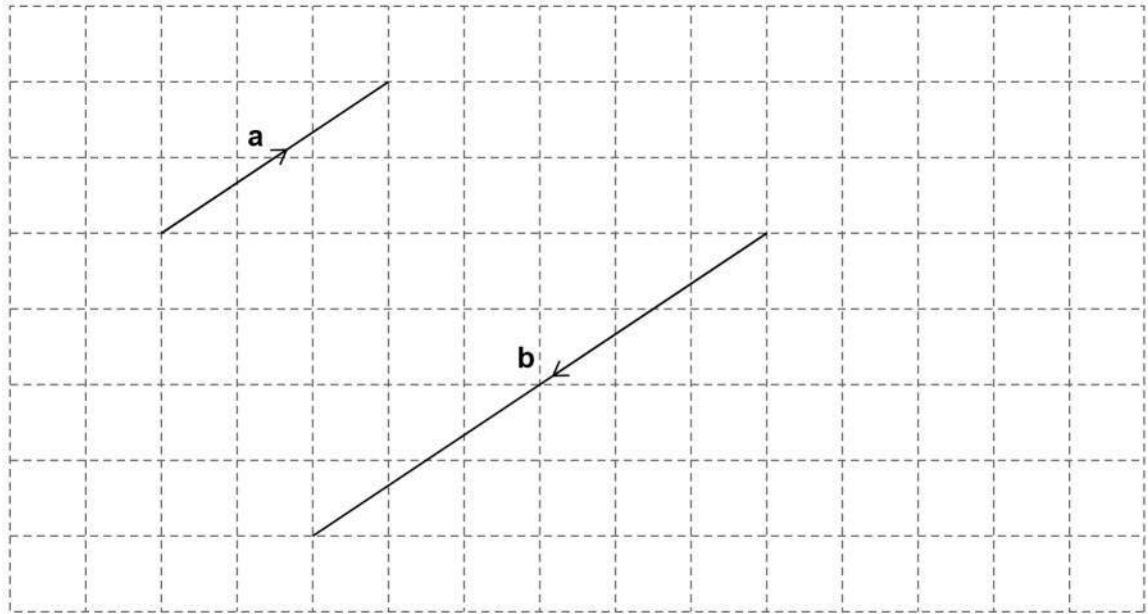
$$y = 4x + 3$$

Turn over for the next question

Turn over ►



13 (a) Vectors **a** and **b** are drawn on a grid.



Write **b** in terms of **a**.

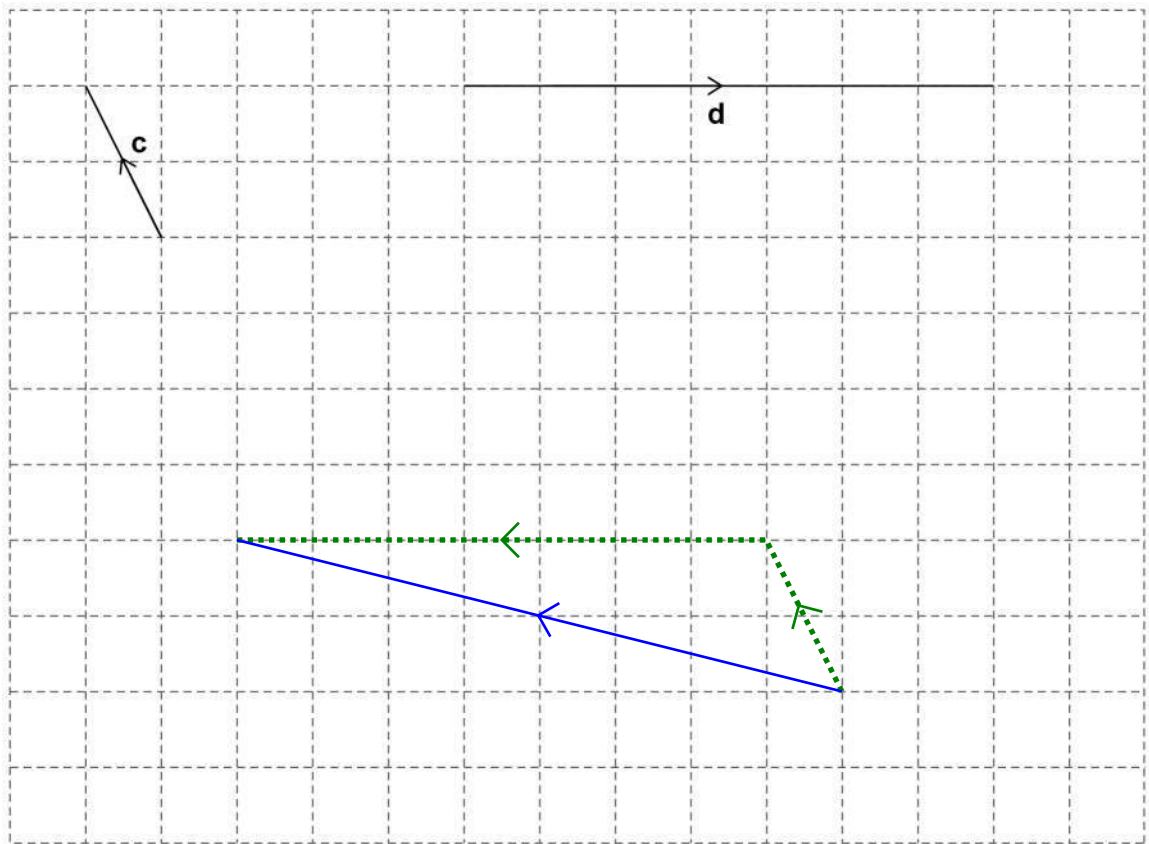
[1 mark]

$$\mathbf{b} = \underline{\hspace{2cm} -2\mathbf{a} \hspace{2cm}}$$

Vector **b** is twice as long as **a** but in the opposite direction



13 (b) Vectors  $c$  and  $d$  are drawn on a grid.



On the grid above, draw a vector representing  $c - d$

[2 marks]

Combining the vectors  $c$  and  $-d$  by imagining them drawn tip to tail.  $-d$  is the same length as  $d$  but in the opposite direction

Turn over for the next question



- 14 For Class X, number of boys : number of girls = 7 : 8  
For Class Y, number of boys : number of girls = 3 : 4

Which statement **must** be true?

Tick **one** box.

[1 mark]

Class X has more boys than class Y

Class X has twice as many girls as class Y

Class X has a greater proportion of boys than class Y

Class X has the same proportion of boys as class Y

The proportion of boys in Class X is  $7/(7 + 8) = 7/15 = 0.46\dots$   
The proportion of boys in Class Y is  $3/(3 + 4) = 3/7 = 0.42\dots$

- 15 Simplify fully  $\frac{a^3b^2}{cd} \times \frac{c}{ab^5}$

[3 marks]

$$\frac{a^3b^2c}{ab^5cd}$$

The fractions are multiplied by multiplying the numerators and denominators

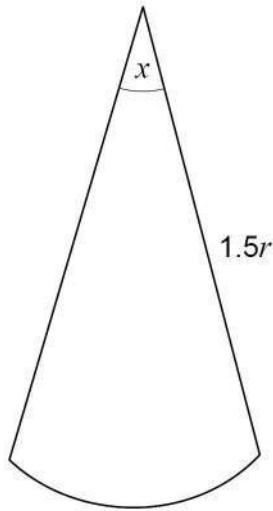
Cancel out common factors to simplify the fraction. Dividing both the numerator and denominator by a cancels out a from the denominator and  $a^3/a = a^2$ . Dividing both the numerator and denominator by  $b^2$  cancels out  $b^2$  from the numerator and  $b^5/b^2 = b^3$ . Dividing both the numerator and denominator by c cancels out c from both.

Answer  $\frac{a^2}{b^3d}$

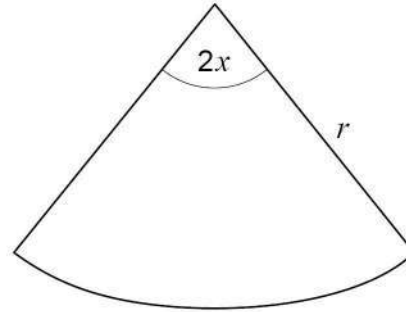


16 Here are two sectors from different circles.

Sector A



Sector B



Not drawn  
accurately

Which sector has the bigger area?

Tick a box.



Sector A



Sector B

Show working to support your answer.

[2 marks]

$$\frac{x}{360} \times \pi \times (1.5r)^2 = \frac{2.25}{360} x \pi r^2 \leftarrow \text{Area of Sector A}$$

$$\frac{2x}{360} \times \pi \times r^2 = \frac{2}{360} x \pi r^2 \leftarrow \text{Area of Sector B}$$

Area of a sector =  $\frac{a}{360} \times \pi r^2$ , where  $a$  is the angle of the sector and  $r$  is the radius.  
2.25/360 is greater than 2/360 and the rest of the expressions are the same



17

A factory makes kettles.

Four samples of kettles are tested for faults.

Each sample has size 200

Here are the relative frequencies of faulty kettles in the samples.

Sample	P	Q	R	S
Relative frequency	0.03	0.035	0.015	0.01

Work out the range of the number of faulty kettles in the four samples.

[3 marks]

$$200(0.035 - 0.01)$$

0.035 - 0.01 works out the range of the relative frequencies. This multiplied by the sample size of 200 kettles works out the range of the number of faulty kettles

Answer \_\_\_\_\_

5



18 (a) Write  $x(3x - 9) = 4$  in the form  $ax^2 + bx + c = 0$  where  $a$ ,  $b$  and  $c$  are integers.

[1 mark]

Expanding the brackets then subtracting 4 from both sides

Answer  $3x^2 - 9x - 4 = 0$

18 (b) Solve  $x(3x - 9) = 4$

Give your answers to 2 decimal places.

[2 marks]

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(3)(-4)}}{2(3)}$$

Using the quadratic formula on the answer to part (a).  $a = 3$ ,  $b = -9$  and  $c = -4$

Type the right side into the calculator exactly as it looks above, except use + instead of  $\pm$  for one of the solutions then go back and change the + to - for the other solution

Answer  $x = 3.39$ ,  $x = -0.39$

Turn over for the next question



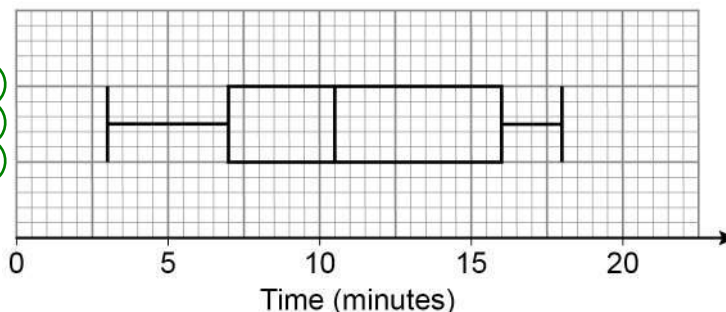


19 Here is some information about the times people took to complete a survey.

Fastest time	3 minutes
Slowest time	18 minutes
Median	11 minutes
Lower quartile	7 minutes
Interquartile range	8 minutes

Ben draws this box plot to show the information.

Time to complete a survey



The scale goes up 5 over 10 boxes.  $5/10 = 0.5$  so the scale is going up in 0.5 every small box

Make **two** criticisms of his box plot.

[2 marks]

Criticism 1 Median is at 10.5

Should be two boxes after 10, at 11

Criticism 2 Upper quartile should be at 15

As adding the interquartile range to the lower quartile works out the upper quartile.  $7 + 8 = 15$



20  $d$  is directly proportional to the square of  $v$ .

$$d = 6 \text{ when } v = 20$$

20 (a) Work out an equation connecting  $d$  and  $v$ .

[3 marks]

$$d = kV^2$$

$$k = \frac{6}{20^2}$$

$v^2$  can be multiplied by anything and still be directly proportional to  $d$

Rearranged to make  $k$  the subject and substituting in 6 for  $d$  and 20 for  $v$

Substituting 0.015 for  $k$  in the original equation

Answer  $d = 0.015v^2$

20 (b) Work out the value of  $d$  when  $v = 30$

[2 marks]

$$0.015 \times 30^2$$

Substituting 30 for  $v$  in the equation found in part (a)

Answer  $13.5$

Turn over for the next question



21 Hanif makes green paint by mixing blue paint and yellow paint in the ratio  
blue : yellow = 7 : 3

He buys blue paint in 50-litre containers, each costing £225

He buys yellow paint in 20-litre containers, each costing £80

He wants to

sell the green paint in 5-litre tins

make 40% profit on each tin.

How much should he sell each tin for?

[5 marks]

$$\left(\frac{7}{10} \times 5 \times \frac{225}{50} + \frac{3}{10} \times 5 \times \frac{80}{20}\right) \times 1.4$$

There are 10 parts in total in the ratio so  $\frac{7}{10}$  of the green paint is blue and  $\frac{3}{10}$  is yellow.

$\frac{7}{10} \times 5$  works out  $\frac{7}{10}$  of the 5 litres so is how many litres of blue paint is in each tin. Multiplying this by  $\frac{225}{50}$  (which is the cost for 1 litre of blue paint as the cost of £225 is divided by the 50 litres in each container) works out the cost of the blue paint needed for one tin of green paint.

$\frac{3}{10} \times 5$  works out  $\frac{3}{10}$  of the 5 litres so is how many litres of yellow paint is in each tin. Multiplying this by  $\frac{80}{20}$  (which is the cost for 1 litre of yellow paint as the cost of £80 is divided by the 20 litres in each container) works out the cost of the yellow paint needed for one tin of green paint.

Adding together the cost of the blue paint and yellow paint needed for one tin works out the cost of making one tin.  $100\% + 40\% = 140\%$ , which is 1.4 as a decimal so multiplying this by 1.4 works out how much he should sell each tin for after the profit is added on

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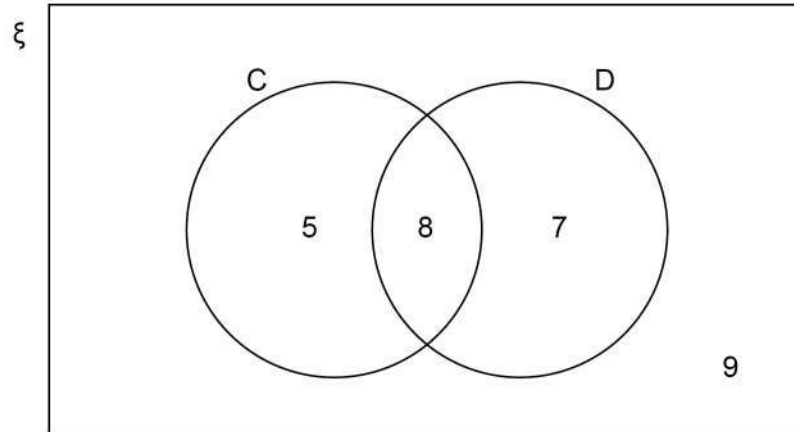


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Answer £ 30.45



- 22**  $\xi = 29$  students in a class  
 C = students who own a cat  
 D = students who own a dog



- 22 (a)** A student is chosen at random.  
 Circle the probability that the student owns a cat or a dog but not both.

[1 mark]

$$\frac{12}{29}$$

$$\frac{13}{29}$$

$$\frac{15}{29}$$

$$\frac{20}{29}$$

5 own a cat but not a dog. 7 own a dog but not a cat.  $5 + 7 = 12$ . There are 12 out of the 29 students who own a cat or a dog but not both

- 22 (b)** A student who owns a dog is chosen at random.  
 Circle the probability that the student also owns a cat.

[1 mark]

$$\frac{7}{15}$$

$$\frac{8}{15}$$

$$\frac{7}{29}$$

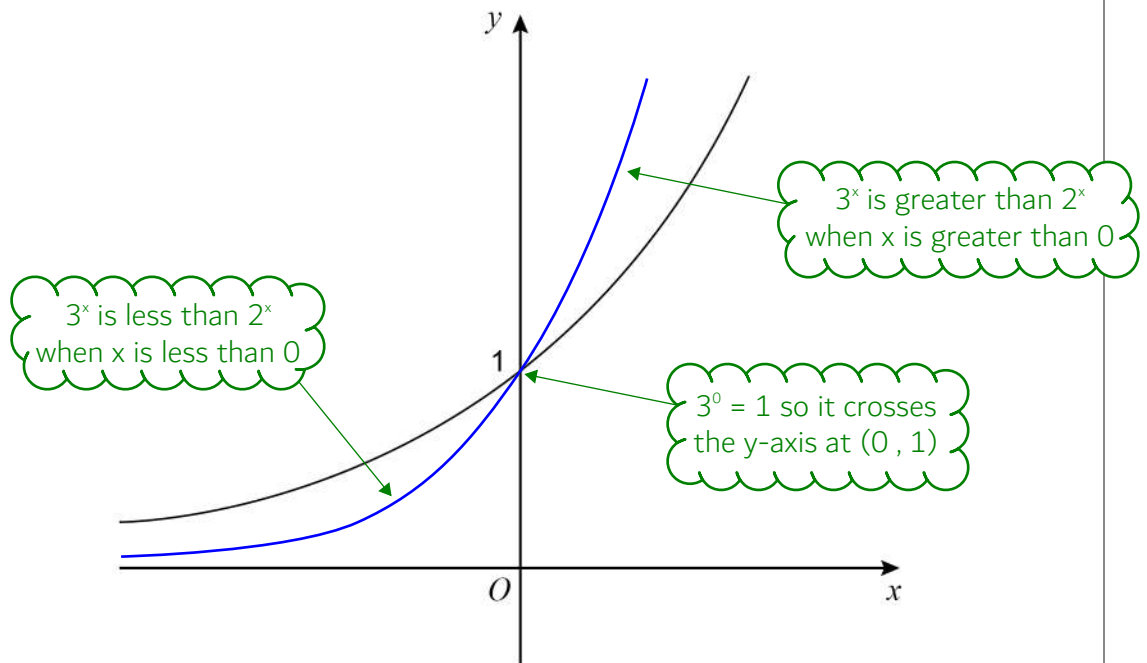
$$\frac{8}{29}$$

Out of the 15 who own a dog, 8 own a cat

Turn over ►



23

Here is a sketch of the curve  $y = 2^x$ On the axes above, sketch the curve  $y = 3^x$ **[2 marks]**

Use table mode on the calculator to create a table of values for  $f(x) = 2^x$  and  $g(x) = 3^x$ .

Press Menu then 3. Set  $f(x) = 2^x$  then press =. Set  $g(x) = 3^x$  then press =. Start: -5. End: 5. Step: 1

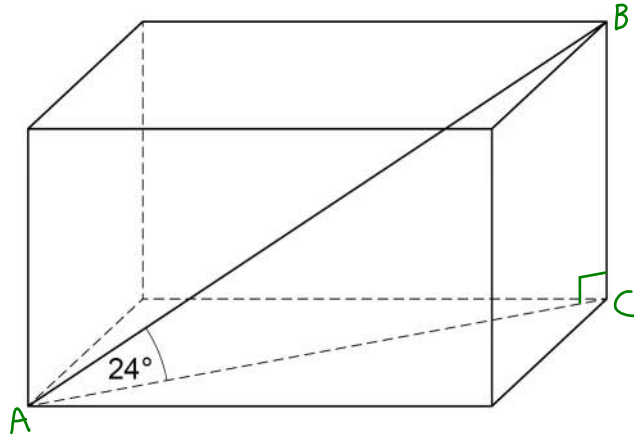


24

The length of a diagonal of a cuboid is 20 cm

The diagonal makes an angle of  $24^\circ$  with the base.

The area of the base is  $150 \text{ cm}^2$



Work out the volume of the cuboid.

[3 marks]

SOH CAHTOA

Using right angled trigonometry to find the length BC on the right angled triangle ABC. AB is the hypotenuse, which is ticked as we have it and BC is the opposite, which is ticked as we are trying to find it. This tells us we can use SOH as it has two ticks

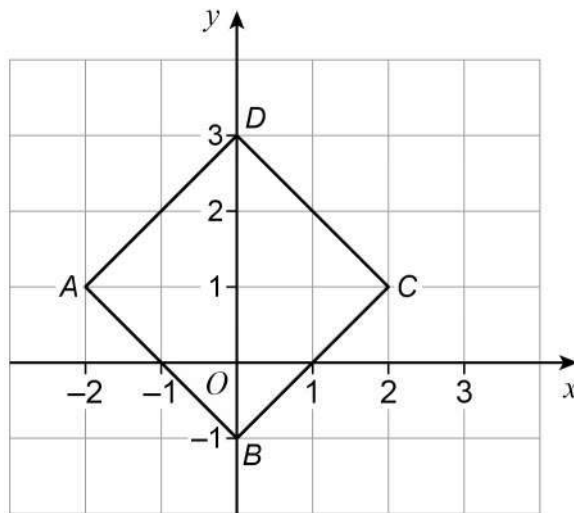
$150 \times \sin 24 \times 20$

The cuboid can be treated like a prism. The base is the cross section and has an area of 150. Multiplying this by the length BC gives the volume. BC is the opposite, which is equal to  $\sin$  of the angle  $\times$  hypotenuse (from the formula triangle)

Answer 1220  $\text{cm}^3$



25

 $ABCD$  is a square. $A$  is  $(-2, 1)$   $B$  is  $(0, -1)$   $C$  is  $(2, 1)$   $D$  is  $(0, 3)$ 25 (a) A **single** transformation of  $ABCD$  is such that $B$  is mapped to  $D$  $D$  is mapped to  $B$  $A$  and  $C$  are invariant points.

Describe fully the transformation.

**[2 marks]**Reflection in the line  $y = 1$ 

25 (b) A different **single** transformation of  $ABCD$  is such that

$B$  is mapped to  $D$

$D$  is mapped to  $B$

the only invariant point is  $(0, 1)$

Describe fully the transformation.

[3 marks]

Rotation by 180 degrees centre  $(0, 1)$

26  $g(x) = 16 - x$      $h(x) = x^3$

Solve  $gh(x) = 24$

[3 marks]

$$16 - x^3 = 24$$

$$x = \sqrt[3]{-8}$$

Substituting  $h(x)$  for  $x$  in  $g(x)$  to get the composite function  $gh(x)$ . Setting it equal to 24

Subtracting 16, flipping the signs (so positive becomes negative and negative becomes positive, which can be done by dividing both sides by  $-1$ ), then cube rooting both sides

$$x = \underline{\quad -2 \quad}$$

Turn over for the next question





27

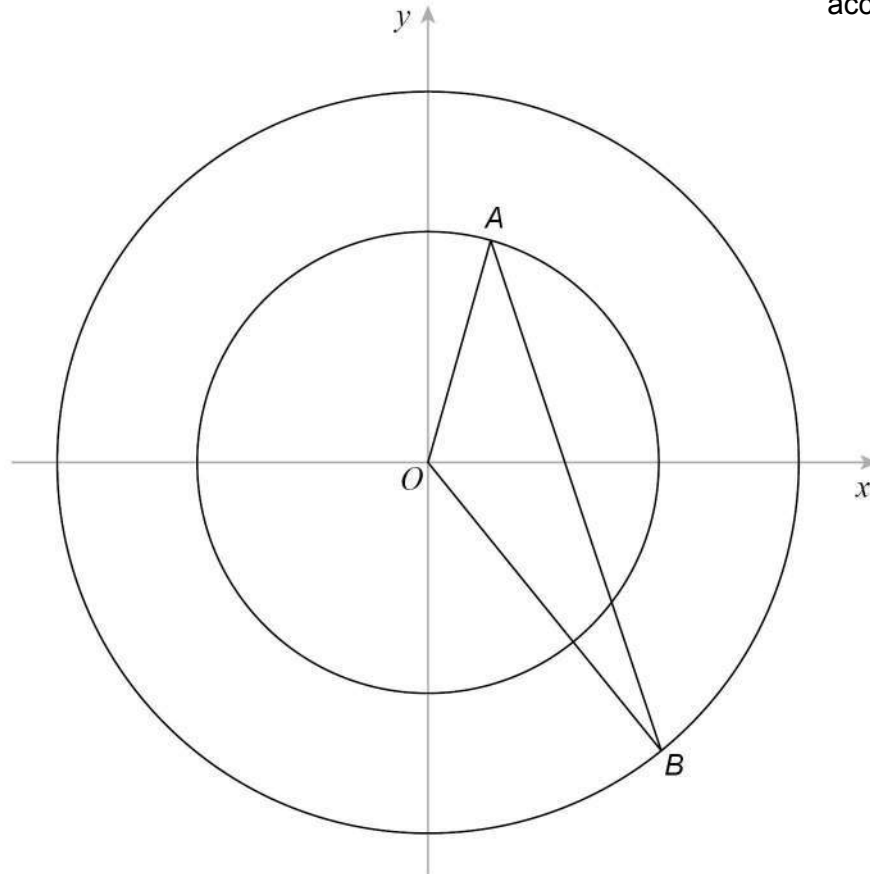
In this question, all lengths are in centimetres.

$A$  is a point on a circle, centre  $O$ .

$B$  is a point on a different circle, centre  $O$ .

$$AB = 20$$

Not drawn  
accurately



The equation of the larger circle is  $x^2 + y^2 = 144$

radius of smaller circle : radius of larger circle = 4 : 5



Work out the size of angle  $AOB$ .

[5 marks]

$$\sqrt{144} = 12$$

From the equation of the larger circle,  $x^2 + y^2 = r^2$ , where  $r$  is the radius. So the radius of the larger circle is 12

$$\frac{12}{5} \times 4 = 9.6$$

5 parts of the ratio represent the radius of the larger circle, 12. So dividing by 5 works out 1 part. Multiplying by 4 works out 4 parts, which represents the radius of the smaller circle

$$a^2 = b^2 + c^2 - 2bc \cos A$$

There aren't two opposite pairs of sides and angles so the sine rule can't be used. So the cosine rule has to be used

$$A = \cos^{-1} \left( \frac{a^2 - b^2 - c^2}{-2bc} \right)$$

Rearranging to make  $A$  the subject, which represents angle  $AOB$

$$= \cos^{-1} \left( \frac{20^2 - 12^2 - 9.6^2}{-2(12)(9.6)} \right)$$

$AB$  is opposite angle  $AOB$  so is set as  $a$ .  $b$  and  $c$  are the other sides of the triangle, which are the radii of the larger and smaller circles

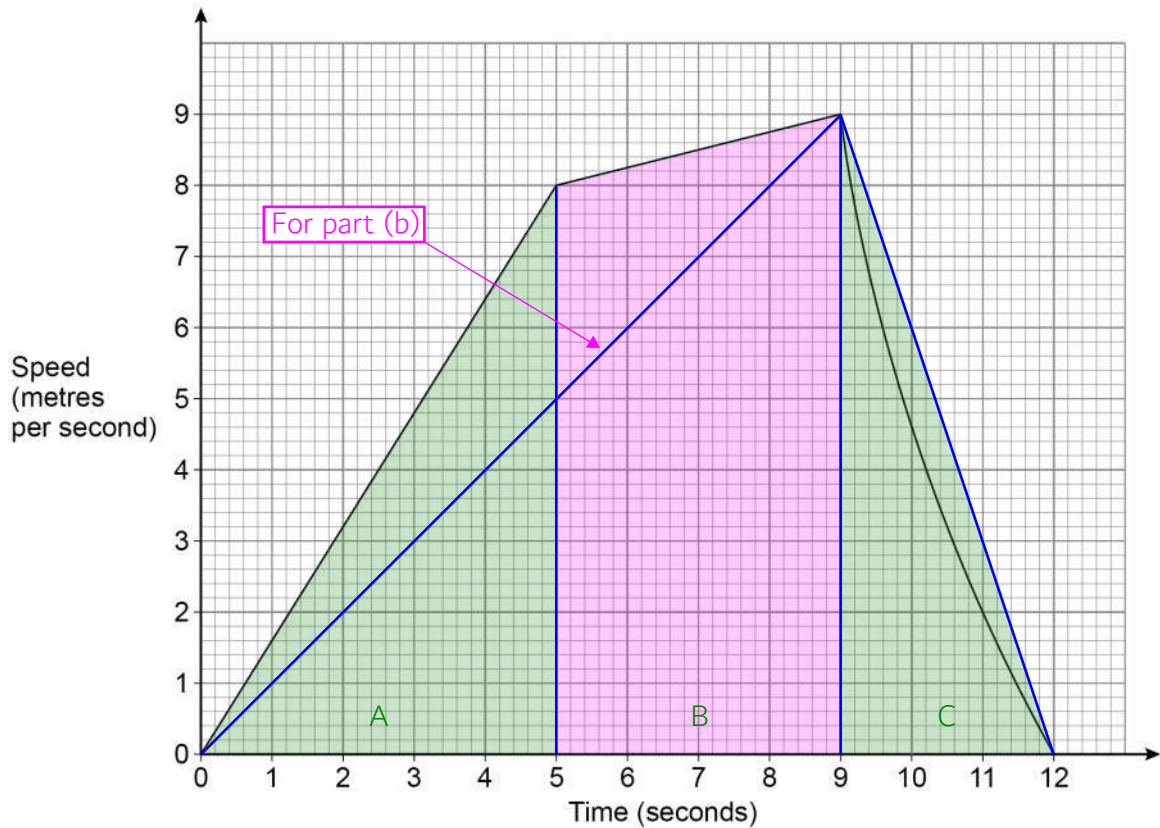
Answer 135 degrees

Turn over for the next question



28

Leo runs for 12 seconds.  
The graph shows his speed.



28 (a) Show that the distance he runs is less than 67.5 metres.

[4 marks]

$$\frac{1}{2} \times 5 \times 8 + \frac{1}{2} (8+9) \times 4 + \frac{1}{2} \times 3 \times 9 = 67.5$$

Area of triangle A =  $\frac{1}{2} \times \text{base} \times \text{height}$

Area of triangle C =  $\frac{1}{2} \times \text{base} \times \text{height}$

Area of trapezium B =  $\frac{1}{2} (a + b) \times h$ ,  
where a and b are the parallel sides  
and h is the distance between them

The distance is the area under the line on a speed-time graph. We have worked out an estimate of the area by using triangle C instead of the curve. As the area under the curve is less than the triangle, the distance must be less than 67.5



- 28 (b) Work out his average acceleration for the first 9 seconds.  
State the units of your answer.

[2 marks]

The average acceleration is the gradient of the straight line which runs from (0, 0) to (9, 9). Gradient = (change in y)/(change in x) =  $9/9 = 1$ . For the units, we have divided change in speed (m/s) by change in time (s).  $\text{m/s} \div \text{s} = \text{m/s}^2$

Answer 1 m/s<sup>2</sup>

END OF QUESTIONS

