Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

GCSE MATHEMATICS

H

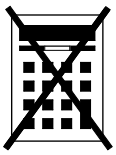
Higher Tier Paper 1 Non-Calculator

Thursday 25 May 2017 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- mathematical instruments.

You must **not** use a calculator.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

.CG Maths.

Answer **all** questions in the spaces provided

1 Simplify $2^5 \times 2^3$

Circle your answer.

https://youtu.be/9J6t2XASAIo

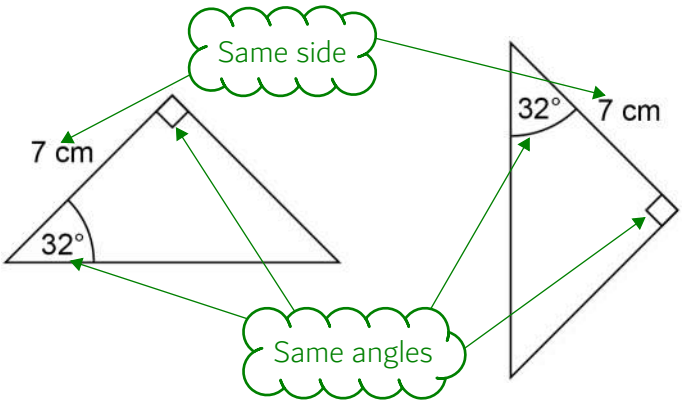
[1 mark]

4^8 2^8 2^{15} 4^{15}

$a^x \times a^y = a^{x+y}$

$2^5 \times 2^3 = 2^{5+3} = 2^8$

2



Not drawn accurately

Circle the reason why these triangles are congruent.

[1 mark]

SSS SAS ASA RHS

https://youtu.be/g2zOFlHYSU8

3 Which of these is a geometric progression?

Circle your answer.

[1 mark]

2, 4, 6, 8, 10 2, 3, 5, 8, 12

Each term is multiplied by 3 to get the next term → 2, 6, 18, 54, 162 2, 6, 10, 14, 18

https://youtu.be/toa4RKrQUDA

4 $a : b = 4 : 3$

Circle the correct statement.

[1 mark]

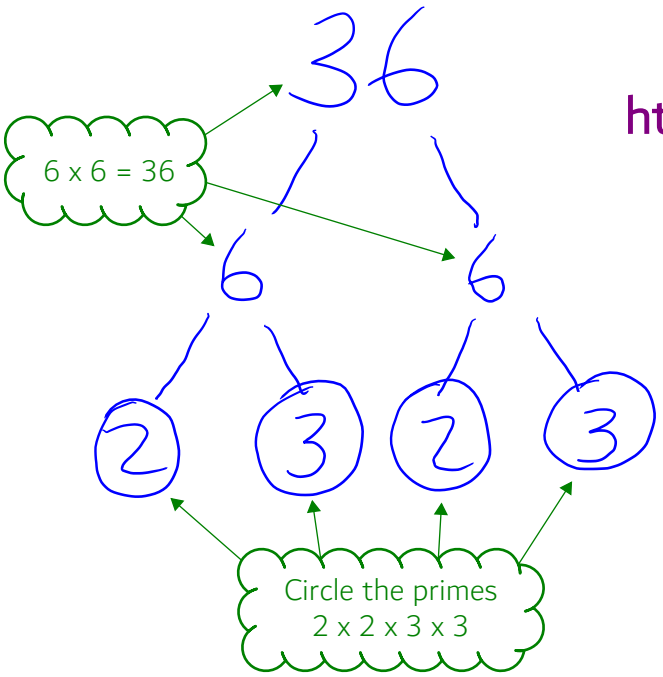
b is $\frac{4}{7}$ of a $\quad$ b is $\frac{3}{7}$ of a $\quad$ b is $\frac{4}{3}$ of a $\quad$ b is $\frac{3}{4}$ of a

a could be 4 and b could be 3. This is the only statement which works with these numbers as 3 is 3/4 of 4

5 Write 36 as a product of prime factors.

Give your answer in index form.

[3 marks]



https://youtu.be/bzXHjrgoTvY

Answer $2^2 \times 3^2$

Check:
$2^2 \times 3^2 = 4 \times 9 = 36$

Turn over for the next question

7

6 The table shows information about the times for 10 people to complete a task.

Time, t (minutes)	**Frequency**
$0 < t \leqslant 20$	1
$20 < t \leqslant 40$	6
$40 < t \leqslant 60$	3

These statements are about the mean and range of the actual times.

Tick the correct box for each statement.

[4 marks]

	True	**False**
The mean could be less than 20 minutes	☐	☐
The mean could be more than 40 minutes	☐	☐
The mean could be less than 40 minutes	☐	☐
The range could be more than 40 minutes	☐	☐
The range could be less than 40 minutes	☐	☐
The range could be more than 60 minutes	☐	☐

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$40 < t \leqslant 60$	3

These statements are about the mean and range of the actual times.

Tick the correct box for each statement.

[4 marks]

	True	False
The mean could be less than 20 minutes Lowest possible results: 0, 20, 20, 20, 20, 20, 20, 40, 40, 40 The mean of six 20s is 20. Three 40s weigh up the mean more than one 0 weighs it down.	☐	☒
The mean could be more than 40 minutes Highest possible results: 20, 40, 40, 40, 40, 40, 40, 60, 60, 60 The mean of six 40s is 40. Three 60s weigh up the mean more than one 20 weighs it down.	☒	☐
The mean could be less than 40 minutes Lowest possible results: 0, 20, 20, 20, 20, 20, 20, 40, 40, 40 The mean of three 40s is 40. Six 20s and one 0 all weigh the mean down.	☒	☐
The range could be more than 40 minutes Highest: 60, Lowest: 0 60 - 0 = 60 The range could be up (but not equal to) 60	☒	☐
The range could be less than 40 minutes Highest: 40, Lowest: 20 40 - 20 = 20	☒	☐
The range could be more than 60 minutes The range could be up (but not equal to) 60	☐	☒

7 $\frac{3}{5}$ of a number is 162

Work out the number.

[2 marks]

$\frac{3}{5}x = 162$

$x = 162 \div \frac{3}{5}$

$= 162 \times \frac{5}{3}$

Dividing by a fraction: multiply by the reciprocal

$3\overline{)162} = 54$

$54 \times 5 = 270$

Multiplying by a fraction: divide by the denominator then multiply by the numerator

Answer 270

https://youtu.be/wdG1sIbsTzQ

8 x km/h = y mph

Use 8 km/h = 5 mph to write a formula for y in terms of x.

[2 marks]

x needs to be divided by 8 to work out how many lots of 8km/h it is. Then multiplying by 5 as every lot of 8km/h is a lot of 5mph

Answer $y = \frac{x}{8} \times 5$

https://youtu.be/DIl3X0Sy_CU

Turn over for the next question

8

9 (a) Density = $\frac{\text{mass} \times 6}{\text{volume} \times 3} = \frac{m}{V} \times \frac{6}{3}$ $\frac{6}{3} = 2$

The mass of solid A is 6 times the mass of solid B.

The volume of solid A is 3 times the volume of solid B.

Complete the sentence.

[1 mark]

The density of solid A is 2 times the density of solid B.

9 (b) Average speed = $\frac{\text{distance} \div 2}{\text{time} \times 2}$ Dividing by twice the amount is basically dividing by 2 ÷2 then ÷2

If the distance is halved and the time is doubled, what happens to the average speed?

Circle your answer.

[1 mark]

× 2 × 4 no change ÷ 2 (÷ 4)

https://youtu.be/7YGLTVslGGw

10 Solve the simultaneous equations.

$2x + y = 18$ — ①

$x - y = 6$ — ②

[3 marks]

2x+x
y+-y=0

18+6

① + ② $3x = 24$

$x = 8$

$\frac{24}{3}$

2x
2x8

$16 + y = 18$

$y = 2$

16 + 2 = 18

Answer $x = 8$, $y = 2$

Turn over for the next question

5

11 Billy wants to buy these tickets for a show.

4 adult tickets at £15 each

2 child tickets at £10 each

A 10% booking fee is added to the ticket price.

3% is then added for paying by credit card.

Work out the **total** charge for these tickets when paying by credit card.

[5 marks]

$4 \times 15 + 2 \times 10 = 60 + 20 = 80$

$80 \div 10 = 8$

$80 + 8 = 88$

$(88 \div 100) \times 3 = 0.88 \times 3 = 2.64$

$88 + 2.64$

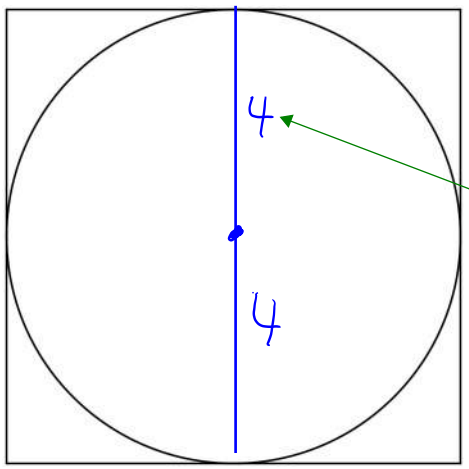
$$\begin{array}{r} 0.88 \\ \times \quad 3 \\ \hline 2.64 \\ {\scriptstyle 2} \end{array}$$

Answer £ 90.64

https://youtu.be/ibGT8CkJFHI

12 Here is a circle touching a square.

$\sqrt{64} = 8$



Not drawn accurately

The length of the sides on the square → 8

The radius is half of the length of the square

$\frac{8}{2} = 4$

The area of the square is 64 cm^2

Work out the area of the circle.

Give your answer in terms of π.

[3 marks]

πr^2

$\pi \times 4^2$

Answer 16π cm^2

https://youtu.be/GGVClNMZ698

Turn over for the next question

8

13 Write the number six million five thousand two hundred in standard form.

[2 marks]

6005200

Its easier to start writing 200, then 5000 first.

Divided by 10 6 times to get an appropriate decimal so it needs multiplying by 10^6 to keep the value the same.

Answer 6.0052×10^6

https://youtu.be/PM-CgGIVTxE

14 Solve $-3x > 6$

[1 mark]

Divide both sides by -3. When multiplying or dividing by a negative number, flip the inequality sign. e.g. 3 > 2, but -3 > -2 isn't true.

Answer $x < -2$

https://youtu.be/lXj30cFLir4

15 $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$ and $\frac{1}{9}$ are four fractions.

How many of these fractions convert to a recurring decimal?

Circle your answer.

[1 mark]

1/8 is the only one which doesn't recur as it terminates

0 1 2 (3) 4

0.125 $8\overline{)1.0^{2}0^{4}0}$

$0.\dot{1}4285\dot{7}$ $7\overline{)1.0^{3}0^{2}0^{6}0^{4}0^{5}0}$

$0.1\dot{6}$ $6\overline{)1.0^{4}0^{4}0}$

$0.\dot{1}$ $9\overline{)1.0^{1}0}$

https://youtu.be/Z389RFZFr08

16 A fair spinner has five equal sections numbered 1, 2, 3, 4 and 5

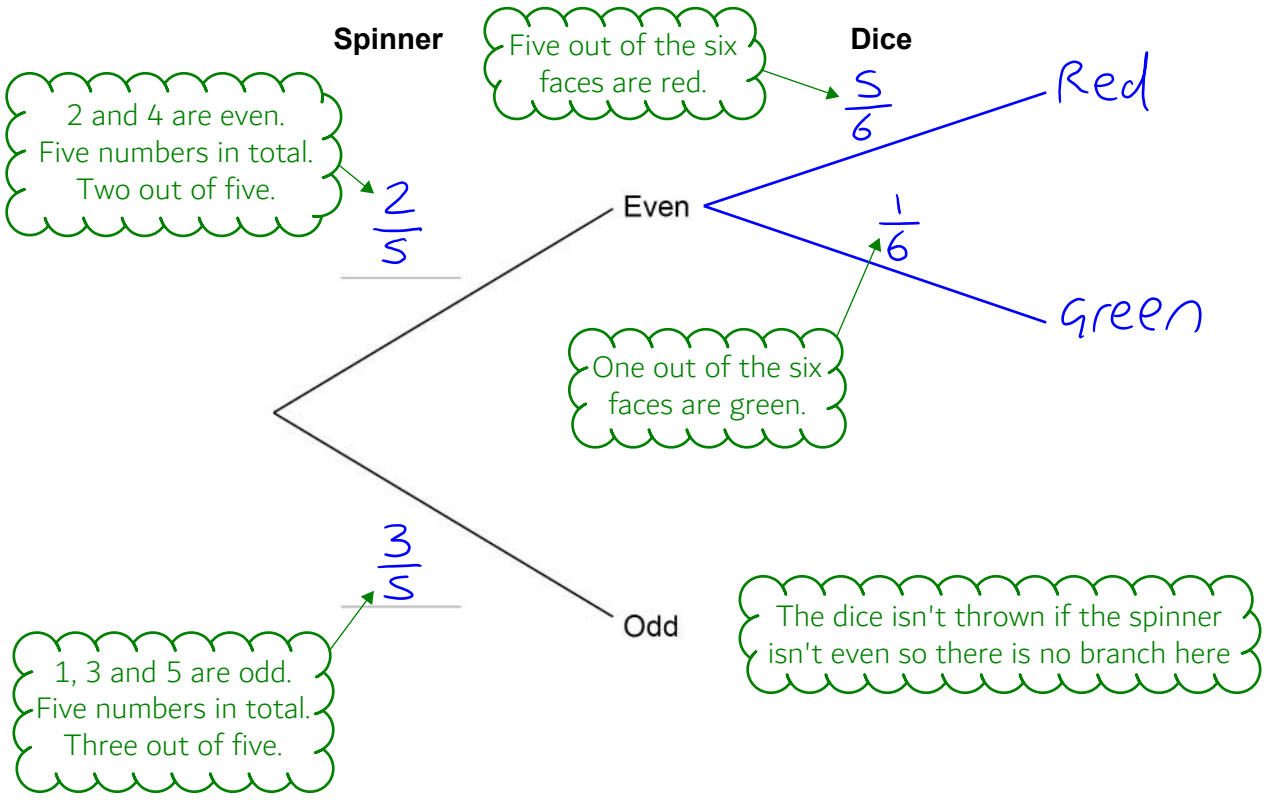
A fair six-sided dice has five red faces and one green face.

The spinner is spun.

If the spinner shows an even number, the dice is thrown.

16 (a) Complete the tree diagram for the spinner and the dice.

[2 marks]



16 (b) Work out the probability of getting an even number and the colour green.

[2 marks]

$\frac{2}{5} \times \frac{1}{6}$

'And' means multiply.

Answer $\frac{2}{30}$

https://youtu.be/BlPwFUAqVUo

8

17 *A* is the point (2, –5)

B is the point (4, –9)

17 (a) Show that the gradient of the straight line passing through *A* and *B* is –2

[2 marks]

$\frac{-5-(-9)}{2-4} = \frac{4}{-2} = -2$

Gradient = (change in y)/(change in x)
Change in y = (y coordinate of A) - (y coordinate of B)
Change in x = (x coordinate of A) - (x coordinate of B)

17 (b) *C* is the point (–301, 601)

Does *C* lie on the straight line passing through *A* and *B*?

You **must** show your working.

[2 marks]

$\frac{601-(-5)}{-301-2} = \frac{606}{-303} = -2$

C must lie on the straight line as both AB and AC have the same gradient and both go through point A

Answer yes

https://youtu.be/NokubQWwhgM

18 Bottles of drink are for sale at three shops.

The normal price of a bottle is the same at each shop.

Shop A	**Shop B**	**Shop C**
Buy 1 bottle Get 2 more bottles at half price	Buy 2 bottles Get 3 more bottles at half price	30% off a bottle

What is the cheapest way to buy **exactly** 8 bottles?

You can buy from more than one shop.

You **must** show your working.

Let x be the normal cost of a bottle.

[3 marks]

Shop A : $x + \frac{1}{2}x + \frac{1}{2}x = 2x$

$\frac{2x}{3} = 0.6x$

Dividing the total cost by the number of bottles gives the average cost of one bottle

Shop B : $x + x + \frac{1}{2}x + \frac{1}{2}x + \frac{1}{2}x = \frac{7}{2}x$

$\frac{7}{2}x \div 5 = \frac{7}{10}x = 0.7x$

Shop C : 70% of $x = \frac{7}{10}x = 0.7x$

It is the cheapest in Shop A so we want to by as many as possible, however we have to buy them in lots of three. 2 x 3 = 6 so we can by six bottles. 8 - 6 = 2 so we need to buy two more bottles.

2 from A : $x + \frac{1}{2}x = \frac{3}{2}x$ $\frac{3}{2}x \div 2 = \frac{3}{4}x = 0.75x$

Answer 6 From A, 2 From C

It is cheapest to buy two more bottles from Shop C (we have to buy in lots of five from Shop B).

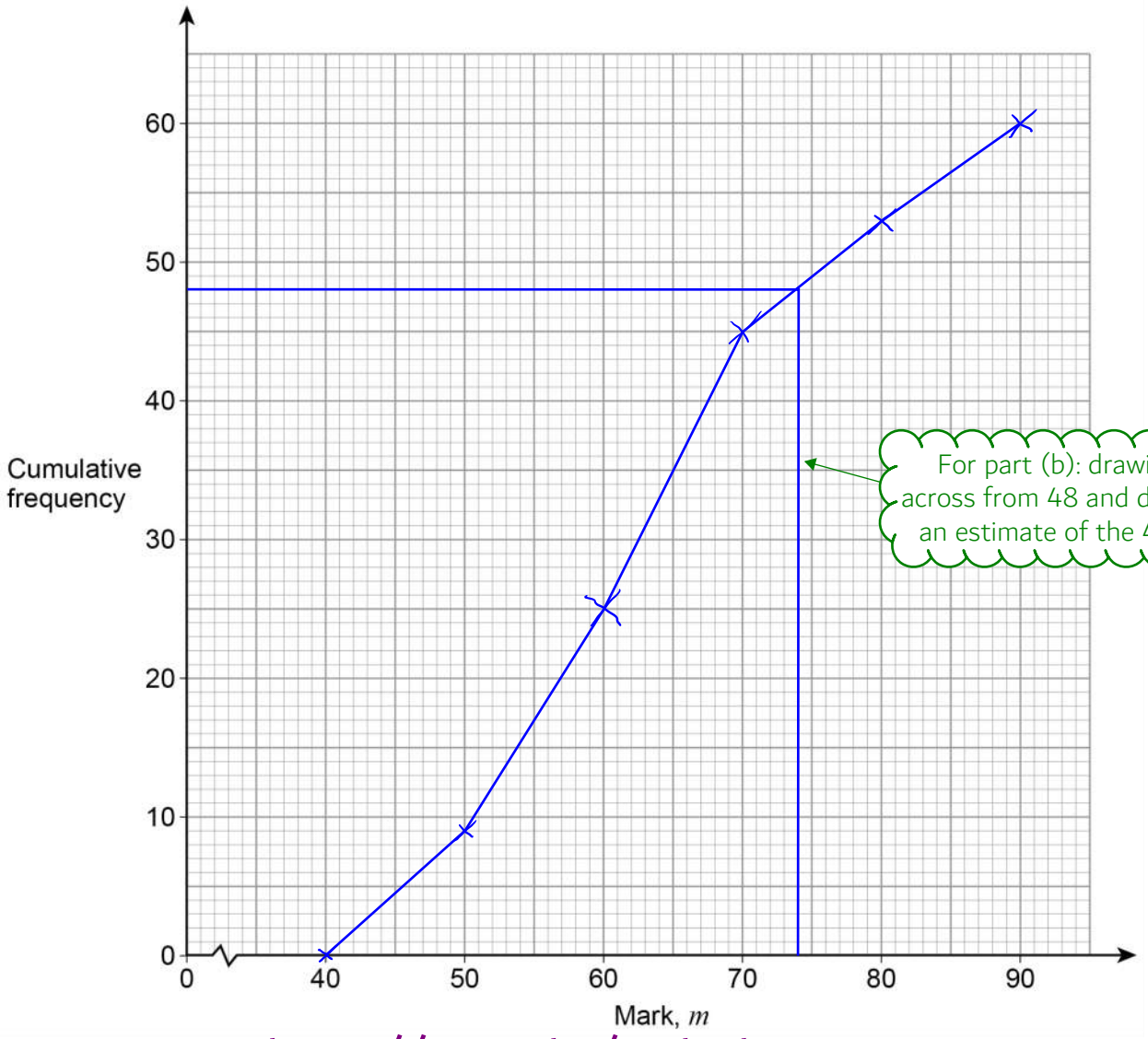
7

19 Here is some information about the marks of 60 students in a test.

Mark, m	Frequency	Cumulative Frequency
$40 < m \leqslant 50$	9	9
$50 < m \leqslant 60$	16	25 ← 9 + 16
$60 < m \leqslant 70$	20	45 ← 25 + 20
$70 < m \leqslant 80$	8	53 ← 45 + 8
$80 < m \leqslant 90$	7	60 ← 53 + 7

19 (a) On the grid, draw a cumulative frequency graph.

[3 marks]



https://youtu.be/jvBbOlnXmG0

19 (b) Use your graph to estimate the lowest mark of the top 20% of students.

[2 marks]

10% of 60 = 6

20% of 60 = 12

60 - 12 = 48

There are 60 students in total. Calculating how many students are in the top 20%.

The 48th value on the graph is an estimate for the lowest mark

Answer 74

https://youtu.be/jvBbOlnXmG0

20 Work out the diameter of the circle $x^2 + y^2 = 64$

Circle your answer.

[1 mark]

8 (16) 32 128

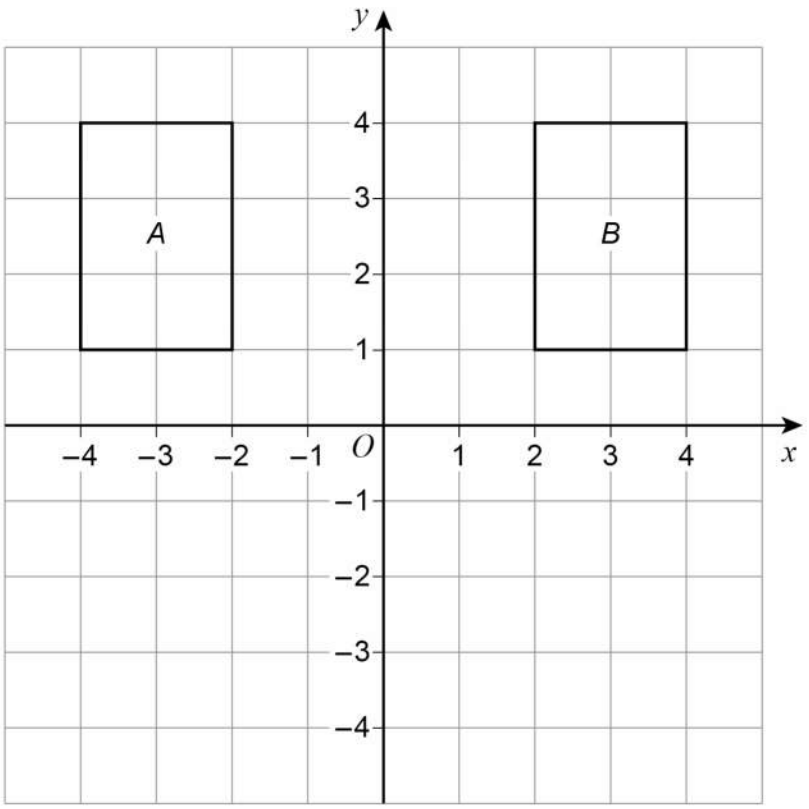
Equation of a circle: $x^2 + y^2 = r^2$, where r is the radius.
$r^2 = 64$, $r = 8$. Diameter is double the radius. 8 x 2 = 16

Turn over for the next question

https://youtu.be/u2ofqywtLs8

6

21 (a) The diagram shows rectangles A and B.



Rectangle A can be mapped to rectangle B by a **single** transformation.

Javed says,

"The **only** single transformation is a reflection in the y-axis because the rectangles are on opposite sides of the y-axis."

Is he correct?

Tick a box.

Yes ☐ No ☒

Give a reason for your answer.

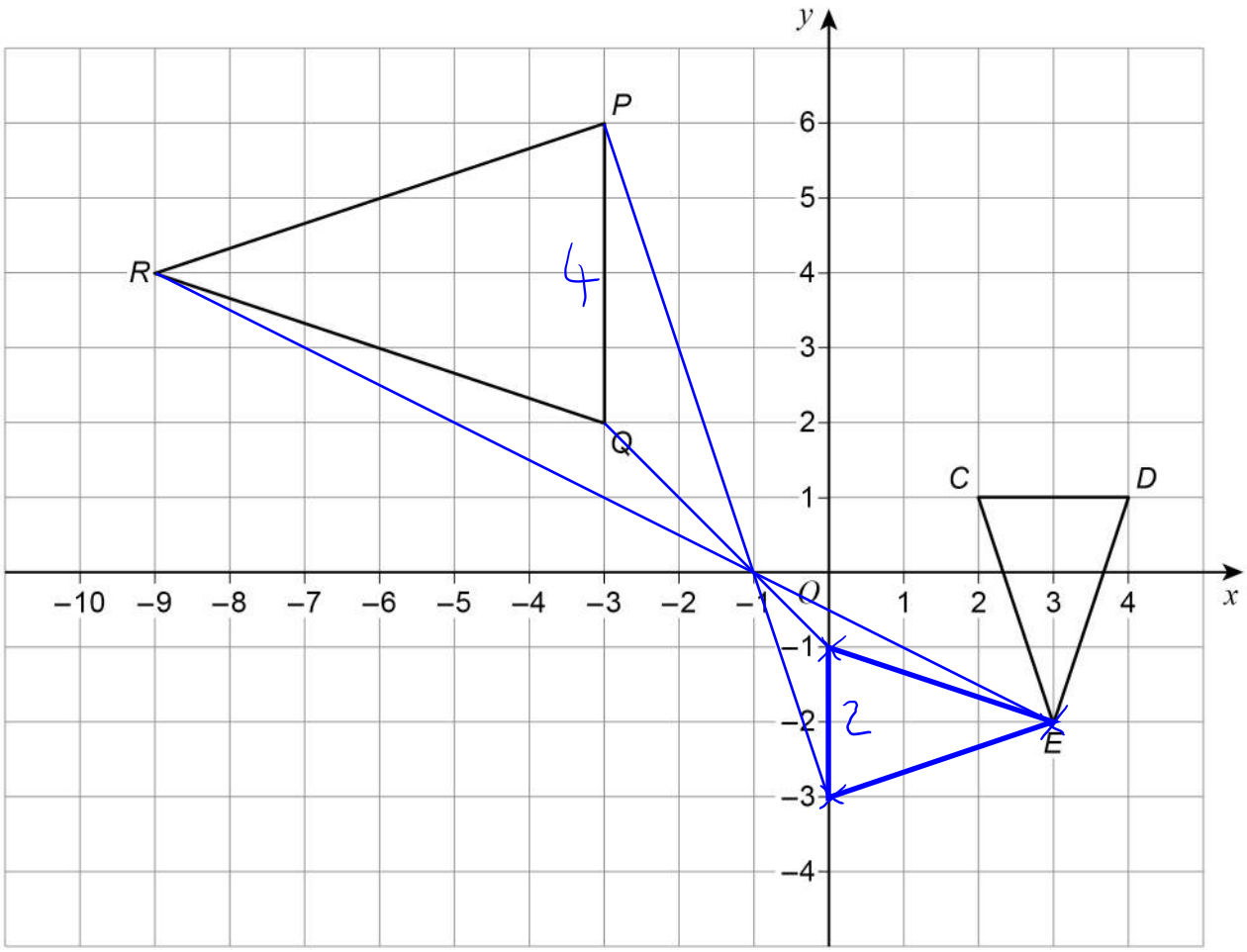
[1 mark]

It can be traslated by $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

Moved 6 to the right

https://youtu.be/cYJWIs9x1Ww

21 (b) This diagram shows triangles *CDE* and *PQR*.



CDE is mapped to *PQR* by combining two single transformations.

The first is a rotation of 90° anticlockwise about *E*.

Describe fully the second transformation.

[3 marks]

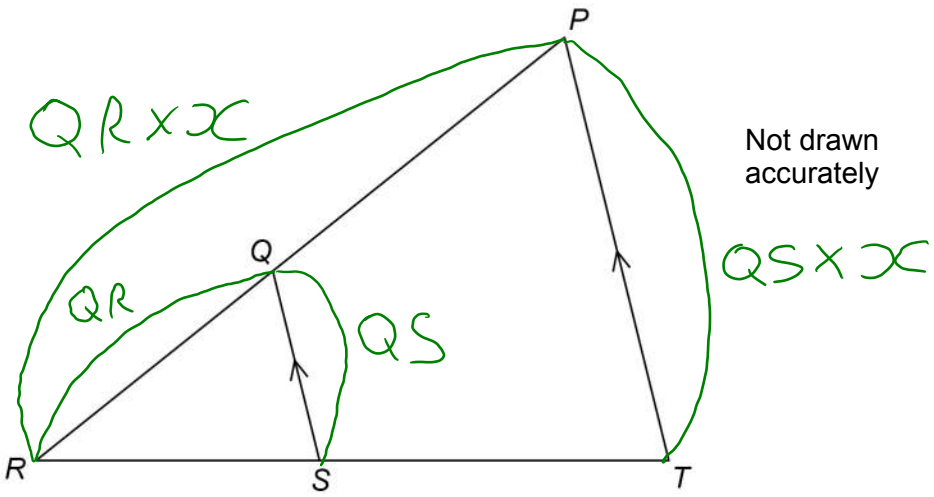
Enlargement, scale factor -2, centre (-1, 0)

https://youtu.be/r1Ot07h70y0

Turn over for the next question

4

22 *PRT* and *QRS* are similar triangles.



Not drawn accurately

Which of these is equivalent to $\frac{QR}{PR}$?

Circle your answer.

QR is the shorter side and is being divided by a longer side, PR, which is a scaled version of QR.

[1 mark]

$\frac{RS}{ST}$ $\frac{QS}{PT}$

$\frac{PT}{QS}$ $\frac{RT}{RS}$

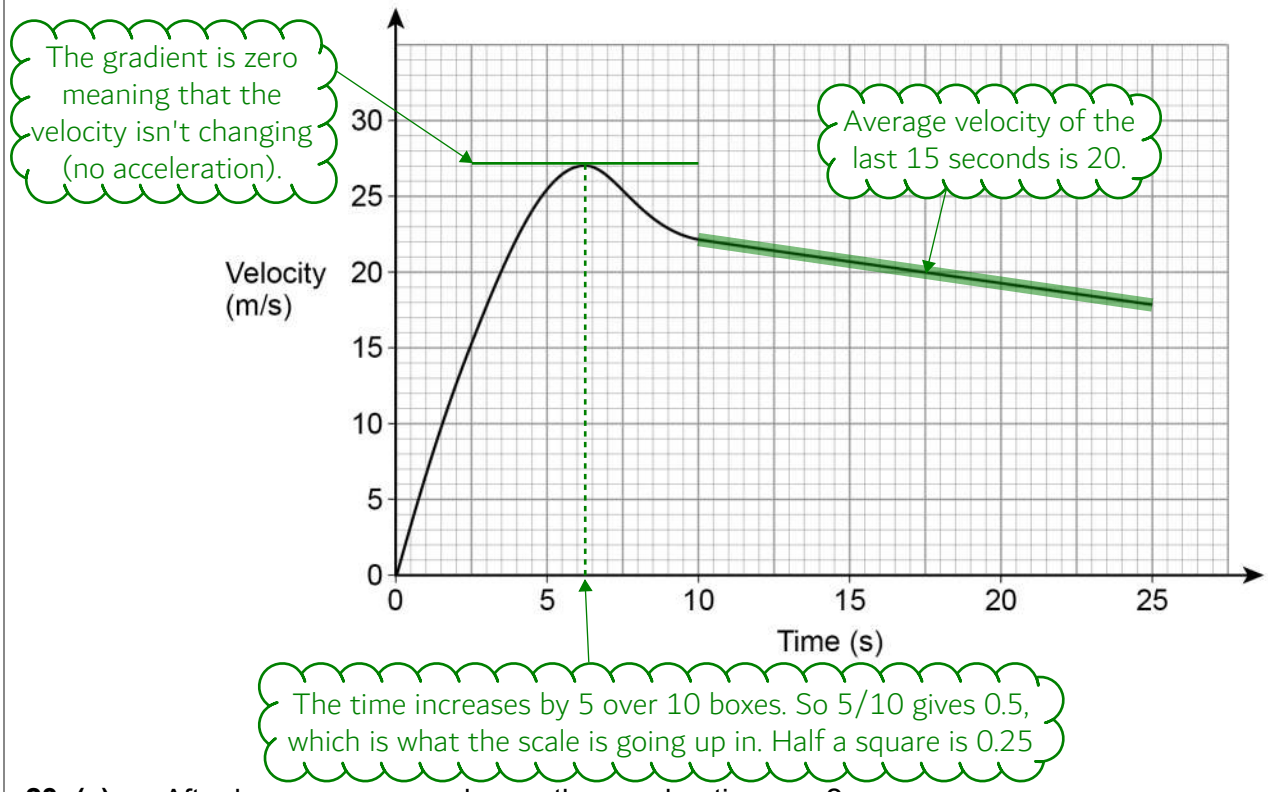
QS is the shorter side and is being divided by a longer side, PT, which is a scaled version of QS.

$\frac{QR}{PR} = \frac{QR}{QR \times x} = \frac{1}{x}$

$\frac{QS}{PT} = \frac{QS}{QS \times x} = \frac{1}{x}$

https://youtu.be/Yslj6TsFAzE

23 Here is a velocity-time graph of a motorbike for 25 seconds.



23 (a) After how many seconds was the acceleration zero?

[1 mark]

Answer 6.25 seconds

23 (b) Work out the distance travelled in the last 15 seconds.

[2 marks]

$d = st$ ← Distance = average speed x time

$d = 20 \times 15$

Answer 300 metres

https://youtu.be/GVtJB-Qx0Sw

4

24 (a) Work out $\sqrt{12\frac{1}{4}}$ as an improper fraction.

[1 mark]

$\sqrt{\frac{48}{4}+\frac{1}{4}}=\sqrt{\frac{49}{4}}=\frac{\sqrt{49}}{\sqrt{4}}$

Answer $\frac{7}{2}$

24 (b) Work out $\sqrt[3]{16}$ as a power of 2

$\sqrt[y]{a^x}=a^{\frac{x}{y}}$

[2 marks]

$16=2\times2\times2\times2=2^4$

Answer $2^{\frac{4}{3}}$

https://youtu.be/szGErNmKkRE

25 In an office there are twice as many females as males.

$\frac{1}{4}$ of the females wear glasses.

$\frac{3}{8}$ of the males wear glasses.

84 people in the office wear glasses.

Let x = total number of people.

Work out the number of people in the office.

[4 marks]

Number of males $= \frac{1}{3}x$, females $= \frac{2}{3}x$

$\frac{1}{4} \times \frac{2}{3}x + \frac{3}{8} \times \frac{1}{3}x = 84$

$\frac{2}{24}x = 84$

$x = 84 \div \frac{2}{24} = 84 \times \frac{24}{2}$

$\frac{1}{4} \times \frac{2}{3}x = \frac{2}{12}x$

$\frac{2}{12}x = \frac{4}{24}x$

$\frac{3}{8} \times \frac{1}{3}x = \frac{3}{24}x$

$\frac{4}{24}x + \frac{3}{24}x = \frac{7}{24}x$

$84 \div 7 = 12$

$12 \times 24 = 12 \times 12 \times 2$

$= 288$

Answer 288

https://youtu.be/YOlxLc-TcmE

Turn over for the next question

7

26 Expand and simplify $(x - 4)(2x + 3y)^2$

[4 marks]

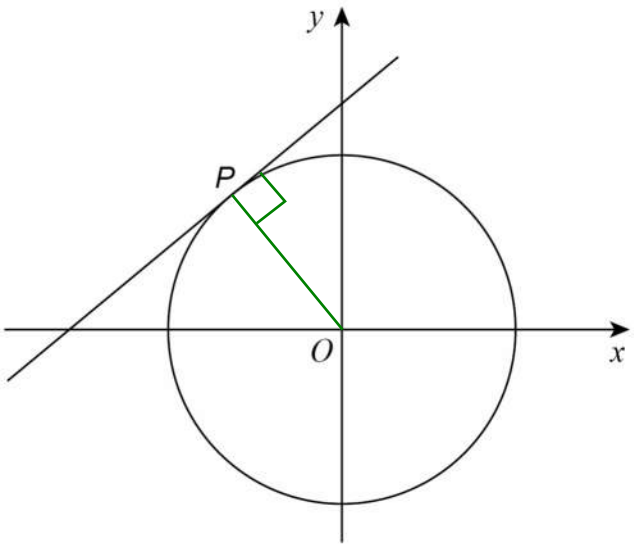
$(x-4)(4x^2+12xy+9y^2)$

Expand out the square bracket first using 'square the first term, double the product of the two terms, square the last term'. Then expand out with the first bracket.

Answer $4x^3+12x^2y+9xy^2-16x^2-48xy-36y^2$

https://youtu.be/eJYo9We1zuU

27 P (–1, 4) is a point on a circle, centre O



Not drawn accurately

Work out the equation of the tangent to the circle at P.

Give your answer in the form $y = mx + c$

The general equation of a straight line is y = mx + c, where m is the gradient and c is the y-intercept.

[4 marks]

Circle theorem: the angle between a tangent and a radius is always 90°. The tangent is perpendicular to the radius so its gradient, m, is the negative reciprocal of the gradient of the radius. c = y - mx and point P lies on the tangent.

gradient of radius $= \frac{0-4}{0--1} = -4$

$m = \frac{1}{4}$ The gradient m is the negative reciprocal of -4

$\frac{up}{across} = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}$

$c = 4 - \frac{1}{4} \times -1 = \frac{17}{4}$

P(-1, 4) means x = -1 and y = 4 at this point on the tangent. It must satisfy the equation.

$\frac{16}{4} - -\frac{1}{4} = \frac{16}{4} + \frac{1}{4}$

Answer $y = \frac{1}{4}x + \frac{17}{4}$

8

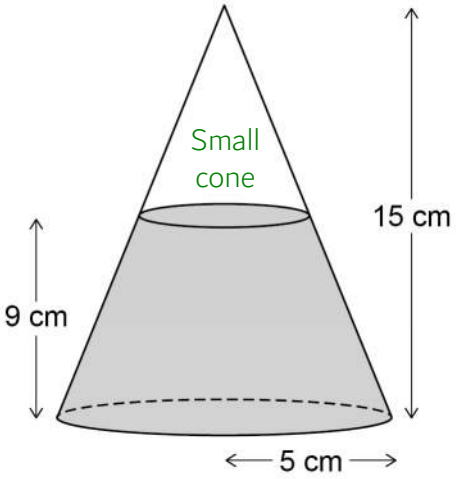
28 Volume of cone = $\frac{1}{3}\pi r^2 h$ where r is the radius and h is the perpendicular height.

A cone has a

horizontal base of radius 5 cm

height of 15 cm

The cone contains water to a depth of 9 cm



The water is 9/15 of the way up the cone, or 6/15 of the way from the top.

Work out the volume of the water, in cm^3

Give your answer in terms of π.

[4 marks]

Volume of water = volume of large cone - volume of small cone

$\frac{1}{3}\pi(5)^2(15) - \frac{1}{3}\pi(2)^2(6)$

$125\pi - 8\pi$

15 − 9 = 6

r is $\frac{6}{15}$ of 5, $\frac{6}{15} \times 5 = \frac{2}{5} \times 5 = 2$

$\frac{1}{3} \times \pi \times 5^2 \times 15$

$\frac{1}{3} \times 15 \times 25 \times \pi$

$5 \times 25 \times \pi = 125\pi$

$\frac{1}{3} \times \pi \times 2^2 \times 6$

$\frac{1}{3} \times 6 \times 4 \times \pi$

$2 \times 4 \times \pi = 8\pi$

Answer 117π cm^3

29 Simplify $\dfrac{2\sin 45^\circ - \tan 45^\circ}{4\tan 60^\circ}$

Give your answer in the form $\dfrac{\sqrt{a}-\sqrt{b}}{c}$ where a, b and c are integers.

[4 marks]

$\dfrac{2\left(\frac{\sqrt{2}}{2}\right)-(1)}{4(\sqrt{3})} = \dfrac{\sqrt{2}-1}{4\sqrt{3}}$

$\dfrac{\sqrt{2}-1}{4\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$ ← Rationalise the denominator.

$\dfrac{(\sqrt{2}-1)\times\sqrt{3}}{4\sqrt{3}\times\sqrt{3}}$ $\quad \sqrt{2}\times\sqrt{3}=\sqrt{6} \quad -1\times\sqrt{3}=-\sqrt{3}$ $\quad 4\times\sqrt{3}\times\sqrt{3}=4\times 3=12$

Answer $\dfrac{\sqrt{6}-\sqrt{3}}{12}$

https://youtu.be/7VTxamgb1Uw

END OF QUESTIONS

8