AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

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Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

GCSE
MATHEMATICS

H

Higher Tier Paper 2 Calculator

Thursday 3 November 2022 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

.CG Maths.

Answer **all** questions in the spaces provided.

1 Work out $\dfrac{4^6 - 11}{\sqrt{625} - 225}$

Circle your answer.

Type it into the calculator exactly as it is above

[1 mark]

–61.6 [✓ –20.425] 204.25 3870.56

2 Work out $(3.1 \times 10^9)^2$

Circle your answer.

Type it into the calculator exactly as it is above

[1 mark]

6.2×10^{18} 6.2×10^{81} [✓ 9.61×10^{18}] 9.61×10^{81}

3 The equation of a line is $y = 3x - 6$

Circle the coordinates of the y-intercept.

[1 mark]

[✓ (0, –6)] (–6, 0) (0, 3) (3, 0)

The x-coordinate must be 0 as the y-intercept is on the y-axis.
When x = 0, y = 3(0) - 6 = -6, so the y-coordinate must be -6

4 $a \times b^4 = c$

Circle the correct expression for a.

[1 mark]

$\frac{c}{\sqrt[4]{b}}$ $\frac{c}{b^{-4}}$ $\left(\frac{c}{b}\right)^4$ [✓ $\frac{c}{b^4}$]

Dividing both sides by b^4 finds that $a = c/b^4$

5 Written as the product of prime factors,

$12\,600 = 2^3 \times 3^2 \times 5^2 \times 7$

and

$14\,112 = 2^5 \times 3^2 \times 7^2$

Work out the highest common factor (HCF) of 12 600 and 14 112

Give your answer as an integer.

[2 marks]

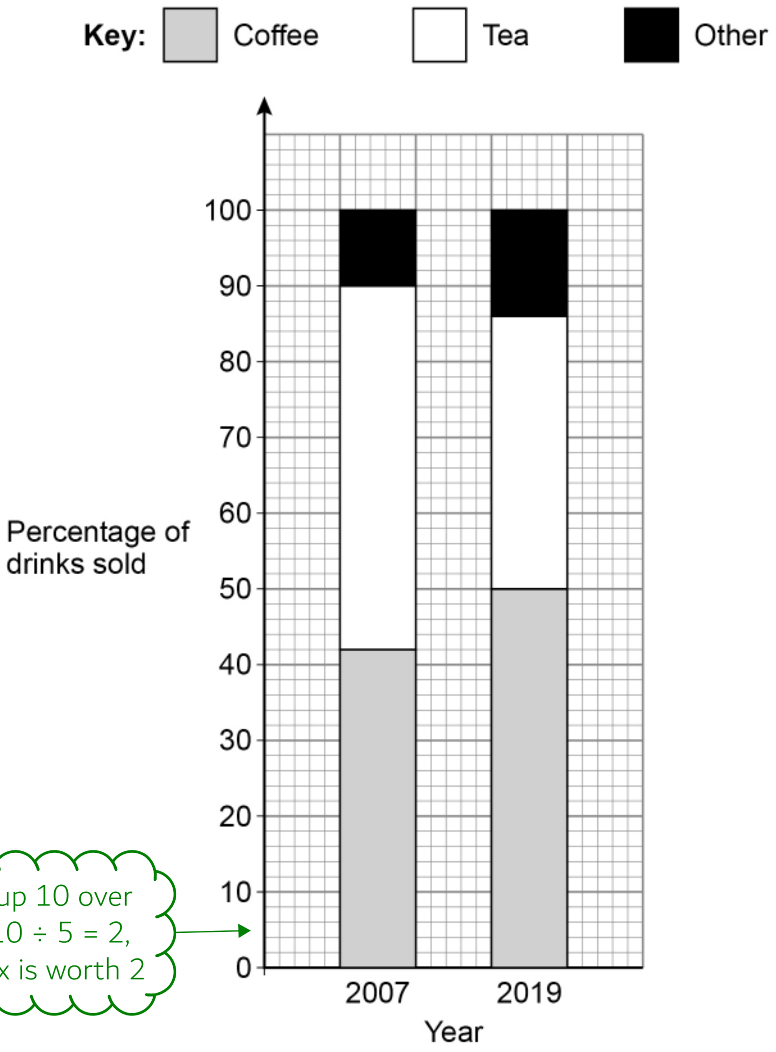
$2^3 \times 3^2 \times 7$ ← Multiplying the lowest power of each prime in both lists works out the HCF

Answer 504

6

Turn over ▶

6 The composite bar chart shows information about the **percentage** of drinks sold by a café in 2007 and 2019



The scale goes up 10 over 5 small boxes. 10 ÷ 5 = 2, so each small box is worth 2

6 (a) In 2007 the café sold a total of 24 000 drinks.

How many **more** teas than coffees were sold?

[2 marks]

90−42 ← Working out that 48% of drinks sold were tea. The bar for tea goes from 42% up to 90%. Subtracting these from each other works out how tall the bar is

48−42 ← Subtracting the 42% for coffee from the 48% for tea works works out that tea had 6% more than coffee

$\frac{6}{100} \times 24000$ ← Doing 6% of the 24000 drinks. Percentage is out of 100

Answer 1440

6 (b) Were more coffees sold at the café in 2019 than in 2007 ?

Tick a box.

☐ Yes ☐ No ☑ Cannot tell

Give a reason for your answer.

[1 mark]

Only given percentages for 2019

Percentage is a proportion, not an amount. There is a higher percentage of coffee in 2019 but there may be fewer drinks sold in total, meaning that there could possibly be less coffees sold in 2019

7 (a) k is a whole number between 40 and 50

The cube root of k is 3, to the nearest whole number.

Work out the **largest** possible value of k.

[2 marks]

Using table mode, enter $f(x) = \sqrt[3]{x}$. Start: 40. End: 50. Step: 1

This lists out the cube roots of all the whole numbers between 40 and 50. $\sqrt[3]{42} = 3.4...$ which rounds to 3. Then $\sqrt[3]{43} = 3.5...$ which rounds to 4. So 42 is the largest k can be

Answer 42

7 (b) Fay tries to solve $x^2 = 100$

She says,

"The only possible value of x is 10"

Give a reason why she is **not** correct.

[1 mark]

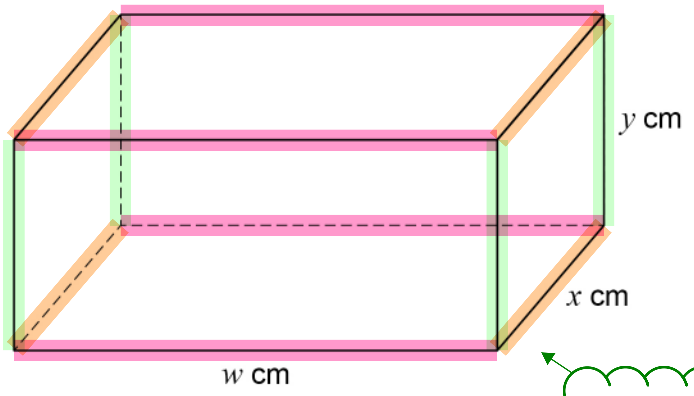
x can be -10

$\sqrt{100}$ is 10 or -10, as $(-10)^2 = 100$

6

8 (a) Here is a cuboid.

w, x and y are **different** whole numbers.



The length w is shown in pink, the length x is shown in orange and the length y is shown in green

The total length of **all** the edges of the cuboid is 80 cm

The volume is **greater** than 200 cm^3

Work out one possible set of values for w, x and y.

[2 marks]

$4w+4x+4y=80$

There are 4w, 4x and 4y which must add up to 80

$w+x+y=20$

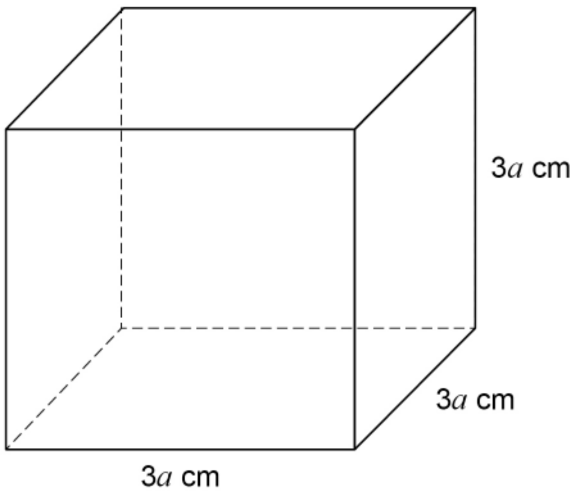
Dividing all terms on both sides by 4 simplifies the equation

$10\times6\times4=240$

Volume of cuboid = length x width x height. So multiplying w, x and y must give a result more than 200. 10, 6 and 4 are different whole numbers which add up to 20 and when multiplied gives 240

$w =$ 10 $x =$ 6 $y =$ 4

8 (b) Here is a solid cube.



Circle the expression for the **total** surface area in cm^2

[1 mark]

$36a$ $54a$ $36a^2$ [✓ $54a^2$]

Each face is a square. Area of square = length^2. The length is 3a. $(3a)^2 = 9a^2$. There are 6 square faces so multiplying the $9a^2$ by 6 works out that the surface area is $54a^2$

9 The 47th triangular number is 1128

The 48th triangular number is 1176

Work out the 49th triangular number.

[1 mark]

1176 − 1128

This works out that the increase from the 47th triangular number to the 48th is 48

48 + 1

Triangular numbers increase by 1 more each time so this works out that the increase needs to by 49 to the next triangular number

1176 + 49

Answer 1225

Adding the increase of 49 to the 48th to get the 49th

Triangular numbers start with 1, then add 2, then add 3, then add 4...

4

Turn over ▶

10 The nth terms of two linear sequences, A and B, are added to give the nth term of a new sequence.

The new sequence starts

8 13 18 23

The nth term of sequence A is $n + 1$

Work out the nth term of sequence B.

[4 marks]

$5n+3-(n+1)$

The new sequence increases by 5 between each term so must involve 5n. The 0th term (the one before the first term) would be 3, so the nth term must be 5n + 3. Subtracting the nth term of sequence A leaves the nth term of sequence B

Answer $4n+2$

Subtracting everything in the bracket. 5n - n = 4n. 3 - 1 = 2

11 A tank contains 40 litres of water.

11 (a) Water leaks out of the tank at a rate of 1.2 litres per minute.

The leak is stopped after 20 minutes.

Show that, when the leak is stopped, the tank contains 16 litres of water.

[1 mark]

1.2×20

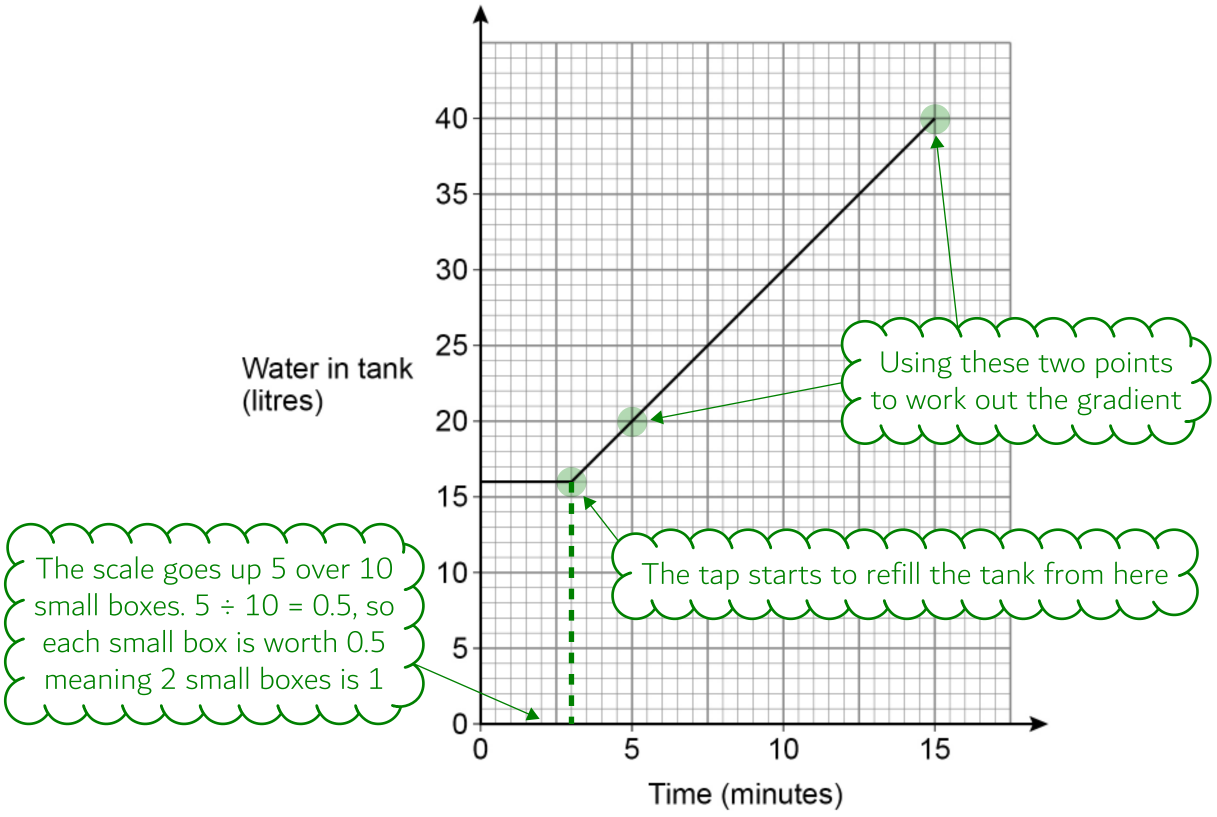
Multiplying the rate of 1.2 litres per minute by the 20 minutes works out that 24 litres had leaked by the time the leak is stopped

$40-24=16$

Subtracting the 24 litres which had leaked from the 40 litres it started with shows that the tank contains 16 litres when the leak is stopped

11 (b) The tank is refilled with water from a tap.

The graph shows the amount of water in the tank **after** the leak is stopped.



Complete this report by writing a number in each answer space.

[3 marks]

6 small boxes after 0, so this is 3 minutes

Report

3 minutes after the leak is stopped, the tap starts to refill the tank.

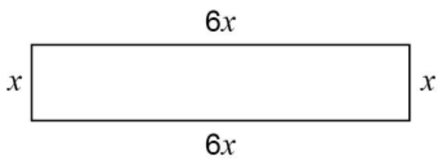
The rate at which the tank refills is 2 litres per minute.

$\frac{40-20}{15-5}$

The gradient of the line is the rate the tank refills. Gradient = (change in y)/(change in x). From (5 , 20) to (15 , 40), change in y is 40 - 20 and change in x is 15 - 5

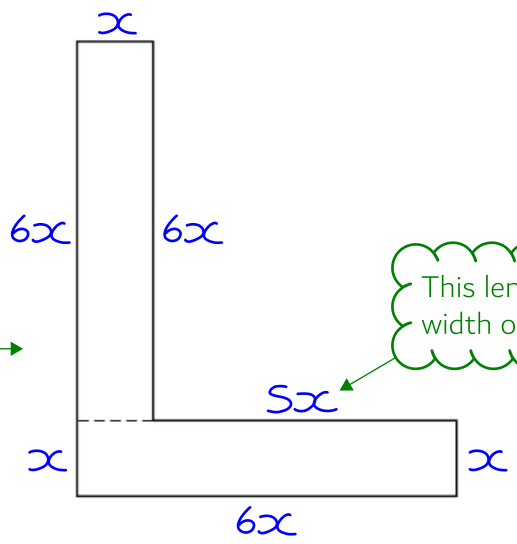
8

12 The length of this rectangle is 6 times the width.



Not drawn accurately

Two of these rectangles are joined, with no overlap, to make this L-shape.



Not drawn accurately

Labelling all of the lengths on the perimeter in terms of x

This length must be 5x as subtracting the width of x from the length of 6x leaves 5x

The perimeter of the L-shape is 98.8 cm

Work out the value of the perimeter of **one** of the rectangles.

[4 marks]

$26x = 98.8$

Adding all of the x on the perimeter of the L-shape gives 26x. This must be equal to the actual perimeter of 98.8cm

$x = 98.8 \div 26$

Dividing both sides by 26 gets x on its own and finds that x is 3.8cm

3.8×14

6x + 6x + x + x = 14x, so there are 14 lots of x on the perimeter of one of the rectangles. Multiplying the value of x by 14 works out the perimeter

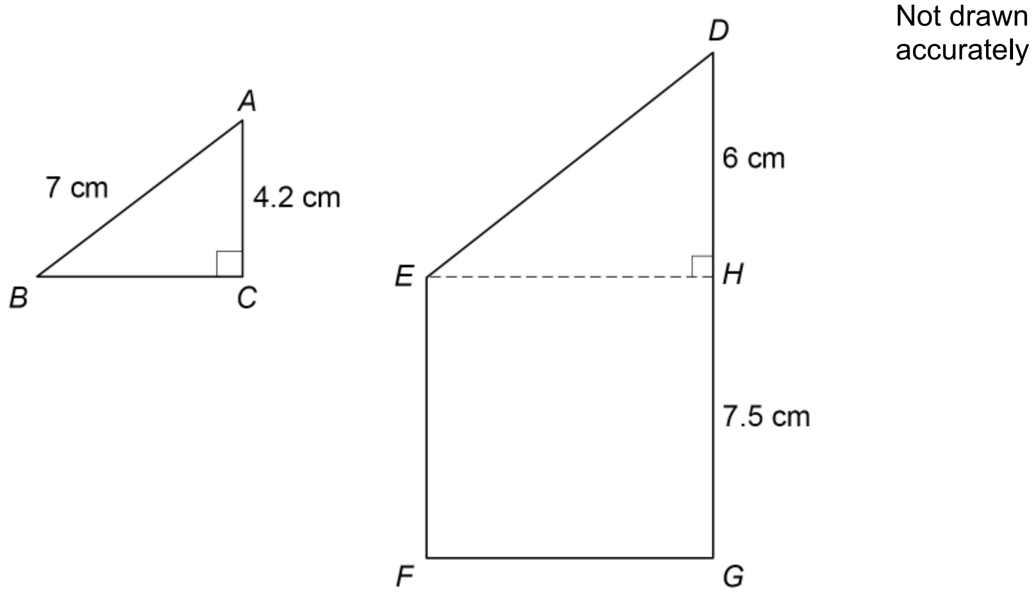
Answer 53.2 cm

13 Trapezium *DEFG* is formed by joining

triangle *DEH*

to

rectangle *EFGH*.



ABC is similar to *DEH*.

Work out the area of *DEFG*.

[5 marks]

$BC^2+4.2^2=7^2$ ← Pythagoras' Theorem can be used to find the missing side in the right-angled triangle ABC. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. Substituting side BC for a, 4.2 for b and 7 for c

$BC=\sqrt{7^2-4.2^2}=5.6$ ← Subtracting 4.2^2 from both sides then square rooting both sides finds that side BC is 5.6cm

$6\div4.2$ ← Side DH (6cm) is the longer version of side AC (4.2cm) so dividing these works out that the scale factor between the two similar triangles is 10/7. This means that all of the sides on the smaller triangle ABC are multiplied by 10/7 to get the sides on the larger triangle DEH

$5.6\times\frac{10}{7}$ ← Multiplying side BC by the scale factor works out that side EH is 8cm

The method continues on the next page

Answer 84 cm^2

9

$\frac{1}{2} \times 8 \times 6 = 24$ ← Area of triangle = 1/2 x base x height. The base of triangle DEH is 8cm and its height is 6cm

$8 \times 7.5 = 60$ ← Area of rectangle = length x width. The length of rectangle EFGH is 8cm and its width is 7.5cm

$24 + 60$ ← Adding the area of the triangle DEH and rectangle EFGH gives the total area of DEFG

14 Fred bought an apartment for £137 500

He made 8% profit when he sold the apartment.

He used all of this profit to pay 40% of the deposit on a house.

The deposit was one sixth of the price of the house.

Work out the price of the house.

[4 marks]

$\frac{8}{100} \times 137500$ ← 8 is put over 100 to convert 8% into a fraction. Multiplying by this fraction finds that 8% of the £137500 is £11000, which is the profit

$\frac{40}{100}x = 11000$ ← Let x be the deposit on the house. 40 is put over 100 to convert 40% into a fraction. Multiplying x by this fraction expresses 40% of the deposit, which is £11000 as this is all of the profit

$x = 11000 \div \frac{40}{100}$ ← Dividing both sides by 40/100 finds that x, the deposit, is £27500

27500×6 ← The deposit is 1/6 of the price of the house, so multiplying the deposit by 6 finds the price of the house

Answer £ 165000

15 Circle the correct statement.

The unit of each is squared, so whatever the conversion scale factor is for the units of length also needs to be squared

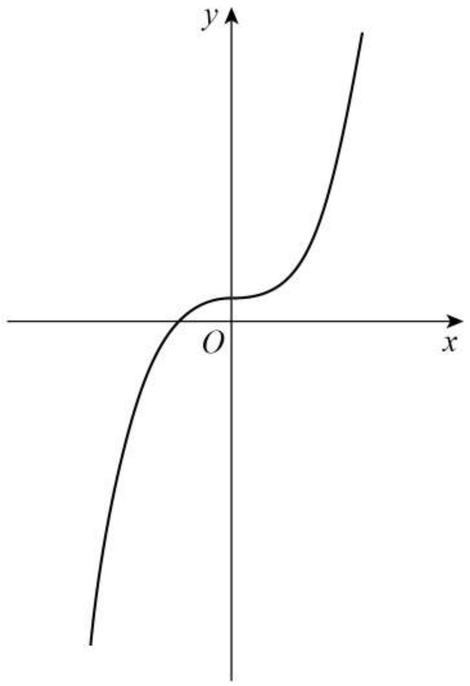
[1 mark]

$1\ m^2 = 100\ mm^2$ [✓ $1\ cm^2 = 100\ mm^2$] $1\ m^2 = 100\ cm^2$ $1\ km^2 = 100\ m^2$

$1 \times 1000^2 = 1000000$
$1 \times 10^2 = 100$
$1 \times 100^2 = 10000$
$1 \times 1000^2 = 1000000$

← There are 1000mm in 1m, so multiplying 1 by 1000^2 converts $1m^2$ into $1000000mm^2$. There are 10mm in 1cm, so multiplying 1 by 10^2 converts $1cm^2$ into $100mm^2$. There are 100cm in 1m, so multiplying 1 by 100^2 converts $1m^2$ into $10000cm^2$. There are 1000m in 1km, so multiplying 1 by 1000^2 converts $1km^2$ into $1000000m^2$

16 Here is a sketch of a graph.



Circle the possible equation of the graph.

[1 mark]

$y = x^2 + 1$ $y = \frac{1}{x} + 1$ [✓ $y = x^3 + 1$] $y = 1 - x^2$

The shape of the graph has to be an x^3 graph. This is a common graph

Alternatively, table mode can be used on the calculator for each equation to give a table of values then the right one can be worked out

17 A sequence of numbers is formed by the iterative process

$$u_{n+1} = \frac{20}{u_n + 3} \quad \text{where} \quad u_1 = 1$$

Work out u_3

Enter 1 then press =. Enter 20/(ANS + 3) and press = twice

This substitutes u_1 into the equation to find u_2 then substitutes u_2 in to find u_3

Circle your answer.

[1 mark]

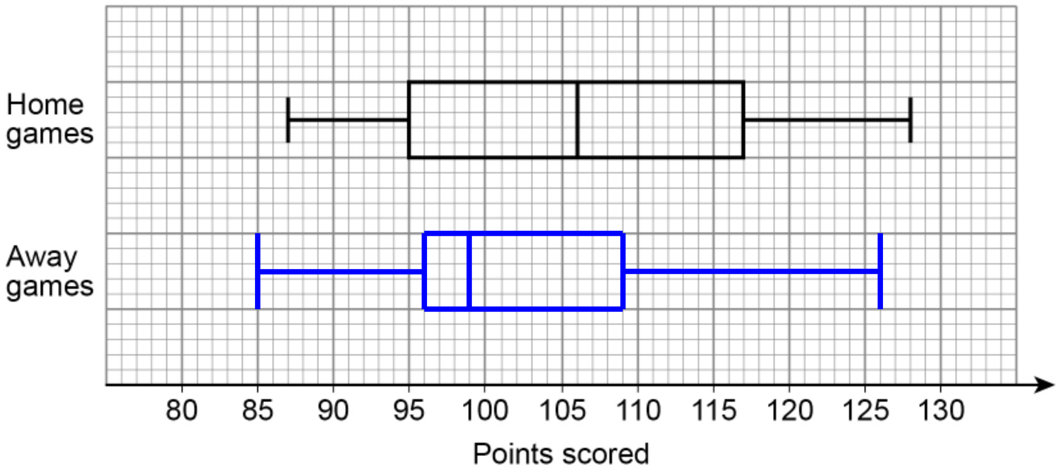
$\frac{40}{11}$ [✓ $\frac{5}{2}$] 7 5

7

Turn over ▶

18 A basketball team plays 19 home games and 19 away games.

The box plot shows information about the points the team scored in **home** games.



Here are the points the team scored in the 19 **away** games.

85	89	93	95	⓽⓺ 96	96	98	98	98	99
100	103	105	107	109	110	114	119	126	

18 (a) On the grid, draw a box plot for the away games.

[4 marks]

$\frac{19+1}{4}=5$

Using the formula (n + 1)/4, where n is the number of games, works out that the 5th value is the lower quartile. The median will be 5 after this so will be the 10th and the upper quartile will be 5 after this so will be the 15th

The lowest value is 85, the lower quartile is 96, the median is 99, the upper quartile is 109 and the highest value is 126. Drawing vertical lines for all of these then joining it up into a box plot

18 (b) On average, did the team score more points in home games or away games?

Use **one** statistical measure to support your decision.

[1 mark]

Home games, as its median is higher

The median for the home games was 106 and the median for the away games was 99

18 (c) Was the number of points scored more consistent in home games or away games?

Use **one** statistical measure to support your decision.

[1 mark]

Away games, as its interquartile range is less

The distance between the lower and upper quartile is less for the away games

19 Using the quadratic formula, or otherwise, solve $3x^2 + x - 5 = 0$

[2 marks]

$$\frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times -5}}{2 \times 3}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The equation is in the form $ax^2 + bx + c = 0$, so the quadratic formula can be used straight away. a = 3, b = 1 and c = -5

Answer 1.1 , -1.5

8

Turn over ▶

20 A vending machine has a different item in each section.

It sells

7 drinks, 3 of which are juice

5 snacks, 2 of which are fruit bars

11 meals, 4 of which are salad.

One drink, one snack and one meal are chosen at random.

Show that the probability of getting a juice, a fruit bar and a salad is **more** than 5%

[3 marks]

$\frac{3}{7} \times \frac{2}{5} \times \frac{4}{11}$ ← Juice AND fruit bar AND salad. AND means to multiply the probabilities. 3 out of the 7 drinks are juice so the probability of juice is 3/7

$\frac{24}{385} \times 100$ ← Multiplying the fraction by 100 converts it into a percentage

6.2% ← This is more than 5%

21 $f(x) = \frac{3x+9}{5}$ and $g(x) = 6x - 1$

21 (a) Show that gf(2) is an integer.

[2 marks]

$\frac{3(2)+9}{5}$ ← Substituting 2 for x in f(x) works out that f(2) is 3

$6(3)-1=17$ ← Substituting 3 (the result from f(2)) for x in g(x) works out that g(3) is 17

21 (b) Show that $f^{-1}(8)$ is **not** an integer.

[2 marks]

$x=\frac{3y+9}{5}$ ← Switching f(x) with x and x with y then rearranging to make y the subject finds the inverse function $f^{-1}(x)$

$5x=3y+9$ ← Multiplying both sides by 5 eliminates the fraction on the right

$5x-9=3y$ ← Subtracting 9 from both sides gets the y term on its own

$\frac{5x-9}{3}=y$ ← Dividing both sides by 3 gets y on its own and makes it the subject

$\frac{5(8)-9}{3}=\frac{31}{3}$ ← What y was equal to is the inverse function $f^{-1}(x)$. Substituting in 8 for x to find $f^{-1}(8)$

31/3 is not an integer

7

Turn over ▶

22 Factorise fully $x^3 - 49x$

[2 marks]

$x(x^2-49)$

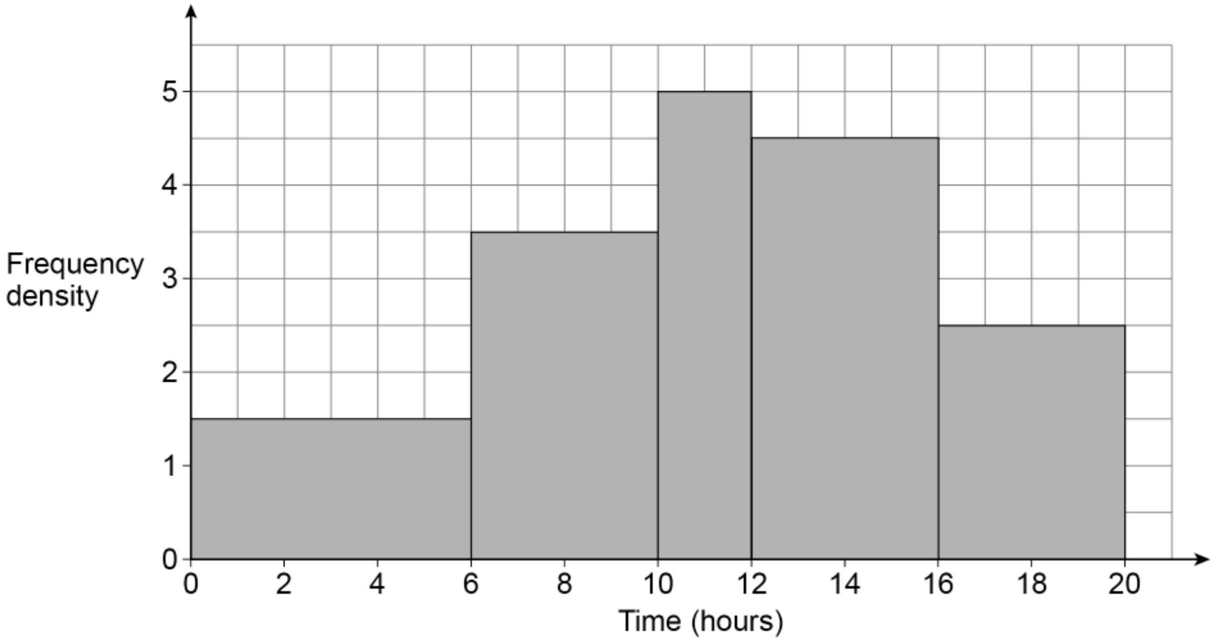
Factorising by bringing out x as a factor. x is in both terms so is a common factor to both terms. This is brought out, the two terms are divided by x and the result is left in a bracket

Answer $x(x+7)(x-7)$

Factorising again by using difference of two squares on the $x^2 - 49$. $A^2 - B^2 = (A + B)(A - B)$. A is x and B is 7

23 61 students recorded how many hours they spent revising for a test.

The histogram represents the results.



Do not write outside the box

23 (a) Work out an estimate of the mean time the 61 students spent revising.

You may use the table to help you.

[4 marks]

Time, x (hours)	**Frequency**	**Midpoint**	
$0 \leqslant x < 6$	9 ×	3 =	27
$6 \leqslant x < 10$	14 ×	8 =	112
$10 \leqslant x < 12$	10 ×	11 =	110
$12 \leqslant x < 16$	18 ×	14 =	252
$16 \leqslant x < 20$	10 ×	18 =	180

6 x 1.5 = 9
4 x 3.5 = 14
2 x 5 = 10
4 x 4.5 = 18
4 x 2.5 = 10

Frequency on a histogram is the area of each box. Multiplying the class widths by the frequency densities works out the frequencies

The midpoints can be worked out by doing the mean of the upper and lower bound of each interval

(0 + 6)/2 = 3
(6 + 10)/2 = 8
(10 + 12)/2 = 11
(12 + 16)/2 = 14
(16 + 20)/2 = 18

Multiplying the frequencies by the midpoints works out estimates for the total time for each interval

$$\frac{27+112+110+252+180}{61}$$

Answer 11.2 hours

Mean = total/number, where total is the total time and number is the total frequency. An estimate for the total time is found by adding all the totals of each interval. The total frequency is 61

23 (b) Give a reason why the answer to part (a) is an estimate.

[1 mark]

Exact times are not used

The midpoints are used and these are estimates of what each of the times are in each interval

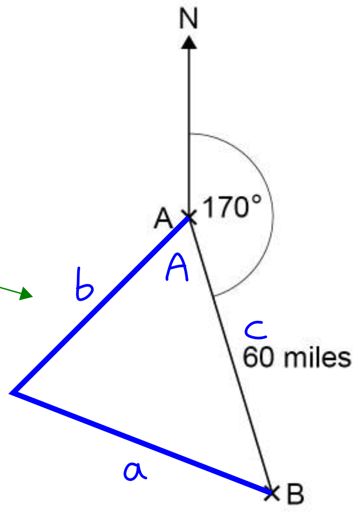
7

Turn over ►

24 B is 60 miles from A on a bearing of 170°

Not drawn accurately

A rough sketch forms a triangle. Labelling the triangle for the cosine rule. The angle we have must be A then side a is opposite angle A. b and c are the other two sides



A ship sails from A on a bearing of 247°

It travels at a constant speed of 23 mph for $1\frac{1}{2}$ hours.

Is the ship now closer to B than it was when it left A?

You **must** show your working.

[5 marks]

$s^{d}t$ ← Writing the formula triangle for distance, speed and time

$23 \times 1\frac{1}{2} = 34.5$ ← From the formula triangle, distance = speed x time. So the ship is now 34.5 miles from A. This is length b

$247 - 170 = 77$ ← Subtracting the bearings works out the angle A

$a^2 = b^2 + c^2 - 2bc\cos A$ ← There are not opposite pairs of sides and angles. So the sine rule cannot be used. Therefore the cosine rule probably needs to be used

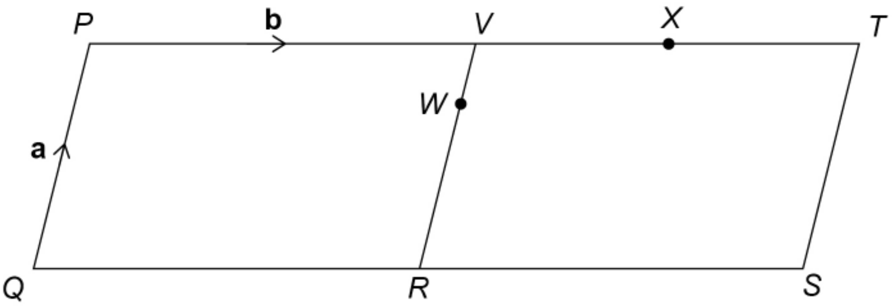
$a = \sqrt{34.5^2 + 60^2 - 2 \times 34.5 \times 60 \times \cos 77}$ ← Square rooting both sides to find side a. Substituting in the values of b, c and A

$= 62.1$

No ← The distance between the ship and A was 60 miles. The distance is now 62.1 miles and this is not closer

25 Two congruent parallelograms, *PQRV* and *VRST*, are joined.

Not drawn accurately



$\overrightarrow{QP} = \mathbf{a}$ $\overrightarrow{PV} = \mathbf{b}$

X is the midpoint of *VT*.

VW : *WR* = 1 : 2

There are 3 parts in total in the ratio and 1 of these is for VW. So VW is 1/3 of VR

Prove that *Q*, *W* and *X* lie on a straight line.

[3 marks]

$\overrightarrow{WX} = \frac{1}{3}a + \frac{1}{2}b$

$\overrightarrow{WX} = \overrightarrow{WV} + \overrightarrow{VX}$. $\overrightarrow{WV} = 1/3\ \overrightarrow{RV} = 1/3\ \overrightarrow{QP} = 1/3$ a.
$\overrightarrow{VX} = 1/2\ \overrightarrow{VT} = 1/2\ \overrightarrow{PV} = 1/2$ b

$\overrightarrow{QW} = a + b - \frac{1}{3}a$

$\overrightarrow{QW} = \overrightarrow{QP} + \overrightarrow{PV} + \overrightarrow{VW}$. $\overrightarrow{QP}$ = a. $\overrightarrow{PV}$ = b. $\overrightarrow{VW} = -1/3$ a

$= \frac{2}{3}a + b$

Simplifying by collecting like terms

$= 2\left(\frac{1}{3}a + \frac{1}{2}b\right)$

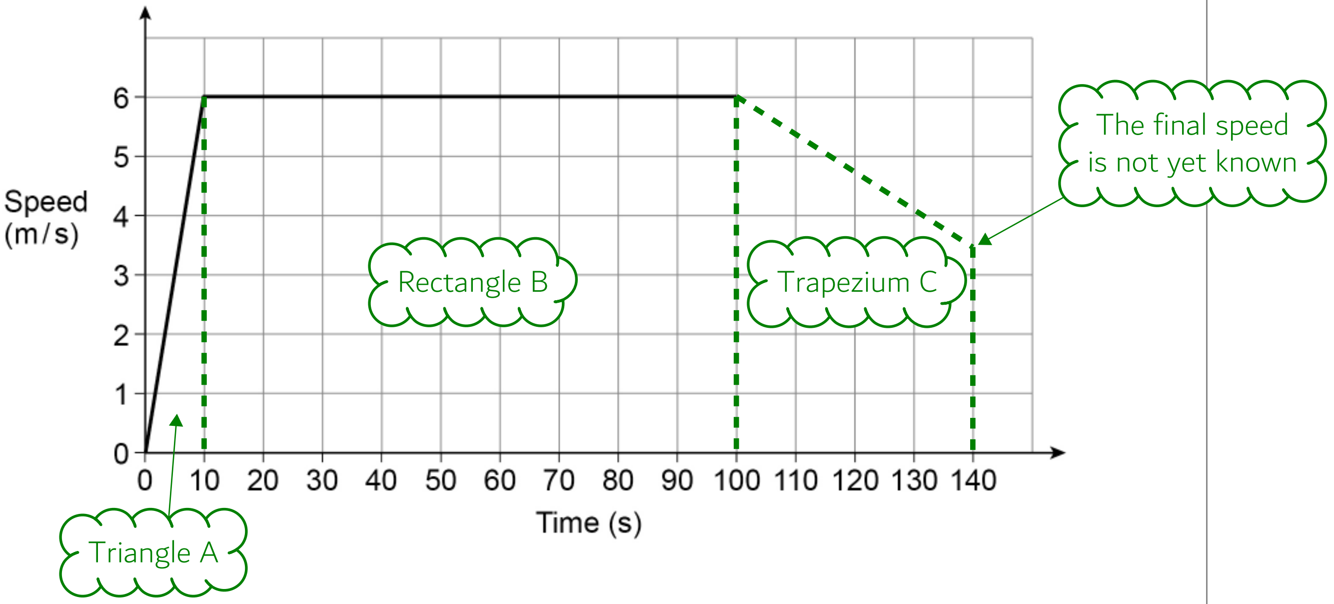
Showing that $\overrightarrow{WX}$ can be multiplied to get $\overrightarrow{QW}$

Therefore Q, W and X must lie on a straight line as the vectors $\overrightarrow{WX}$ and $\overrightarrow{QW}$ are in the same direction and both go through W

8

26 Helena ran an 800-metre race in 140 seconds.

The speed-time graph represents the first 100 seconds of her run.



Helena ran the last 40 seconds with constant deceleration.

Work out her speed as she finished the race.

[4 marks]

Distance is the area under the line on a speed-time graph

$\frac{1}{2} \times 10 \times 6 = 30$

Area of triangle = 1/2 x base x height. The base of triangle A is 10 and its height is 6

90×6

Area of rectangle = length x width. The length of rectangle B is 90 and its width is 6

$800 - 540 - 30$

Subtracting the area of triangle A and rectangle B from the total distance of the race works out how much distance is done in the last 40 seconds

$\frac{1}{2}(6 + x) \times 40 = 230$

The shape of the last part of the graph will be a trapezium, however the final speed is not currently known so using x to represent this. Area of trapezium = 1/2 (a + b)h, where a and b are the parallel sides and h is the distance between them

$x = \frac{230}{\frac{1}{2} \times 40} - 6$

Answer 5.5 metres per second

Rearranging to find x by dividing both sides by the 1/2 and 40 then subtracting 6 from both sides

27 In a class there are

n boys

a total of 25 students.

Two of the students are chosen at random.

The probability that both students are boys is $\frac{7}{20}$

Work out the value of n.

[4 marks]

$\frac{n}{25} \times \frac{n-1}{24}$

Boy AND boy. AND means to multiply the probabilities. The probability of the first boy is n/25 as there are n boys out of 25 students. The probability of the second boy is (n - 1)/24 as there is 1 less boy and 1 less student in total in the class after the first boy has been chosen

$\frac{n^2-n}{600} = \frac{7}{20}$

Multiplying the two fractions by multiplying the numerators and multiplying the denominators. Setting the probability equal to the actual probability of 7/20

$n^2-n=210$

Multiplying both sides by 600 eliminates the denominator on the left

$n^2-n-210=0$

Subtracting 210 from both sides brings it into the quadratic form

$$\frac{-(-1)\pm\sqrt{(-1)^2-4\times1\times(-210)}}{2\times1}$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The equation is in the form $an^2 + bn + c = 0$ so can be solved using the quadratic formula. a = 1, b = -1 and c = -210

$n =$ 15

There is also a solution of n = -14 but this is ignored as n cannot be negative

8

Turn over ▶

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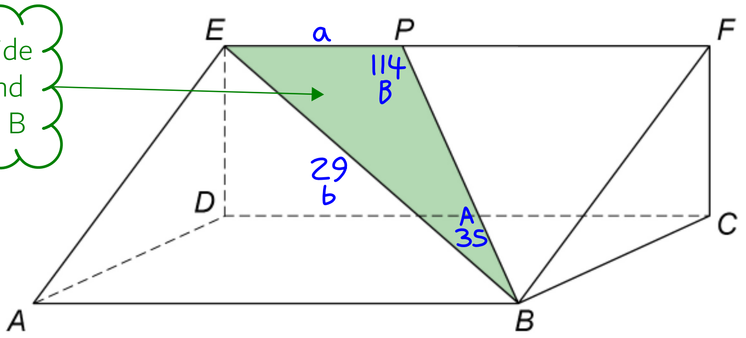
28 *ABCDEF* is a triangular prism.

P is a point on *EF*.

Labelling the triangle. Side a is opposite angle A and side b is opposite angle B



EB = 29 cm

Angle *EBP* = 35°

Angle *EPB* = 114°

Work out the length of *EP*.

[2 marks]

$\frac{a}{\sin A} = \frac{b}{\sin B}$

There are opposite pairs of sides and angles so the sine rule can be used. Quoting it with the sides on top as we are looking for a side

$EP = \frac{29 \sin 35}{\sin 114}$

Multiplying both sides by sinA makes a the subject. Then substituting in the values

Answer 18.2 cm

END OF QUESTIONS

2

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