

Thursday 4 November 2021 – Morning**GCSE (9–1) Mathematics****J560/05 Paper 5 (Higher Tier)****Time allowed: 1 hour 30 minutes****You can use:**

- geometrical instruments
- tracing paper

Do not use:

- a calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

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Candidate number

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First name(s)

Last name

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **20** pages.

ADVICE

- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** the questions.

1 Work out.

$$3\frac{4}{7} \times \frac{7}{10}$$

Give your answer as a mixed number in its simplest form.

$$\frac{25}{7} \times \frac{7}{10}$$

Converting the mixed fraction into an improper fraction by multiplying the whole number by the denominator and adding the result to the numerator. $3 \times 7 = 21$. $4 + 21 = 25$

$$\frac{25}{3} \times \frac{7}{10}$$

Multiplying the numerators together and the denominators together gives $175/70$

$$\frac{035}{1175}$$

Dividing both the numerator and denominator by 5 gives $35/14$

$$\frac{14}{170}$$

Dividing both the numerator and denominator by 7 gives $5/2$

$$\frac{5}{2}$$

$$2\frac{1}{2}$$

[3]

2 goes into 5 with a remainder of 1.
The 2 becomes the whole number and the 1 remainder is left in the fraction

2 (a) Azmi is given this question.

Write 40 as a product of prime factors.
Give your answer in index form.

Azmi's answer is $2 \times 2 \times 2 \times 5$.

Is Azmi correct?

Explain your answer.

No, it isn't in index form

The answer should be $2^3 \times 5$

[1]

(b) Find the value of x .

$$\frac{1}{16} = 2^x$$

$16 = 2 \times 2 \times 2 \times 2 = 2^4$. A negative power means '1 over'

(b) $x =$ -4 [1]

- 3 A car mechanic has a tin containing 5 litres of engine oil. Each week they use 450 millilitres of this oil for their vehicles.

The car mechanic says

After 9 weeks I will have used over 80% of the oil in this tin.

Are they correct?

Show how you decide.

$$\begin{array}{r} 450 \\ \times 9 \\ \hline 4050 \end{array}$$

Multiplying the 450 millilitres used each week by the 9 weeks works out that they use 4050 millilitres after 9 weeks

$$\begin{array}{r} 500 \\ \times 8 \\ \hline 4000 \end{array}$$

There are 1000 millilitres in 1 litre so multiplying the 5 litres by 1000 works out that there are 5000 millilitres. Dividing this by 10 works out that 10% of the tin is 500 millilitres. Multiplying this by 8 works out that 80% of the tin is 4000 millilitres

Yes

The 4050 millilitres used is more than the 4000 millilitres which is 80% of the tin. Therefore they are correct

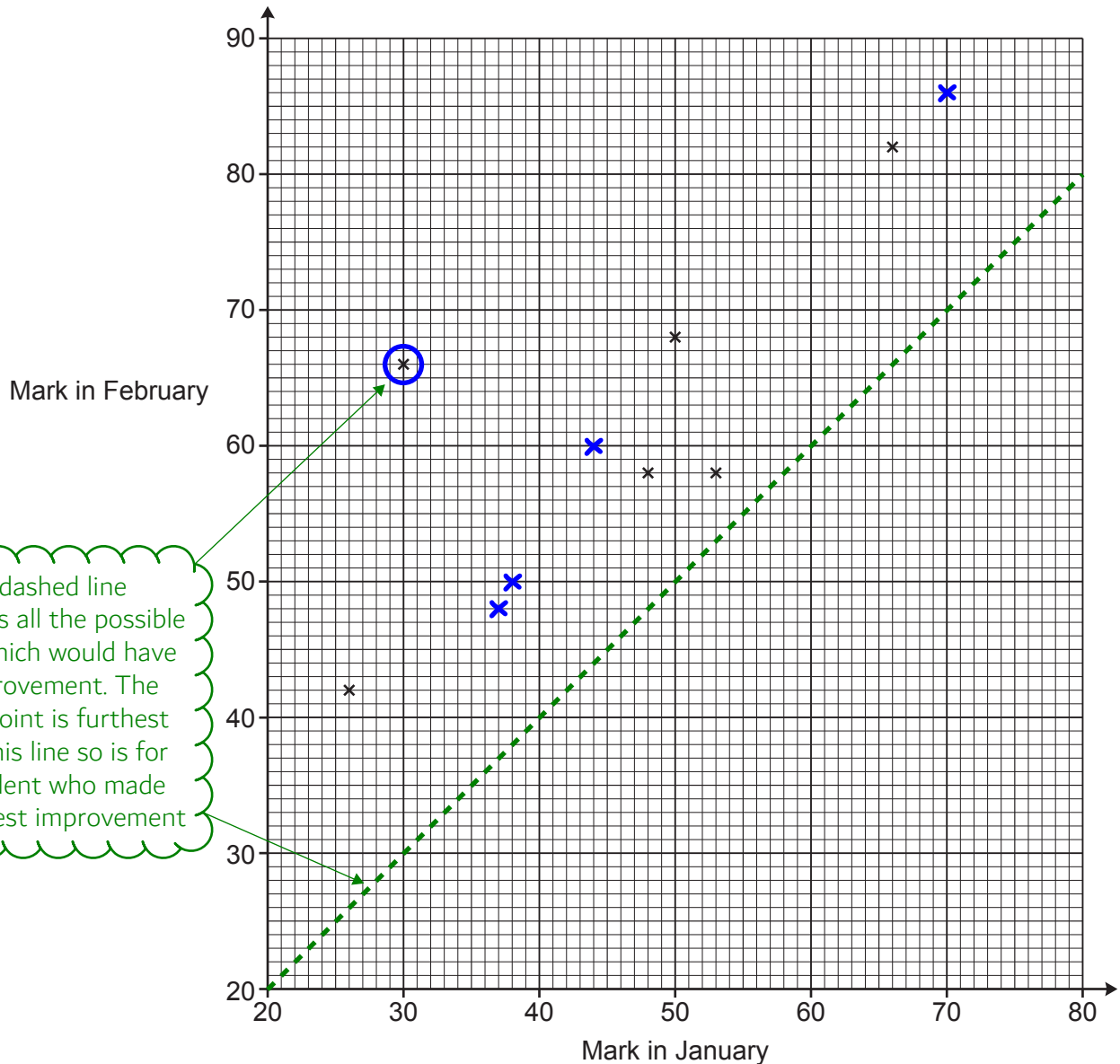
.....

..... [5]

- 4 The table shows the marks obtained by 10 students in spelling tests in January and February.

Mark in January	26	53	50	48	30	66	70	44	37	38
Mark in February	42	58	68	58	66	82	86	60	48	50

The marks for the first six students are plotted on the scatter diagram.



- (a) Plot the marks for the remaining four students. [2]
- (b) Describe the type of correlation shown in the completed scatter diagram.

Positive

As the mark in February increases as the mark in January increases

[1]

- (c) (i) On the scatter diagram, **circle** the student that made the greatest improvement in their marks from January to February. [1]

- (ii) Work out the percentage change in this student's marks from January to February.

$$\begin{array}{r} 66 \\ -30 \\ \hline 36 \end{array}$$

Percentage change = (new - original)/original x 100. The new is the mark in February which is 66. The original is the mark in January which is 30. First working out that the (new - original) is 36

$$30 \overline{) 36.0} \begin{array}{l} 0.12 \\ \underline{30} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

Dividing the result by the original

$$1.2 \times 100 = 120$$

(c)(ii) 120 % [3]

- (d) Another student, Kai, scored 79 marks in the test in January but was absent for the test in February.

Kai says

I could use a line of best fit on the scatter diagram to estimate the marks I may have achieved in the test in February.

Is Kai's method reliable?

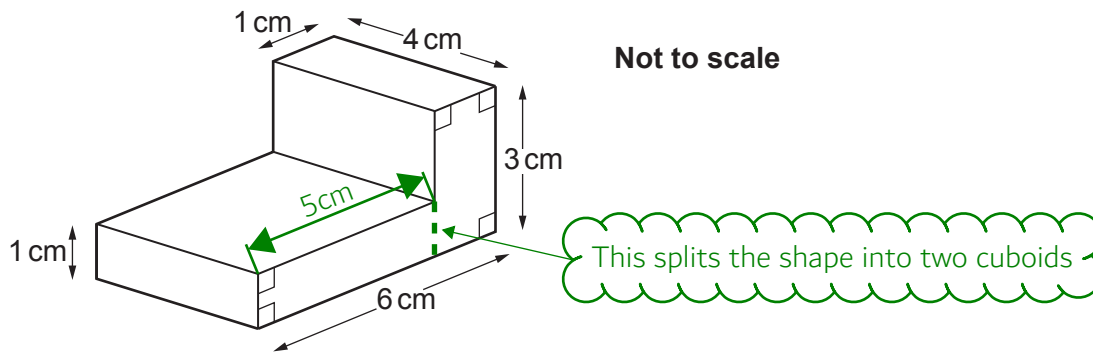
Give a reason for your answer.

No, this is outside of the range of data given

The trend may not continue to rise in a straight line

[1]

5 Work out the volume of this prism.

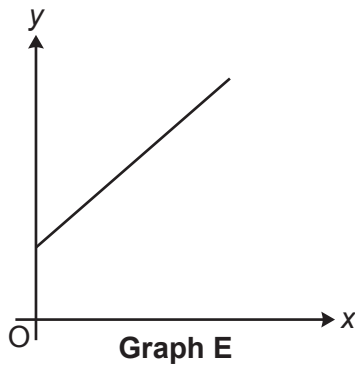
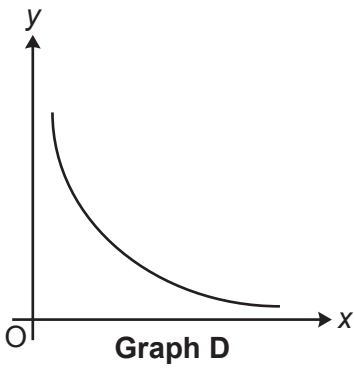
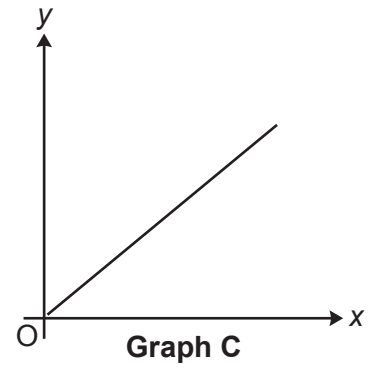
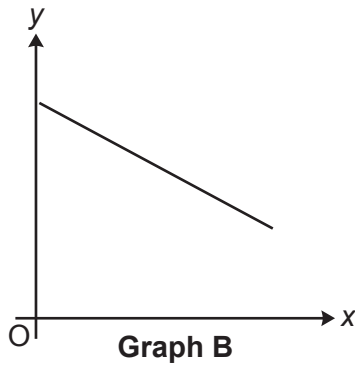
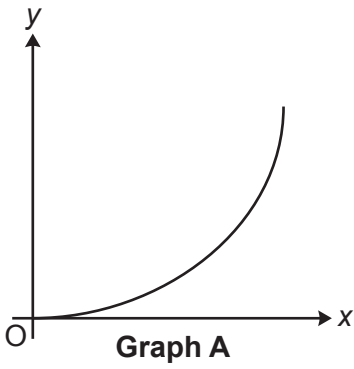


20+12

Volume of cuboid = length \times width \times height. For the cuboid on the left: the length is 5cm, the width is 4cm and the height is 1cm. $5 \times 4 \times 1 = 20$. For the cuboid on the right: the length is 4cm, the width is 1cm and the height is 3cm. $4 \times 1 \times 3 = 12$. Adding the two volumes together gives the volume of the prism

.....32..... cm^3 [4]

6 Here are sketches of five graphs.



Write the letter of the graph that represents the following relationships.

(a) y is directly proportional to x .

Doubling x doubles y →

(a) C [1]

(b) y is inversely proportional to x .

Doubling x halves y →

(b) D [1]

7 (a) Solve the inequality.

$$4(x - 3) < x$$

$$4x - 12 < x$$

Expanding the brackets

$$3x - 12 < 0$$

Subtracting x from both sides to get all the x on the side with the most

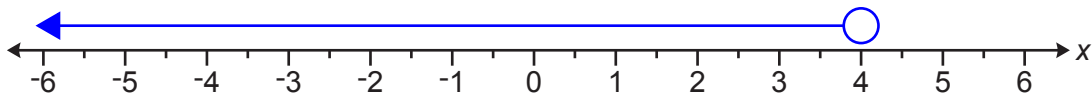
$$3x < 12$$

Adding 12 to both sides to get the x term in its own

Dividing both sides by 3 to get x on its own

(a) $x < 4$ [3]

(b) Show your answer to **part (a)** on the number line.



A hollow circle above 4 indicates that it cannot be equal to 4. The arrow going to the left means that it is all values less than 4

[2]

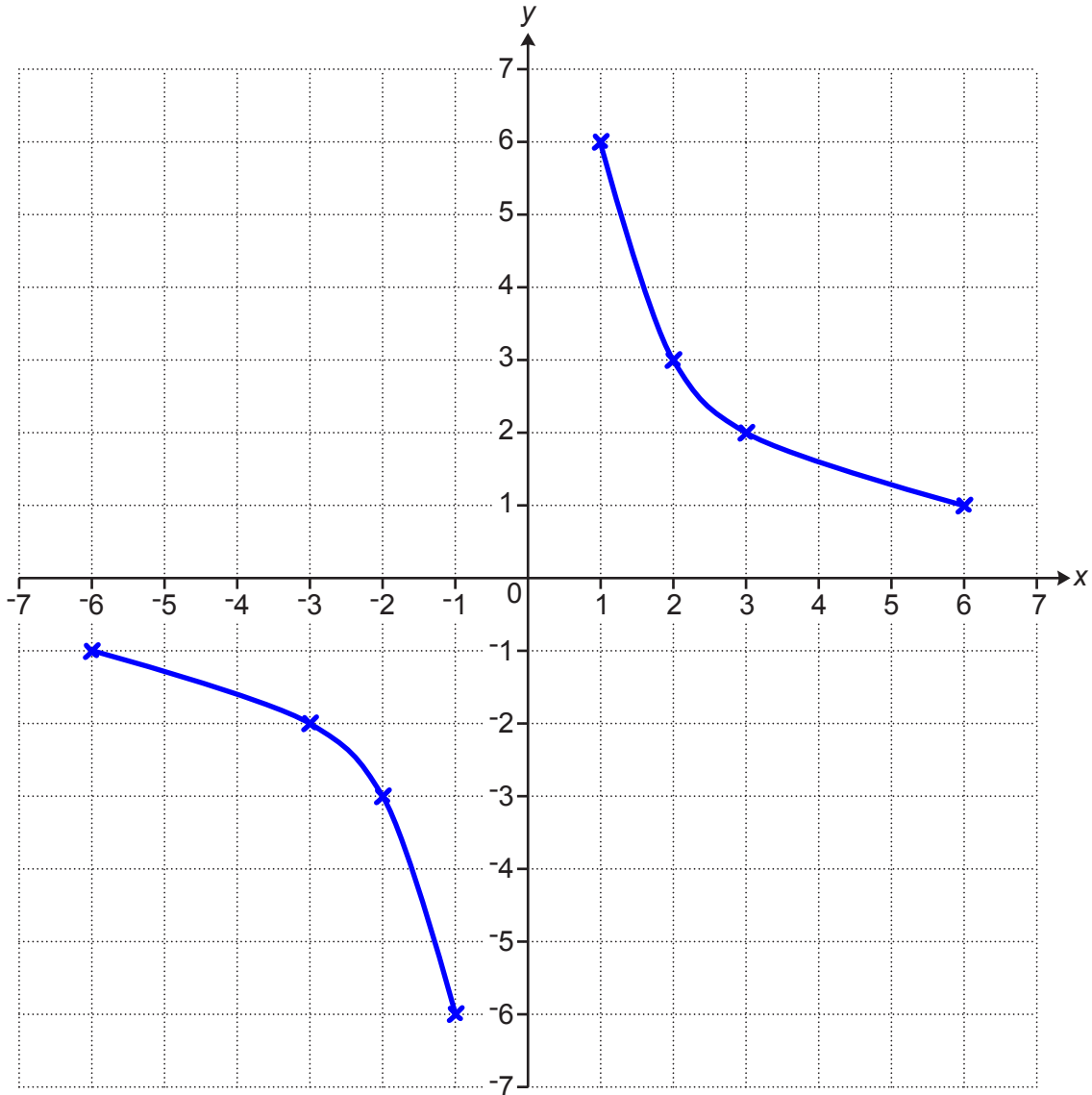
- 8 (a) Complete the table for $y = \frac{6}{x}$.

x	-6	-3	-2	-1	1	2	3	6
y	-1	-2	-3	-6	6	3	2	1

$6/-2 = -3$

[1]

- (b) Draw the graph of $y = \frac{6}{x}$ for $-6 \leq x \leq 6$, $x \neq 0$.



[3]

- 9 A worker received a 10% pay increase in 2017 and a further 10% pay increase in 2018. The worker says

Over these two years, my pay increased by $10\% + 10\% = 20\%$.

The worker is incorrect.

Work out the correct percentage increase.

You must show your working.

$$\begin{array}{r} 1.1 \\ \times 1.1 \\ \hline 110 \\ 121 \\ \hline 1.21 \end{array}$$

100% + 10% increases to 110%. Dividing this by 100 converts it into 1.1 as a decimal multiplier. Multiplying the 1.1 by 1.1 represents an increase of 10% then another increase of 10%. As there is 1 decimal place in both 1.1s, there should be 2 decimal places in the answer

21%

Converting 1.21 into a percentage by multiplying it by 100 gives 121%, which is an increase of 21%

[5]

- 10 Force is measured in newtons (N).
A force of 198.5N is applied to a rectangular surface of length 4.9cm and width 4.1 cm.

Work out an **estimate** of the pressure, in N/cm^2 , applied to this rectangular surface.

[The formula for pressure is: $\text{Pressure} = \frac{\text{Force}}{\text{Area}}$]

$$\frac{200}{5 \times 4}$$

Rounding all values to 1 significant figure gives a pressure of 200N, a length of 5cm and a width of 4cm. Area of rectangle = length x width.
Substituting the force and area into the formula to find pressure

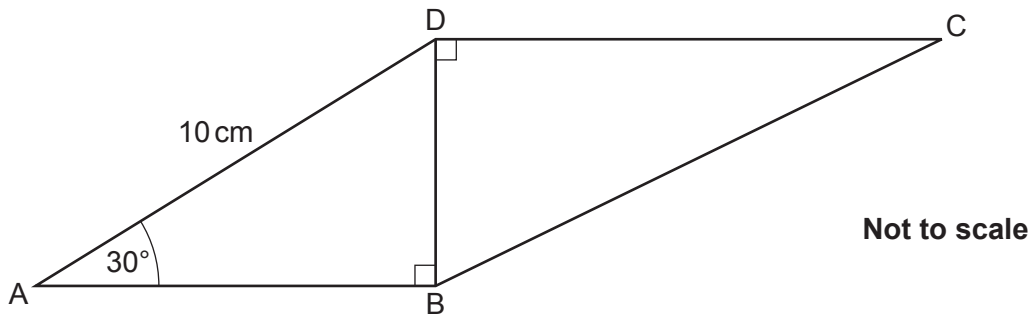
$$\begin{array}{l} 5 \times 4 = 20 \\ 200/20 = 10 \end{array}$$

10

..... N/cm^2 [4]

11 The diagram shows a quadrilateral ABCD.

AD = 10 cm, angle BAD = 30° and angle ABD = angle BDC = 90°.



The ratio of length BD to length DC is 1 : 2.4 .

Work out length BC.

You must show your working.

SOHCAHTOA

Right-angled trigonometry can be used to work out the length BD in triangle ABD. Writing out SOH CAH TOA as formula triangles. 10cm is the hypotenuse so ticking H and we are looking for the opposite so ticking O. There are two ticks on the SOH formula triangle so this one can be used

0 30 45 60 90
0 1

The angles we need to remember the trig values for are 0, 30, 45, 60 and 90. Writing these out and writing 0, 1, 2, 3, 4 under these for the sin values. Square rooting the 1 gives 1 and putting the result over 2 works out that $\sin 30 = 1/2$

$\frac{1}{2} \times 10$

From the formula triangle, opposite = (sin of the angle) x hypotenuse. This works out that the opposite, side BD, is 5cm

$\frac{2.4}{1.2} \times 5$

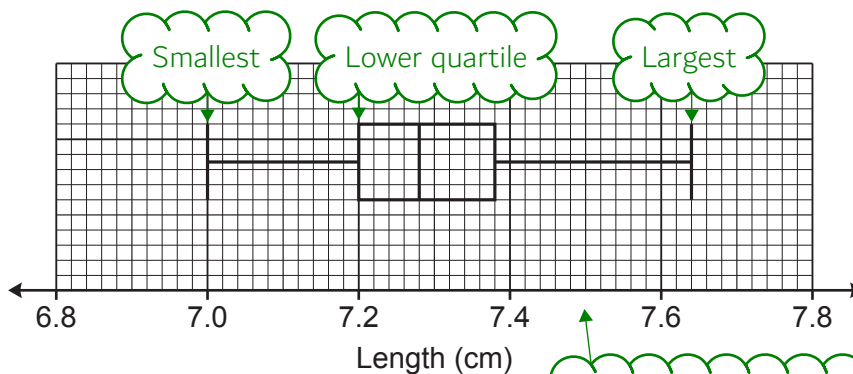
From the ratio, side DC is 2.4 times the length of BD. This works out that side DC is 12cm

$\frac{144}{169}$

Using Pythagoras' Theorem to work out side BC as it is the missing side in right-angled triangle BCD. $a^2 + b^2 = c^2$, where a and b are the two shorter sides and c is the longest side. Adding 12^2 and 5^2 works out c^2

$c^2 = 169$. Square rooting both sides gives that $c = 13$ cm [7]

- 12 The box plot shows the distribution of the lengths, in cm, of 60 full-grown mice owned by a pet shop.



The scale goes up 0.2 over 10 small boxes.
 $0.2/10 = 0.02$ so each small box is worth 0.02

- (a) Find the range.

$7.64 - 7$

Range = largest - smallest

(a) 0.64 cm [2]

- (b) Work out the number of these mice that have a length of at least 7.2 cm.

$$\begin{array}{r} 15 \\ 4 \overline{) 60} \\ \underline{45} \\ 15 \end{array}$$

The lower quartile is 7.2. So $3/4$ of the mice have a length of at least 7.2 cm

(b) 45 [2]

- (c) Sam owns 5 full-grown mice.
 Sam picks the third longest mouse and measures its length.
 Sam then looks at the box plot.

Sam says

This mouse is 7.35 cm long.

Therefore, the mice I own are longer than the full-grown mice owned by the pet shop.

- (i) Give a mathematical reason to support Sam's conclusion.

The median of Sam's mice is more than the median of the pet shop's mice
 [1]

- (ii) Give a mathematical reason why Sam's conclusion may be unreliable.

Only the median length was measured
 [1]

- 13 (a) Write $\frac{3}{11}$ as a recurring decimal.

$$\begin{array}{r} 0.\dot{2}\dot{7} \\ 11 \overline{) 3.00} \\ \underline{22} \\ 80 \\ \underline{77} \\ 30 \\ \underline{27} \\ 30 \\ \underline{27} \\ \dots \end{array}$$

Dividing the numerator by the denominator converts the fraction into a decimal. The remainder of 3 repeats so the digits 2 and 7 must recur

(a) $0.\dot{2}\dot{7}$ [2]

- (b) Write $0.4\dot{8}$ as a fraction in its simplest form.

$$x = 0.4\dot{8}$$

Let x be the recurring decimal

$$10x = 4.8\dot{8}$$

Multiplying both sides by 10 once as there is 1 recurring digit allows the recurring digit to be written in the same decimal place

$$9x = 4.4$$

Subtracting x from $10x$ cancels out the recurring digit

$$x = \frac{4.4}{9}$$

Dividing both sides by 9 expresses x as a fraction, but there is a decimal in the fraction

$$= \frac{44}{90}$$

Multiplying the numerator and denominator by 10 eliminates the decimal from the fraction

Dividing both the numerator and denominator by 2 simplifies the fraction. It cannot be simplified any further

(b) $\frac{22}{45}$ [3]

- 14 Two solid ornaments are mathematically similar.
The larger ornament is twice as tall as the smaller ornament.

The smaller ornament has a volume of 50 cm^3 .

Work out the volume of the larger ornament.

50×2^3

The scale factor of the lengths is 2 so the scale factor of the volume is 2^3

..... 400 cm^3 [2]

- 15 Write $\sqrt{20} + \sqrt{45}$ in the form $k\sqrt{5}$.

$\sqrt{4}\sqrt{5} + \sqrt{9}\sqrt{5}$

Using the rule $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ to split $\sqrt{20}$ and $\sqrt{45}$ into two surds, one of which is the square root of a square number

$2\sqrt{5} + 3\sqrt{5}$

Square rooting the 4 to get 2 and square rooting the 9 to get 3

$2x + 3x = 5x$. Surds behave in a similar way when added

$5\sqrt{5}$

..... [3]

- 16 In a group of 60 students, 40 own a smartphone, 27 own a tablet and 8 own neither.

A student is chosen at random from those that own a tablet.

Find the probability that they also own a smartphone.

You must show your working.

$$60 - 8 = 52$$

Subtracting the 8 who have neither from the 60 students finds that 52 either have a smartphone or a tablet or have both

$$40 + 27 = 67$$

Assuming that no student owns both, adding the 40 who own a smartphone and the 27 who own a tablet works out that there would be 67 students who either have a smartphone or a tablet

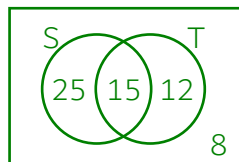
$$67 - 52 = 15$$

Working out that 67 is 15 more than 52. Therefore there must be 15 who own both a smartphone and a tablet. Every one student who owns both takes one off the total

15 out of the 27 who own a tablet also own a smartphone

$$\frac{15}{27}$$

[5]



A Venn diagram would look like this

17 Simplify fully.

$$\frac{2x^2 - 50}{x^2 + 7x + 10}$$

$$2(x^2 - 25)$$

Factorising the numerator by bringing out 2 as a factor

$$\frac{2(x+5)(x-5)}{(x+5)(x+2)}$$

Fully factorising the numerator by using difference of two squares. Factorising the denominator. 5 and 2 multiply to 10 and add to 7 so putting these in brackets with x

Cancelling out the $(x + 5)$ from the numerator and denominator as it is a common factor to both

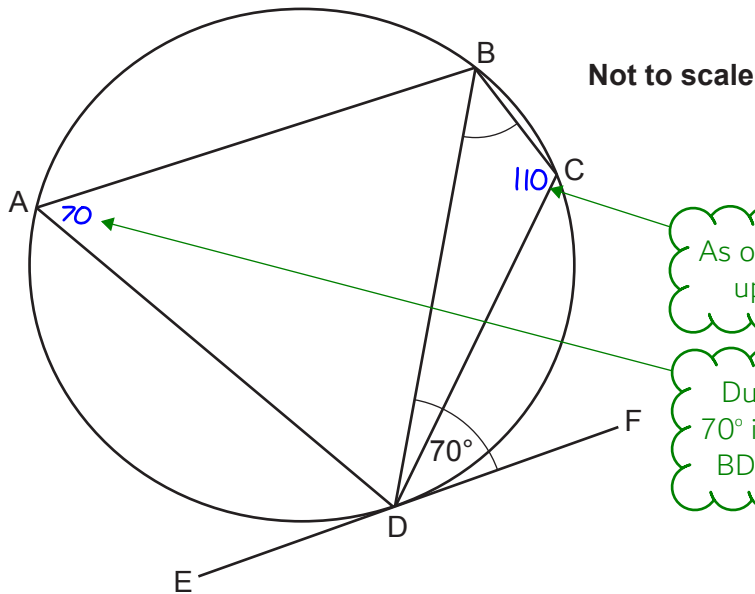
$$\frac{2(x-5)}{x+2}$$

..... [5]

18 A, B, C and D are points on the circumference of a circle.

EF is the tangent to the circle at D.

Angle BDF = 70° .



As opposite angles in a cyclic quadrilateral add up to 180° . ABCD is a cyclic quadrilateral

Due to the alternate segment theorem. The 70° is the angle between tangent EF and chord BD. Angle BAD is the interior opposite angle

The ratio angle BCD : angle CBD is 5 : 2.

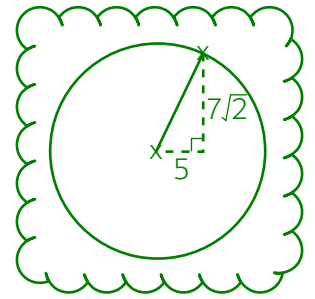
Work out angle CBD.

You must show your working.

$$\frac{022 \times 2}{5110}$$

5 parts of the ratio represent angle BCD, which is 110° . So dividing the 110° by 5 works out that 1 part of the ratio represents 22° . 2 parts of the ratio represent angle CBD so multiplying the value of 1 part by 2 works out what the 2 parts representing angle CBD are worth

.....44..... $^\circ$ [5]



- 19 The point $(5, 7\sqrt{2})$ lies on the circumference of a circle, centre $(0, 0)$.

Find the equation of the circle.

$$5^2 + (7\sqrt{2})^2$$

The radius of the circle goes from the centre to the point given. The distance of 5 from the centre to the point in the x-direction and the distance of $7\sqrt{2}$ from the centre to the point in the y-direction forms a right-angled triangle. So Pythagoras' Theorem can be used to work it out. $a^2 + b^2 = c^2$, where a and b are the shorter sides and c is the longest side. Substituting 5 for a and $7\sqrt{2}$ for b expresses c^2 , which is the radius²

$$\begin{array}{r} 49 \\ \times 2 \\ \hline 98 \\ + 25 \\ \hline 123 \end{array}$$

$$(7\sqrt{2})^2 = 7 \times 7 \times \sqrt{2} \times \sqrt{2} = 49 \times 2$$

Then adding the 5^2

The general equation of a circle with its centre at the origin is $x^2 + y^2 = r^2$, where r is the radius. $r^2 = 123$

$$x^2 + y^2 = 123$$

[4]

- 20 $x^2 - 2y = 5$ and $4y + z = 7$.

Write z in terms of x.

Give your answer in its simplest form.

$$z = 7 - 4y$$

Rearranging the second equation to make z the subject by subtracting 4y from both sides. This is z in terms of y, so y needs to be found

$$-2y = 5 - x^2$$

Rearranging the first equation to make y the subject. First subtracting x^2 from both sides

$$2y = x^2 - 5$$

Dividing both sides by -1 changes the signs of all of the terms on both sides

$$y = \frac{x^2 - 5}{2}$$

Dividing both sides by 2 gets y on its own. y is now in terms of x

$$z = 7 - 2x^2 + 10$$

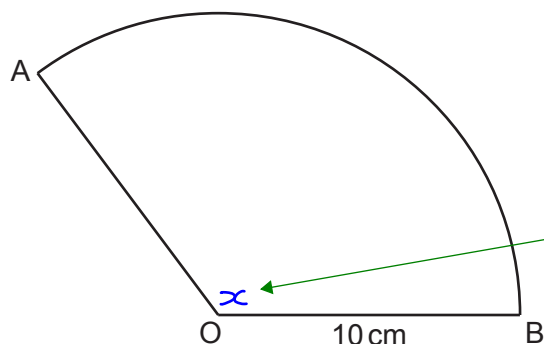
Substituting y in term of x for y in z in terms of y. The -4 and the denominator of 2 cancel out to -2 so it becomes $z = 7 - 2(x^2 - 5)$

Collecting like terms to simplify

$$z = 17 - 2x^2$$

[4]

21 AOB is a sector of a circle, centre O and radius 10 cm.



Not to scale

Let x be the angle of the sector

The area of the sector is $40\pi \text{ cm}^2$.

Work out the perimeter of the sector.

Give your answer in the form $a + b\pi$, where a and b are integers.

You must show your working.

$$\frac{x}{360} \times \pi \times 10^2 = 40\pi$$

Area of circle = $\pi \times \text{radius}^2$. The radius is 10cm. There are 360° around the centre of a circle so $x/360$ is the fraction of the circle the sector is. Doing this fraction of the area of the circle must give the area of the sector

$$x = \frac{40\pi \times 360}{\pi \times 10^2}$$

Rearranged to find the angle x by multiplying both sides by 360 and dividing both sides by $(\pi \times 10^2)$

$$\frac{36 \times 4}{144}$$

The π on the numerator and denominator cancel out. Dividing the 40 by 10 to get 4 and the 360 by 10 to get 36 divides the numerator by 10^2 . Working out the 4×36 finds that the angle of the sector is 144°

$$10 + 10 + \frac{144}{360} \pi \times 10 \times 2$$

Expressing the perimeter of the sector by adding two lots of the radius and the arc length. The arc length is $144/360$ of the circumference. Circumference = $\pi \times \text{diameter}$. Diameter = $2 \times \text{radius}$

$$\frac{12}{30} = \frac{2}{5}$$

Simplifying $144/360$ by dividing both the numerator and denominator by 12 gives $12/30$, which simplifies to $2/5$ by dividing both the numerator and denominator by 6

$$\begin{aligned} 10 + 10 &= 20 \\ 10 \times 2 &= 20 \\ 2/5 \times 20 &= 8 \\ 8 \times \pi &= 8\pi \end{aligned}$$

$$20 + 8\pi$$

..... cm [6]

Turn over for Question 22

22 n is a positive integer.

Prove that $(2n + 1)(n - 3)(n + 2) + 3n(n + 7)$ is always even.

$$2n^2 - 6n + n - 3$$

Expanding out the first two brackets

$$(2n^2 - 5n - 3)(n + 2)$$

Collecting like terms then writing it multiplied by the third bracket

$$2n^3 + 4n^2 - 5n^2 - 10n - 3n - 6 + 3n^2 + 21n$$

Expanding out these two brackets and now expanding out the $3n(n + 7)$ at the end

$$2n^3 + 2n^2 + 8n - 6$$

Collecting like terms

Bringing out 2 as a factor shows that it must be even

$$2(n^3 + n^2 + 4n - 3)$$

[6]

END OF QUESTION PAPER

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