

Please write clearly in block capitals.

Centre number

--	--	--	--	--

Candidate number

--	--	--	--

Surname

---

Forename(s)

---

Candidate signature

---

I declare this is my own work.

# GCSE MATHEMATICS

# H

Higher Tier Paper 2 Calculator

Wednesday 7 June 2023

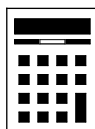
Morning

Time allowed: 1 hour 30 minutes

## Materials

For this paper you must have:

- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).



## Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
<b>TOTAL</b>	

## Advice

In all calculations, show clearly how you work out your answer.



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to [curtis@cgmaths.co.uk](mailto:curtis@cgmaths.co.uk)

Answer **all** questions in the spaces provided.

Do not write  
outside the  
box

1 Write  $30 : 12$  in the form  $n : 1$

[1 mark]

The 12 is divided by 12 to get 1 so the 30 must also be divided by 12.  
This can be left as a fraction as the result will not be a whole number

Answer  $\frac{30}{12} : 1$

2 Four consecutive triangular numbers are 6 10 15 21

+4 +5 +6

Write down the next triangular number.

[1 mark]

1 more is added between each term in the sequence.  $21 + 7 = 28$

Answer  $28$



- 3 Write down the reciprocal of  $\frac{4}{7}$

[1 mark]

Answer \_\_\_\_\_  $\frac{7}{4}$  \_\_\_\_\_

Reciprocal means to do 1 divided by, which flips the fraction

- 4 The price of a toy increases by 12.5% to £19.53

Work out the **original** price of the toy.

[2 marks]

$$100 + 12.5$$

The original price is 100%. Adding the increase of 12.5% works out that it had been increased to 112.5%

$$19.53 \div 112.5$$

Dividing the increased price by the percentage the original had increased to works out that £0.1736 is 1% of the original

$$0.1736 \times 100$$

Multiplying the value of 1% by 100 works out the 100%, which is the original price

Answer £ \_\_\_\_\_  $17.36$  \_\_\_\_\_

Turn over for the next question

Turn over ►



5 Jess saves 2p, 5p and 10p coins.

She has

- 45 10p coins
- 8 times as many 2p coins as **10p coins**
- £17.70 in total.

Work out total **value** of 2p coins : total **value** of 5p coins

Give your answer in its simplest form.

[4 marks]

$$45 \times 0.10 = 4.50$$

10p is £0.10. Multiplying this by 45 works out that the value of the 45 10p coins is £4.50

$$8 \times 45$$

This works out that there are 360 2p coins

$$360 \times 0.02 = 7.20$$

2p is £0.02. Multiplying this by 360 works out that the value of the 360 2p coins is £7.20

$$17.70 - 4.50 - 7.20$$

Subtracting the value of the 10p coins and the value of the 2p coins from the total value of all the coins works out that the value of the 5p coins is £6

$$\frac{7.20}{6} = \frac{6}{5}$$

Ratios simplify in a similar way to fractions. Putting the value of the 2p coins over the value of the 5p coins in the calculator simplifies it to 6/5. So the ratio must be 6 : 5 in its simplest form

Answer   6   :   5  



- 6 (a) Part of a regular polygon is shown.



Not drawn  
accurately

Assume that the polygon is an octagon.

Work out the size of an **exterior** angle.

[2 marks]

$360 \div 8$

Angle x is an exterior angle. The exterior angles of any polygon add up to  $360^\circ$ . As there are 8 sides in an octagon there are also 8 exterior angles. As it is regular all the exterior angles are the same so dividing  $360^\circ$  by the 8 exterior angles works out each one

Answer 45 °

- 6 (b) In fact, the polygon has **more** sides than an octagon.

What does this mean about the size of an exterior angle?

Tick **one** box.

[1 mark]

It is more than the answer to part (a)

It is the same as the answer to part (a)

It is less than the answer to part (a)

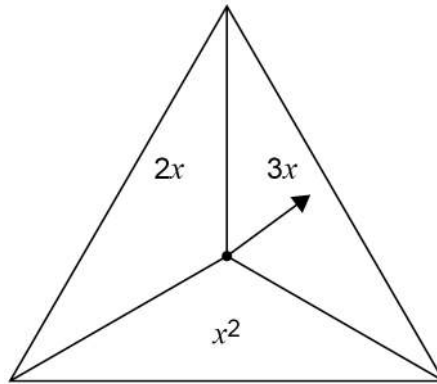
It could be any of the above

There are more than 8 exterior angles so the  $360^\circ$  will be divided by more than 8. Dividing by more will make the exterior angles less



7 In a game,

- an ordinary fair six-sided dice is rolled
- the fair spinner shown is spun.



The score is the dice number **substituted** into the spinner expression.

7 (a) Complete the table to show all of the possible scores.

[2 marks]

	1	2	3	4	5	6
$2x$	2	4	6	8	10	12
$3x$	3	6	9	12	15	18
$x^2$	1	4	9	16	25	36

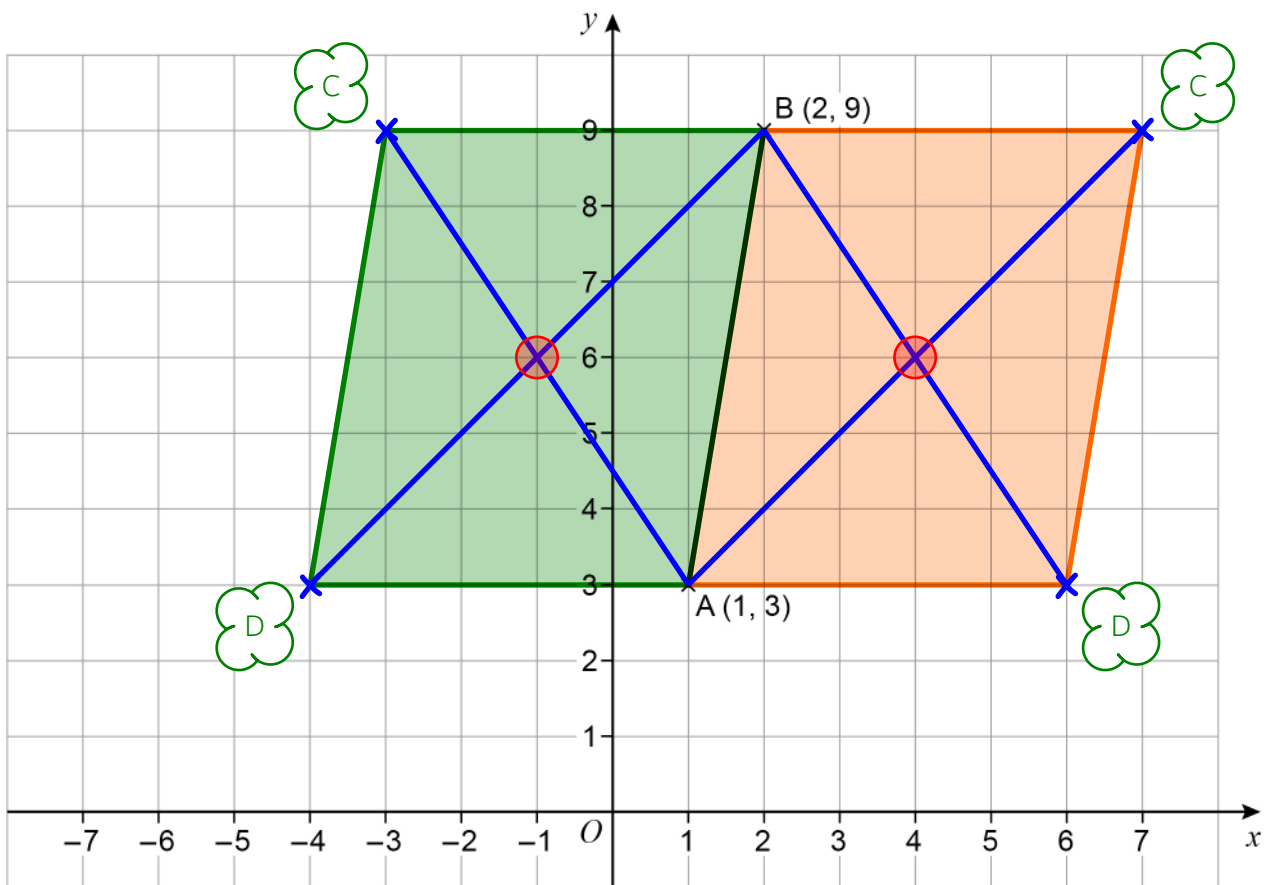
For the first row, all the numbers on the dice are multiplied by 2. For the second row, all of the numbers on the dice are multiplied by 3. For the third row, all of the numbers on the dice are squared







- 9 A (1, 3) and B (2, 9) are points on a centimetre grid.



ABCD is a parallelogram.

AD and BC are **horizontal** and each has length 5 cm

The diagonals of ABCD cross at E.

Work out the **two** possible pairs of coordinates of E.

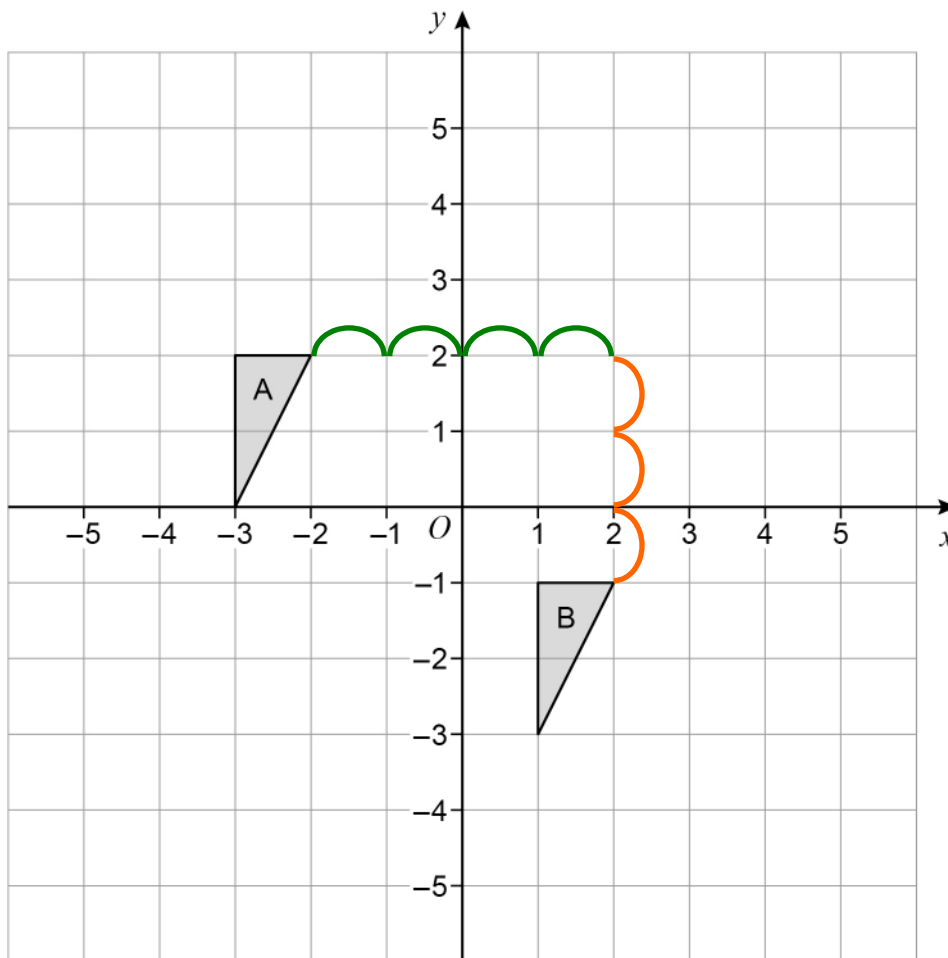
[4 marks]

There are two possible parallelograms (one shown in green and the other shown in orange, which do not have to be drawn). The diagonals of both are shown in blue and where they cross is circled in red

Answer ( -1 , 6 ) and ( 4 , 6 )



10 Write down the translation vector that maps shape A onto shape B. [2 marks]



Answer  $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

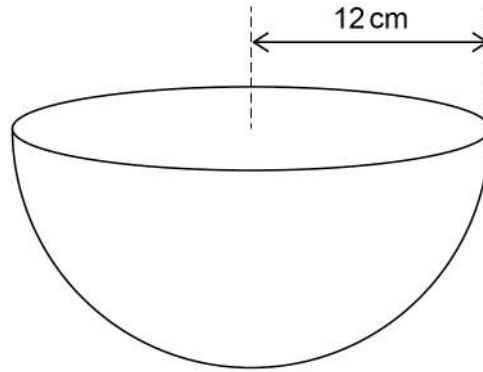
Vectors are in the form  $\begin{pmatrix} x \\ y \end{pmatrix}$ . It has translated 4 in the x-direction (shown in green) and -3 in the y-direction (shown in orange)



11

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

A bowl is a hemisphere with radius 12 cm



Water is poured into the bowl  
at a rate of  $325 \text{ cm}^3$  per second  
for 8 seconds.

Does the water fill **more than** 70% of the bowl?

You **must** show your working.

[4 marks]

$$325 \times 8 = 2600$$

Multiplying the rate in which the water is poured into the bowl by the amount of time it is poured for works out that  $2600 \text{ cm}^3$  is poured into the bowl

$$\frac{1}{2} \times \frac{4}{3} \pi \times 12^3 \times \frac{70}{100} = 2533.3$$

Substituting in the radius of 12 cm into the formula for the volume of a sphere, doing half of this to find the volume of the hemisphere then doing 70% of this finds that 70% of the bowl is  $2533.3... \text{ cm}^3$ . Percentage is out of 100 so putting the 70% over 100 converts it into a fraction, which when multiplied by finds 70%

Yes

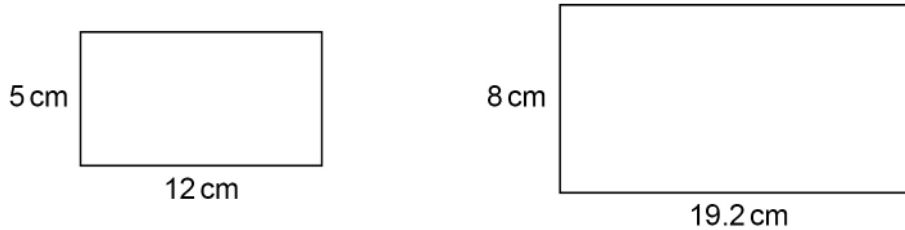
The  $2600 \text{ cm}^3$  poured into the bowl is more than the  $2533.3... \text{ cm}^3$  which is 70% of the bowl



12 Show that these two rectangles are similar.

[2 marks]

Not drawn  
accurately



$$8 \div 5 = \frac{8}{5}$$

$$19.2 \div 12 = \frac{19.2}{12} = \frac{8}{5}$$

Dividing the sides on the larger rectangle by the same sides on the smaller rectangle shows that the same scale factor has been used for all of the sides and therefore they must be similar

13 A factory packs  $x$  boxes of teabags per hour.  
Each box contains 80 teabags.

Show that the factory packs  $\frac{4x}{3}$  teabags per minute.

[2 marks]

$$\frac{80x}{60} = \frac{4x}{3}$$

Multiplying  $x$  by the 80 teabags in each box expresses how many teabags are packed per hour. There are 60 minutes in an hour so dividing this expression by 60 expresses how many teabags are packed per minute. This simplifies to  $4x/3$

Turn over for the next question

Turn over ►



- 14 A company has 123 employees.  
Information about their hourly rates of pay is shown in the table.

Hourly rate, £ $p$	Number of employees
$10 \leq p < 14$	66
$14 \leq p < 20$	32
$20 \leq p < 40$	15
$40 \leq p < 100$	10
	Total = 123

The owner of the company uses the data to make two statements.

**Statement A**

“Over 30% of employees have an hourly rate that is more than £17”

**Statement B**

“The average hourly rate of pay is more than £20”

- 14 (a) Show working that supports **Statement A**.

[3 marks]

$$\frac{3}{6} \times 32$$

Estimating that  $\frac{3}{6}$  of the second interval ( $14 \leq p < 20$ ) is more than £17 as the class width is 6 and 17 is 3 from the end of the interval. Doing this fraction of the 32 to estimate that 16 employees have an hourly rate  $17 < p < 20$

$$16 + 15 + 10$$

Adding the estimated 16 employees who have an hourly rate  $17 < p < 20$  to the 15 who have hourly rate  $20 \leq p < 40$  and the 10 who have hourly rate  $40 \leq p < 100$  as all of these intervals are more than £17. This estimates that 41 employees have an hourly rate of more than £17

$$\frac{41}{123} \times 100 = 33.3\%$$

Putting the estimated 41 employees who have an hourly rate more than £17 over the total 123 employees expresses the fraction estimated to have an hourly rate more than £17. Multiplying this by 100 converts it into a percentage, which is over 30%



14 (b) Why might **Statement A** not be true?

[1 mark]

$$\frac{15+10}{123} \times 100 = 20.3\ldots\%$$

All of the employees with hourly rate  $14 \leq p < 20$  could have hourly rates less than £17 so repeating the calculation done in (a) but with just  $20 \leq p < 40$  and  $40 \leq p < 100$  (which must both be more than £17) to show that the lowest possible percentage is not more than 30%

All of the employees with hourly rate  $14 \leq p < 20$  could have hourly rates less than £17

14 (c) Work out an estimate of the mean to support **Statement B**.

[3 marks]

$$\frac{\frac{10+14}{2} \times 66 + \frac{14+20}{2} \times 32 + \frac{20+40}{2} \times 15 + \frac{40+100}{2} \times 10}{123} = 20.21$$

Doing the mean of the upper bound and the lower bound for each interval works out the midpoint of each interval. These means are found by adding the upper bound to the lower bound then dividing by 2. Multiplying the midpoints by the frequency of each interval works out an estimate for the total hourly rate of all of the employees in each interval. Adding all these estimates gives an estimate of the total hourly rate of all the employees. Dividing this by the 123 employees works out an estimate of the mean. £20.21 is more than £20 so this supports Statement B

14 (d) Why is the mean **not** the best average to represent the data?

[1 mark]

Most of the employees had an hourly rate less than £20

So the mean is not a very representative average. It is skewed higher because of the high midpoint of the last interval which goes up to 100



15 Expand  $(x^2 - 9xy)(2x + 5y)$

[2 marks]

$$\begin{aligned} x^2 \times 2x &= 2x^3 \\ x^2 \times 5y &= 5x^2y \\ -9xy \times 2x &= -18x^2y \\ -9xy \times 5y &= -45xy^2 \end{aligned}$$

Answer  $2x^3 + 5x^2y - 18x^2y - 45xy^2$

There is no need to simplify by collecting the like terms

16 Line A

has equation  $y = ax - 1$

passes through the point (7, 13)

Line B has equation  $5y - 3x = 4$

Show that line A has a greater gradient than line B.

[3 marks]

The general equation of a straight line is  $y = mx + c$ , where  $m$  is the gradient and  $c$  is the y-intercept

$$a = \frac{y+1}{x}$$

Rearranged to make  $a$  the subject by adding 1 to both sides then dividing both sides by  $x$ , as  $a$  represents the gradient of line A

$$= \frac{13+1}{7}$$

Substituted in the  $x$  and  $y$ -coordinates of the point (7, 13) as this is on the line so must satisfy the equation

$$= 2$$

$a = 2$  so the gradient of line A is 2

$$5y = 3x + 4$$

Adding  $3x$  to both sides of the equation of line B to get the  $y$  term on its own

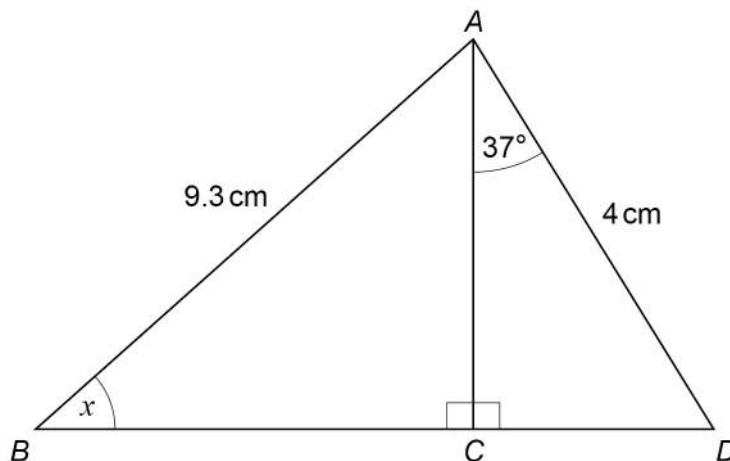
$$y = 0.6x + 0.8$$

Dividing all terms on both sides by 5 makes  $y$  the subject and therefore puts it into the form  $y = mx + c$

Line A has gradient of 2  
Line B has gradient of 0.6



17

Not drawn  
accuratelyWork out the size of angle  $x$ .**[4 marks]**

$$\begin{array}{c} \text{O} \\ \text{H} \\ \text{C} \end{array} \begin{array}{c} \text{A} \\ \text{H} \\ \text{T} \end{array} \begin{array}{c} \text{O} \\ \text{A} \end{array}$$

Using right-angled trigonometry in triangle ACD to work out side AC.  
Ticking H as side AD is the hypotenuse. Ticking A as side AC is the adjacent

$$\cos 37 \times 4 = 3.1... \text{ (A)}$$

There are two ticks on the CAH formula triangle so this one can be used.  
From the formula triangle, adjacent = cos of the angle  $\times$  hypotenuse.  
Storing the exact value as A on the calculator

$$\begin{array}{c} \text{O} \\ \text{H} \\ \text{C} \end{array} \begin{array}{c} \text{A} \\ \text{H} \\ \text{T} \end{array} \begin{array}{c} \text{O} \\ \text{A} \end{array}$$

Using right-angled trigonometry in triangle ABC to work out angle  $x$ .  
Ticking O as side AC is the opposite. Ticking H as side AB is the hypotenuse

$$\sin x = \frac{3.1...}{9.3}$$

There are two ticks on the SOH formula triangle so this one can be used.  
From the formula triangle, sin of the angle = opposite/hypotenuse.  
Using the exact value of side AC which was stored as A on the calculator

$$x = \sin^{-1}\left(\frac{3.1...}{9.3}\right)$$

Rearranging to find  $x$  by doing  $\sin^{-1}$  of both sides

$$x = \underline{\hspace{2cm} 20.1 \hspace{2cm}}^{\circ}$$

Turn over ►





18 Rearrange  $y = \frac{x+8}{x}$  to make  $x$  the subject.

[3 marks]

$$xy = x + 8 \quad \leftarrow \text{Multiplying both sides by } x \text{ to eliminate it as the denominator}$$

$$xy - x = 8 \quad \leftarrow \text{Subtracting } x \text{ from both sides to get all the terms involving } x \text{ on the same side}$$

$$x(y-1) = 8 \quad \leftarrow \text{Factorising by bringing } x \text{ out as a factor}$$

Dividing both sides by  $(y-1)$  to make  $x$  the subject

$$x = \frac{8}{y-1}$$

Answer \_\_\_\_\_



19

Here are the first four terms of a quadratic sequence.

3      20      47      84

Work out an expression for the  $n$ th term of the sequence.**[4 marks]**Quadratic sequences are in the form  $an^2 + bn + c$ 

17   27

Listing out the first differences. The difference between 3 and 20 is 17 and the difference between 20 and 47 is 27

10

The second difference (the difference of the first differences) is 10 as this is the difference between 17 and 27

 $5n^2: 5, 20$  $a$  is half of the second difference so  $a$  is 5. Listing out the sequence of  $5n^2$ .  $n$  is 1 on the first term.  $5(1)^2 = 5$ .  $n$  is 2 on the second term.  $5(2)^2 = 20$  $-2, 0: 2n-4$ Listing what needs to be added to each term of the  $5n^2$  sequence to get the original sequence.  $5 + -2 = 3$  and  $20 + 0 = 20$ . The sequence  $-2, 0$  increases by 2 so must involve  $2n$ . The 0th term (the one before the first term) would be  $-4$  so the  $n$ th term of this sequence must be  $2n - 4$ Adding the sequence of  $2n - 4$  to the sequence of  $5n^2$  gives the original sequence

Answer \_\_\_\_\_

$$5n^2 + 2n - 4$$

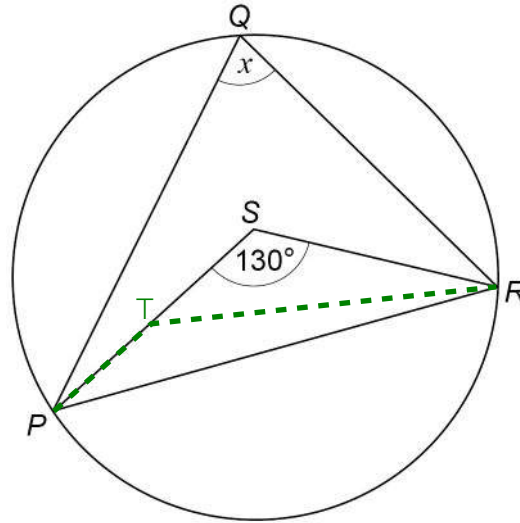
7

Turn over ►



- 20 (a)  $P$ ,  $Q$  and  $R$  are points on a circle.  
 $S$  is a point inside triangle  $PQR$ .

Not drawn  
accurately



Assume that  $S$  is the centre of the circle.

Work out the size of angle  $x$ .

[1 mark]

$$130 \div 2$$

The angle at the circumference is half the angle at the centre

$$x = \underline{\quad 65 \quad}^{\circ}$$

- 20 (b) In fact, the centre of the circle is on  $PS$  but **not** at  $S$ .

What does this mean about the size of angle  $x$  ?

Tick **one** box.

[1 mark]

It is the same as the answer to part (a)

It is greater than the answer to part (a)

It is smaller than the answer to part (a)

It is impossible to tell

Moving the centre of the circle away from  $S$  and toward  $P$  makes the angle at the centre greater than  $130^{\circ}$  and will therefore make angle  $x$  greater. Angle  $PTR$  is greater than angle  $PSR$



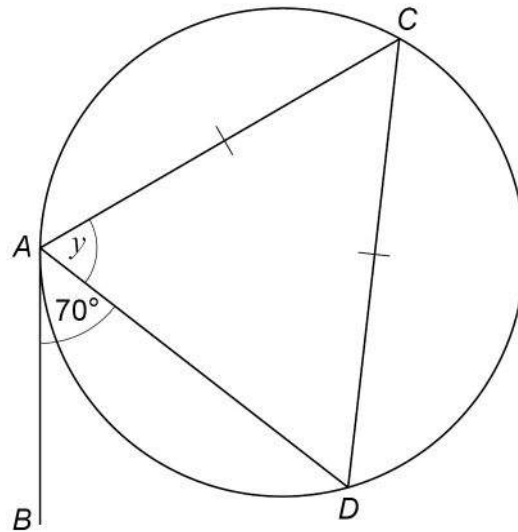
20 (c) For a different circle,

$AB$  is a tangent at  $A$

$C$  and  $D$  are on the circumference of the circle

$AC = CD$

Not drawn  
accurately



Here is Simon's method to work out the size of angle  $y$ .

Angle  $ADC = 70^\circ$  (alternate segment theorem)  
Therefore  $y = 70^\circ$  (angles in an isosceles triangle)

Is he correct?

Give a reason for your answer.

[1 mark]

No, he used the alternate segment theorem incorrectly

Using the alternate segment theorem shows that angle  $ACD = 70^\circ$ , not angle  $ADC$



- 21 Magana decides to put £500 into an account that pays compound interest. She wants to have **at least** £560 in the account after 3 years.

Work out to 1 decimal place the **minimum** annual interest rate she needs.

[3 marks]

$$500 \times \left(\frac{100+x}{100}\right)^3 = 560$$

Using the compound interest formula. The original amount is 100%. Adding x% (where x is the minimum annual interest rate) to this increases it by x%. Putting this over 100 converts the percentage to a fraction, which when multiplied by increases the £500 by x%. Raising the fraction to the power of 3 as it needs to be increased 3 times. This is set equal to £560 as this is the minimum amount she wants to have

$$\frac{100+x}{100} = \sqrt[3]{\frac{560}{500}}$$

Rearranging to find x. Dividing both sides by 500 then cube rooting

$$x = \sqrt[3]{\frac{560}{500}} \times 100 - 100$$

Multiplying both sides by 100 then subtracting 100

Answer \_\_\_\_\_ **3.9** \_\_\_\_\_ %

The answer of 3.84... must be rounded up to 3.9% as 3.8% would not be enough



- 22 An approximate value of a root of an equation,  $x$ , can be found using the iterative formula

$$x_{n+1} = \sqrt[3]{5(x_n)^2 - 2x_n - 3}$$

The starting value is  $x_1 = 4$

- 22 (a) Work out the values of  $x_2$  and  $x_3$

[2 marks]

Enter 4 then press = (or EXE). Enter  $\sqrt[3]{5\text{Ans}^2 - 2\text{Ans} - 3}$ . Press = (or EXE) to get  $x_2$ . Press it again to get  $x_3$

This sets 4 as  $x_1$  then substitutes it in the right side of the iteration formula to get  $x_2$ . Then  $x_2$  is substituted into the right side of the iteration formula to get  $x_3$

$$x_2 = \underline{\hspace{2cm} 4.10 \hspace{2cm}}$$

$$x_3 = \underline{\hspace{2cm} 4.18 \hspace{2cm}}$$

- 22 (b) By continuing the iteration, show that the value of  $x$  is more than 4.25

[1 mark]

$$x_4 = 4.23$$

$$x_5 = 4.28$$

Carrying on from  $x_3$ , enter  $\sqrt[3]{5\text{Ans}^2 - 2\text{Ans} - 3}$ . Keep pressing = (or EXE)

This gives the values of  $x_4$  and  $x_5$  and so on. We can write down the next 2 after  $x_3$  to show that it is rising and that it goes over 4.25



23

Here are three sets of cards.

**Set A**

1
---

1
---

3
---

5
---

5
---

5
---

6
---

8
---

**Set B**

1
---

2
---

4
---

6
---

8
---

8
---

9
---

**Set C**

3
---

4
---

5
---

6
---

In a game, a player has two options.

**Option 1**  
Pick two cards from Set A

**Option 2**  
Pick one card from Set B  
and  
pick one card from Set C

The cards are picked at random.

The player wins if the total of their two cards is exactly 10



Which option gives a better chance of winning?

Option 1

Option 2

Show working to support your answer.

[4 marks]

① 5 & 5

Listing out the cards which can be picked for option 1 to give a total of 10

$$\frac{3}{8} \times \frac{2}{7} \times 100 = 10.7\%$$

The probability of the first 5 is  $\frac{3}{8}$  as 3 out of the 8 cards are a 5. The probability of the second 5 is  $\frac{2}{7}$  as there is one fewer 5 after the first pick and there is one fewer card in total. 'And' means to multiply. Multiplying by 100 converts the probability into a percentage, which makes it easier to compare

② 4 & 6 or 6 & 4

Listing out the cards which can be picked for option 2 to give a total of 10

$$\left(\frac{1}{7} \times \frac{1}{4} + \frac{1}{7} \times \frac{1}{4}\right) \times 100 = 7.1\%$$

The probability of picking a 4 for the first pick is  $\frac{1}{7}$  as 1 out of the 7 cards are a 4 in set B. The probability of picking a 6 on the second pick is  $\frac{1}{4}$  as 1 out of the 4 cards are a 6 in set C. The probabilities for the 6 then 4 are the same. 'And' means to multiply, 'or' means to add. Multiplying by 100 converts the probability into a percentage, which makes it easier to compare

10.7% is greater than 7.1%, so option 1 has a better chance of winning by getting a total of 10

Turn over for the next question

Turn over ►





24

 $a = 65$  to the nearest integer $b = 30$  to 1 significant figureWork out the **upper bound** for  $2a^2 - b^2$ You **must** show your working.**[3 marks]**

$$2\left(65 + \frac{1}{2}\right)^2 - \left(30 - \frac{10}{2}\right)^2$$

Substituting in the upper bound of  $a$  and the lower bound of  $b$  (as a greater amount will be given if subtracting less). The upper bound of  $a$  is expressed by adding half of the resolution of  $a$  (which is 1 as it is to the nearest integer and integers go up in 1s). The lower bound of  $b$  is expressed by subtracting half of the resolution of  $b$  (which is 10 as the 1st significant figure is in the 10s place)

Answer 7955.5



25 Show that  $\frac{x-5}{x-2} + \frac{x+5}{x+2}$

simplifies to  $\frac{ax^2-b}{x^2-4}$  where  $a$  and  $b$  are integers.

[3 marks]

$$\frac{(x-5)(x+2)}{(x+2)(x-2)} + \frac{(x+5)(x-2)}{(x+2)(x-2)}$$

To add fractions the denominators need to be the same. Multiplying both the numerator and denominator of the first fraction by  $(x+2)$  and both the numerator and denominator of the second fraction by  $(x-2)$

$$\frac{x^2+2x-5x-10+x^2-2x+5x-10}{x^2-2x+2x-4}$$

Expanding all the brackets. The two fractions are added by adding the numerators and the shared denominator stays the same

$$\frac{2x^2-20}{x^2-4}$$

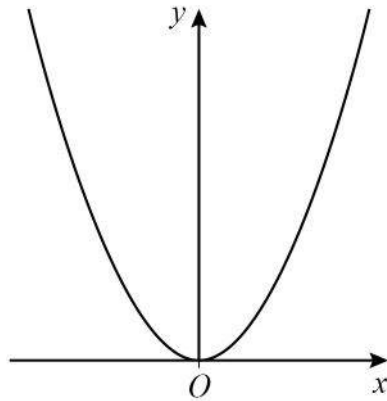
Simplifying by collecting like terms

Turn over for the next question

Turn over ►



26 Here is a sketch of  $y = x^2$



26 (a) The minimum point of  $y = x^2$  is at  $(0, 0)$

Write down the coordinates of the minimum point of  $y = x^2 + 2$

2 has been added to the right side so y is increased by 2

[1 mark]

Answer ( 0 , 2 )

26 (b) The graph  $y = x^2$  is reflected in the  $x$  axis.

Write down the equation of the graph after this transformation.

[1 mark]

Answer  $y = -x^2$

Making the whole of the right side negative reflects it in the x axis

26 (c)  $y = x^2$  is now transformed to give  $y = (x + 3)^2$

Describe fully this single transformation.

[2 marks]

Translation 3 to the left

3 is added to x so this moves 3 to the left

END OF QUESTIONS

