OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

H

Friday 20 May 2022 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number						Candidate number				

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** the questions.

1 (a) Write 6 050 000 in standard form.

6050000 must be divided by 10 6 times to get 6.05, which is a decimal between 1 and 10. So it must be multiplied by 10^6 to keep it equal

(a) 6.05×10^6 [1]

(b) Write 4.58×10^{-3} as an ordinary number.

x 10^{-3} means to divide by 10 3 times

(b) 0.00458 [1]

2 Calculate.

$$\frac{270}{2.5^2} - \frac{4.6 + 17.2}{8.4 - 6.8}$$

Type it into the calculator exactly as it is above

29.575 [2]

3 In January 2018, an art collector bought an antique painting.
In January 2020, he sold it for £17 640.

Assume the value of the painting increased by 5% each year.

Calculate the art collector's profit.
You must show your working.

$x \times \left(\frac{100+5}{100}\right)^2 = 17640$

Let x be the original price in January 2018. Adding 5% to 100% expresses the percentage it increases to. Putting this over 100 converts it into a fraction. Multiplying x by this fraction increases it by 5%. The fraction is raised to the power of 2 as it needs to increase by 5% twice as 2020 is 2 years after 2018. This must be equal to what it was sold for in January 2020

$x = \frac{17640}{\left(\frac{100+5}{100}\right)^2}$

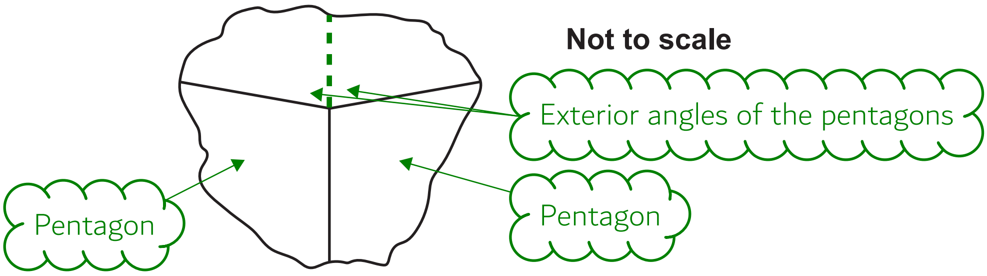
Rearranging to find x by dividing both sides by what x was multiplied by

$17640 - 16000$

Subtracting the original price in January 2018 from the price it was sold for in January 2020 works out the profit

£ 1640 [5]

4 Three **regular** polygons meet at a point.



Two of the polygons are pentagons.

Find the number of sides of the third polygon.
You must show your working.

$360 \div 5$ ← The exterior angles of any polygon add up to 360°. So dividing 360° by the 5 sides of each pentagon (which is the same as the number of exterior angles) works out that their exterior angles are 72°

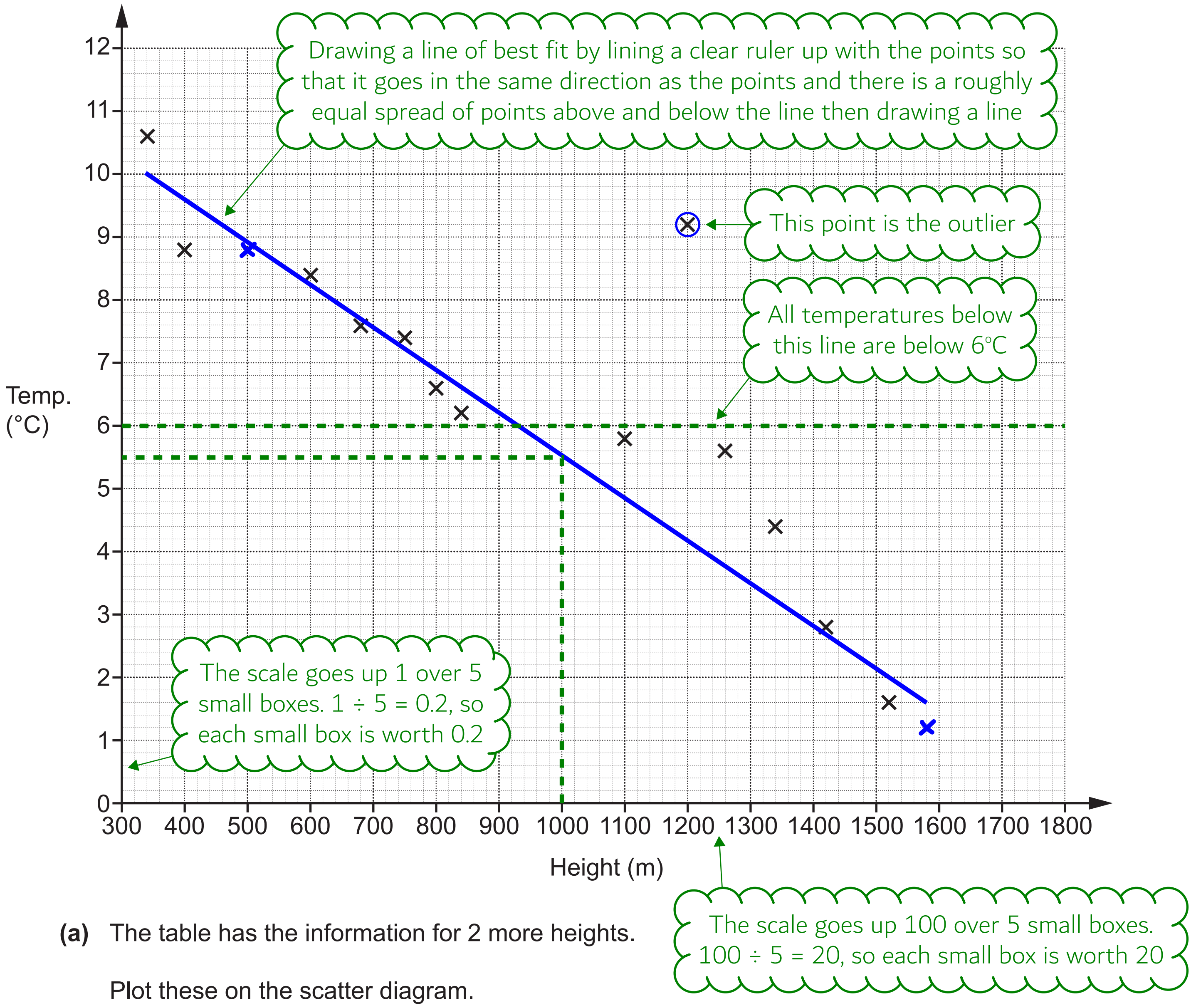
72×2 ← The interior angle of the third polygon is equal to the total of two of the exterior angles of the pentagons

$180 - 144$ ← Interior angles lie on a straight line with exterior angles and angles around a point on a straight line add up to 180°, so subtracting the interior angle of the third polygon from 180° works out the exterior angle

$360 \div 36$ ← The exterior angles of any polygon add up to 360°. So dividing 360° by each exterior angle works out that there are 10 exterior angles, and therefore 10 sides on the third polygon

.......................10....................... **[6]**

5 The scatter diagram shows the midday temperature at 13 different heights on a mountain.



(a) The table has the information for 2 more heights.

Plot these on the scatter diagram.

Height (m)	500	1580
Temperature (°C)	8.8	1.2

[2]

(b) Describe the type of correlation shown in the scatter diagram.

(b) Negative **[1]**

As the temperature decreases as the height increases

(c) By drawing a line of best fit, estimate the temperature at 1000 m.

(c) ....5.5.......... °C **[2]**

(d) Circle the outlier on the scatter diagram. **[1]**

(e) Explain why using the scatter diagram to estimate the temperature at 1800 m may be unreliable.

Outside the range of the data

The data given only goes up to 1580m. The trend may not continue

.... **[1]**

(f) Find the percentage of the 15 temperatures which are below 6 °C.

$\frac{6}{15} \times 100$ ← 6 out of the 15 temperatures are below 6°C. Expressing this as a fraction. Converting it into a percentage by multiplying by 100

(f) ....40.......... % **[3]**

6 A machine can dig, on average, 2 cm of tunnel each minute.
It operates 24 hours each day.

(a) Work out how many days it should take to dig a tunnel of length 3.5 km.
Give your answer to the nearest day.

s d t ← Speed = distance ÷ time. Writing this as a formula triangle

3.5×1000 ← The speed is in cm per minute. The distance needs to be in cm to be compatible with this. Converting the 3.5km into metres using the fact there are 1000m in 1km

3500×100 ← Converting 3500m into centimetres using the fact there are 100cm in 1m

350000÷2 ← From the formula triangle, time = distance ÷ speed. The distance is 350000cm and the speed is 2cm per minute

175000÷60 ← Converting the time in minutes into hours using the fact there are 60 minutes in an hour

$2916.\dot{6}\div24$ ← Converting the time in hours into days using the fact there are 24 hours in a day

The answer of 121.5... is rounded to the nearest day

(a) ....122............ days **[4]**

(b) The machine actually digs an average of 2.5 cm of tunnel each minute for most of the time and an average of 1.5 cm each minute for the rest of the time.

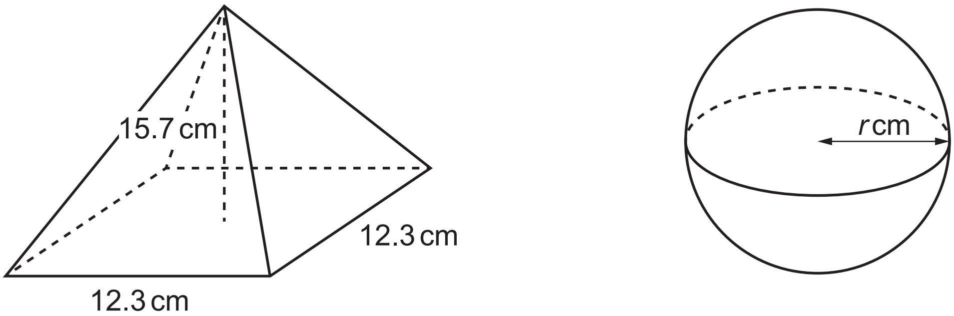
How would this affect your answer to part **(a)**?

It would be less ....

.... **[1]**

As the speed is more by 0.5cm per minute for most of the time and is less by 0.5cm per minute for some of the time. Both differences are the same but it is more for more of the time

7 The diagram shows a square-based pyramid and a sphere.



The pyramid has base length 12.3 cm and perpendicular height 15.7 cm.
The sphere has radius r cm.

The pyramid and the sphere have the same volume.

Work out the radius of the sphere.
You must show your working.

[The volume of a pyramid is $\frac{1}{3}$ × area of base × perpendicular height.

The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$\frac{4}{3}\pi r^3 = \frac{1}{3} \times 12.3^2 \times 15.7$ ← Setting the volume of the sphere equal to the volume of the pyramid. The base is a square. Area of square = length²

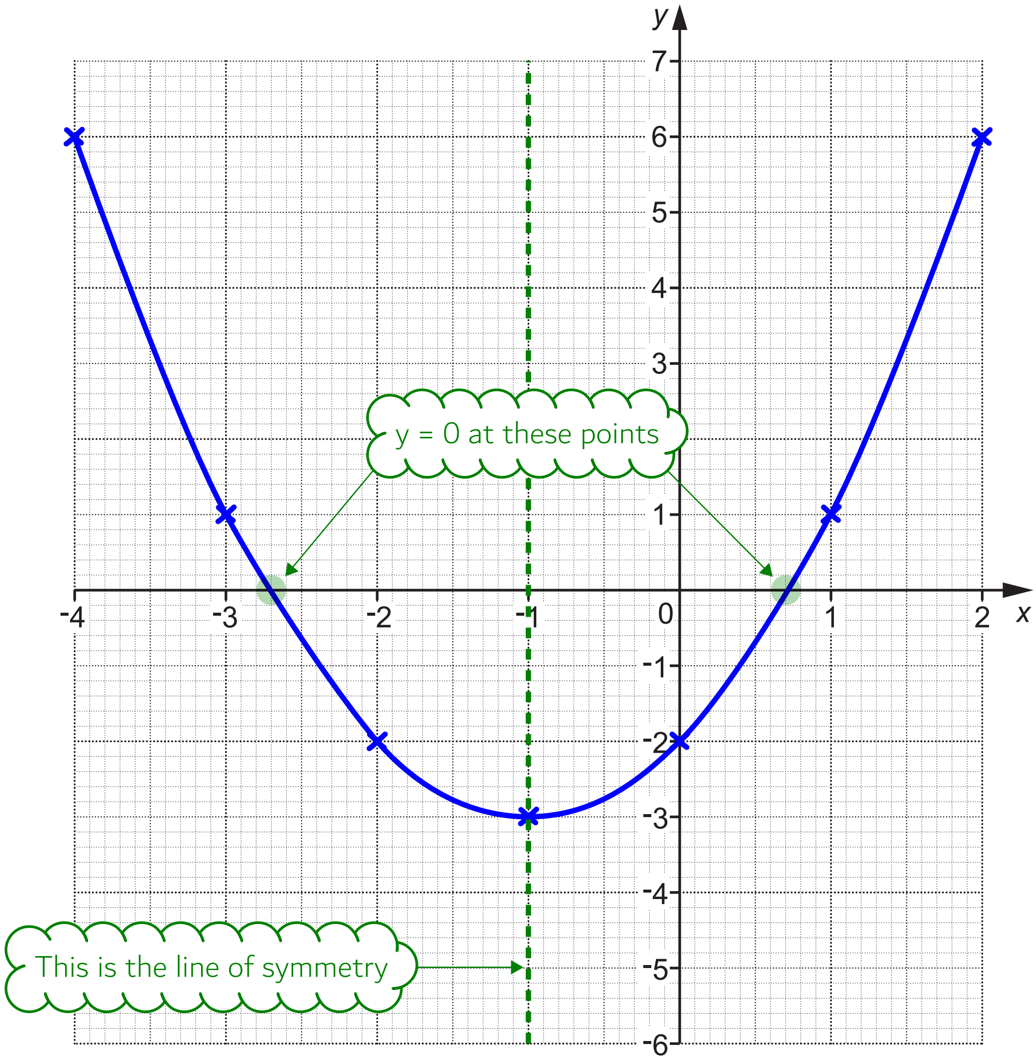
$r = \sqrt[3]{\dfrac{\frac{1}{3} \times 12.3^2 \times 15.7}{\frac{4}{3}\pi}}$ ← Rearranging to find r by dividing both sides by 4/3 and π then cube rooting both sides

.............5.7............. cm **[5]**

8 Here is a table of values for $y = x^2 + 2x - 2$.

x	-4	-3	-2	-1	0	1	2
y	6	1	-2	-3	-2	1	6

(a) Draw the graph of $y = x^2 + 2x - 2$ for $-4 \leqslant x \leqslant 2$.



[3]

(b) Write down the equation of the line of symmetry of the graph.

Every point on the line of symmetry has an x-coordinate of -1

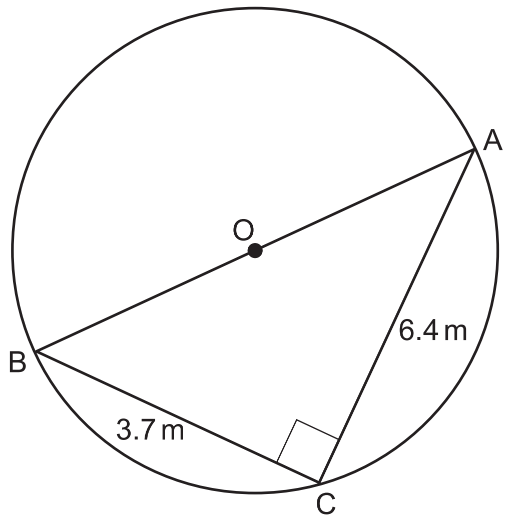
(b) .... x=-1 .... **[1]**

(c) Use the graph to solve the equation $x^2 + 2x - 2 = 0$.
Give your answers to **1** decimal place.

It is basically asking what x is when y = 0

(c) $x =$ .... -2.7 .... or $x =$ .... 0.7 .... **[2]**

9 Points A, B and C lie on the circumference of a circle, centre O.



Not to scale

Angle ACB = 90°, AC = 6.4 m and BC = 3.7 m.

Work out the circumference of the circle.
You must show your working.

$a^2+b^2=c^2$ ← Pythagoras' Theorem can be used to work out the missing side in a right-angled triangle

$AB=\sqrt{3.7^2+6.4^2}$ ← Square rooting both sides to make c the subject then substituting AB for c (as it is the longest side), 3.7 for a and 6.4 for b

$\pi \times 7.3...$ ← Circumference = π x diameter. AB is the diameter. Using the exact answer to the previous calculation

.......................23.2....................... m **[5]**

10 A student is researching the difference in how much exercise adults and children do.
To collect their data, the student interviews the first 25 people found in the High Street at 11 am on one Monday morning.

(a) Make **three** different criticisms of the student's method of collecting data.

1 Small sample

Only 25 people were asked. Ideally it would be more than this

2 Not random

Many people will be at work or at school at 11am on a Monday morning so will be excluded from the sample

3 Might be no children in the sample

Children may be unlikely to be in the High Street at 11am on a Monday morning

[3]

(b) Here is the data collection table that the student used.

Hours exercised in a week (*h*)	**Adult tally**	**Child tally**
$0 \leqslant h \leqslant 2$		
$2 \leqslant h \leqslant 4$		
$4 \leqslant h \leqslant 8$		
$8 \leqslant h \leqslant 12$		
$12 \leqslant h \leqslant 20$		

Make **one** criticism of the student's table.

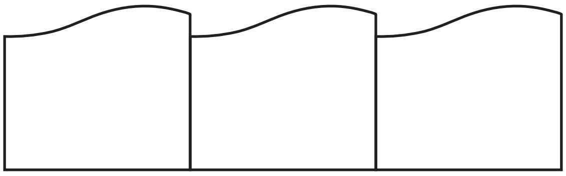
The groups overlap

2 hours can be in the first and second group as both inequalities allow the hours exercised in a week to be equal to 2

[1]

11 Jamie buys fence panels that fit tightly together.

Not to scale



Each panel has a length of 1.8 m, correct to 1 decimal place.
Jamie measures the length of a garden as 42 m, correct to the nearest metre.

Work out the minimum number of panels Jamie should buy in order to be certain that there are enough panels for the length of the garden.
Show how you decide.

The worst case scenario needs to be considered. This will be when the garden is as long as possible and the panels are as short as possible. So the upper bound of the length of the garden and the lower bound of the length of the panels needs to be used

$42 + \frac{1}{2} = 42.5$ ← Adding half of the resolution works out the upper bound of the length of the garden. The resolution is 1 as it is to the nearest 1 metre

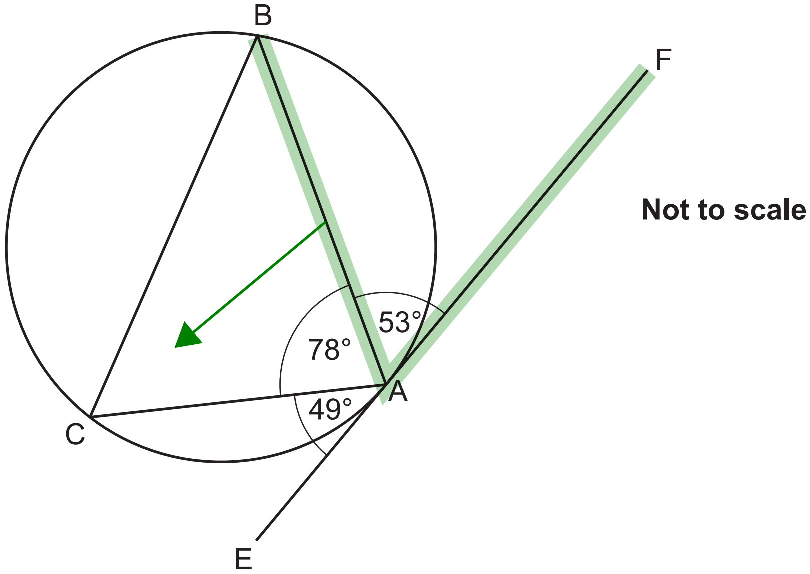
$1.8 - \frac{0.1}{2} = 1.75$ ← Subtracting half of the resolution works out the lower bound of the length of the panels. The resolution is 0.1 as it is correct to 1 decimal place

$42.5 \div 1.75$ ← Dividing the upper bound of the length of the garden by the lower bound of the length of the panels works out the maximum amount of panels he could need and therefore how many he needs to be certain that there will be enough

The answer of 24.2... is rounded up the next whole number as there needs to be a whole number of panels and 24 might not be enough

25 **[4]**

12 (a) Points A, B and C lie on the circumference of a circle.
EAF is a tangent to the circle.



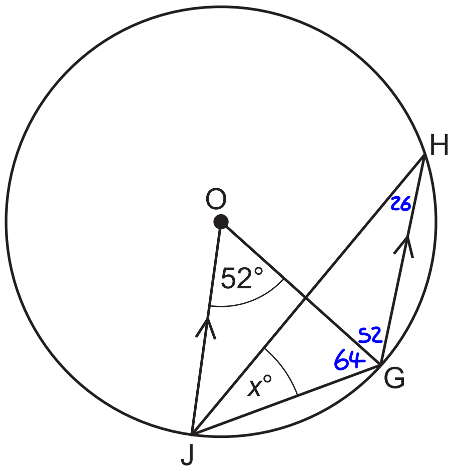
Write down the value of angle BCA giving a reason for your answer.

Angle BCA = ...53........° because ...alternate segment theorem...

...The angle between a tangent and a chord is equal to the interior opposite angle...

.... **[2]**

(b) Points G, H and J lie on the circumference of a circle, centre O.



Not to scale

Angle GOJ = 52° and angle GJH = x°.
Lines JO and GH are parallel.

Find the value of x.
You must show your working.

56÷2 ← The works out that angle JHG is 26° as the angle at the circumference is half the angle at the centre

$\frac{180-52}{2}$ ← This works out that angle OGJ is 64° as the base angles in an isosceles triangle are equal and there are 180° in total in a triangle. Triangle OJG is isosceles as both OJ and OG are radii so are equal in length

180−26−52−64 ← Subtracting the other angles in triangle JHG from the total of 180° in a triangle works out angle x

(b) x = ...38........................ **[5]**

13 Here is a restaurant's menu.

Starter	**Main**	**Dessert**
Prawn Cocktail	Hunter's Chicken	Trifle
Duck Spring Rolls	Beef Curry	Ice Cream
Lamb Meatballs	Steak	Cheesecake
Leaf Salad (V)	Fish Pie	Chocolate Cake
Mushroom Soup (V)	Lasagne	Bakewell Tart
	Egg Salad (V)	Fruit Salad (V)
	Vegetable Hot Pot (V)	Cherry Pie (V)
	Macaroni Cheese (V)	

(V) denotes vegetarian

(a) A 3-course meal consists of one starter, one main and one dessert.

Work out how many different 3-course meals can be chosen from the menu.

$5 \times 8 \times 7$

Using the product rule for counting. There are 5 starters, 8 mains and 7 desserts. Multiplying all these together works out how many options there are in total

(a) ....280........ **[2]**

(b) Find the fraction of the 3-course meals which are completely vegetarian (V).

$2 \times 3 \times 2$

Using the product rule for counting. There are 2 starters which are vegetarian, 3 mains which are vegetarian and 2 desserts which are vegetarian. Multiplying all these together works out that 12 options are vegetarian in total

12 out of the 280 options are vegetarian

(b) ....$\frac{12}{280}$........ **[2]**

14 $(x+2)(3x+a)(bx+3) = 6x^3 + 11x^2 - 17x - 30$

Find the value of *a* and the value of *b*.

$3b=6$ ← There is no need to expand out all the brackets on the left. $x \times 3x \times bx = 3bx^3$, which must be equal to $6x^3$ as this will be the only x^3 term and there is $6x^3$ on the right. Equating the coefficients gives 3b = 6. Dividing both sides by 3 finds that b = 2

$6a=-30$ ← 2 x a x 3 = 6a, which must be equal to -30 as this will be the only constant term and there is -30 on the right. Dividing both sides by 6 finds that a = -5

a = ...-5........

b = ...2........ **[2]**

15 Use algebra to prove that an odd number multiplied by a different odd number always gives an answer that is an odd number. **[4]**

$(2n+1)(2m+1)$ ← n is an integer. Multiplying it by 2 makes it even. Adding 1 to it makes it odd. Doing the same with m, which represents another integer. Multiplying the two expressions for different odd numbers

$4mn+2n+2m+1$ ← Expanding the brackets

$2(2mn+n+m)+1$ ← Bringing out 2 as a factor and leaving the +1 to show it is 1 more than an even number

Adding 1 to an even gives an odd

2mn + n + m will be an integer. Multiplying this by 2 must give an even number. Adding 1 to this gives an odd number

16 Li bought a house at the start of 2016.
Li assumes the value of the house, £V, can be predicted using the formula

$V = 185\,000 \times 1.035^n$

where n is the number of years after the start of 2016.

(a) Explain how you know that the value of the house is predicted to increase each year.

$1.035 > 1$ .... So multiplying by it will increase the value ....

.... **[1]**

(b) Write down the percentage increase per year that is used in the formula.

1.035 x 100 = 103.5. So it is increasing to 103.5% each year

(b) .... 3.5 .... % **[1]**

(c) Write down the value of the house at the start of 2016.

n = 0 at the start of 2016. $1.035^0 = 1$ so the 185000 will be multiplied by 1 and stay the same

(c) £ .... 185000 .... **[1]**

(d) Calculate the predicted value of the house at the start of 2020, giving your answer correct to **4** significant figures.

2020−2016 ← This works out that the start of 2020 is 4 years after the start of 2016

185000×1.035^4 ← Substituting 4 for n in the formula

Writing 212291.7551 correct to 4 significant figures →

(d) £ .... 212300 .... **[2]**

(e) **(i)** Compared with its value at the start of 2016, show that the formula predicts the house will have doubled in value at some point during 2036. **[3]**

2036−2016 ← This works out that the start of 2036 is 20 years after the start of 2016

$1.035^{20} = 1.9...$ ← Raising the multiplier to the power of 20 shows that after exactly 20 years the value of the house will be less than 2 times the value at the start of 2016

$1.035^{21} = 2.0...$ ← Raising the multiplier to the power of 21 shows that after exactly 21 years the value of the house will be more that 2 times the value at the start of 2016

(ii) Give **one** reason why this may not happen.

The rate of increase may not continue ....

.... It may not continue rising by 3.5% per year forever .... **[1]**

17 There are 15 sweets in a bag.
10 of the sweets are toffee and 5 are mint.
Reece takes two of the sweets at random.

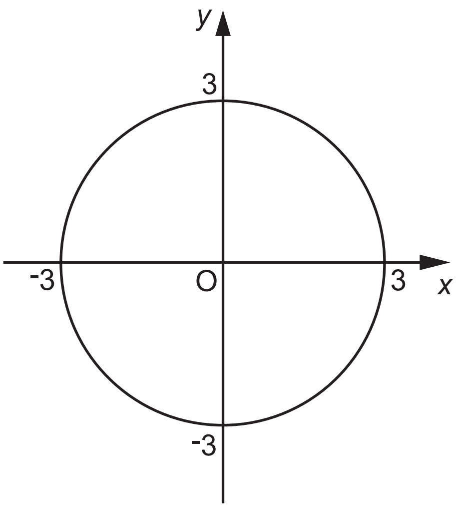
Work out the probability that Reece takes one of each type of sweet.

$\frac{10}{15}\times\frac{5}{14}+\frac{5}{15}\times\frac{10}{14}$

Toffee AND mint OR mint AND toffee. AND means to multiply the probabilities, OR means to add the probabilities. The probability of the first sweet being toffee is 10/15 as 10 out of the 15 sweets are toffee. The probability of the second sweet being mint is 5/14 as there is 1 fewer sweet in total after the first one is picked so 5 out of the 14 sweets is mint

.... $\frac{10}{21}$ .... **[4]**

18 The diagram shows a circle, centre the origin.



Write down the equation of the circle.

The general equation of a circle with its centre at the origin is $x^2 + y^2 = r^2$, where r is the radius. The radius is 3 and $3^2 = 9$

.... $x^2+y^2=9$ .... **[2]**

19 **(a)** Write as a single fraction in its simplest form.

$$\frac{4}{2n+3} - \frac{2n}{n^2+1}$$

$\frac{4(n^2+1)}{(2n+3)(n^2+1)} - \frac{2n(2n+3)}{(2n+3)(n^2+1)}$ ← Making the denominators the same. Multiplying the numerator by whatever the denominator is multiplied by keeps the fractions equivalent

$4n^2+4$ ← Expanding the brackets for the numerator of the first fraction

$4n^2+6n$ ← Expanding the brackets for the numerator of the second fraction

$4n-6$ ← Subtracting the second numerator from the first numerator

$2(2-3n)$ ← Factorising to check if there are any common factors between the numerator and denominator

Putting the result of the numerators subtracted over the common denominator. There is no need to expand the brackets on the denominator as it can be left in factorised form

(a) $\frac{4-6n}{(2n+3)(n^2+1)}$ **[4]**

(b) Simplify.

$$\frac{x^2 - x - 12}{2x^2 - 3x - 20}$$

Fractions can be simplified by cancelling out common factors to the numerator and denominator. Both the numerator and denominator should be factorised to express them as factors so the common factors can be found

$(x-4)(x+3)$

Factorising the numerator. -4 and 3 multiply to the -12 and add to the -1 (the coefficient of x). Putting these in brackets with x

$2x^2-8x+5x-20$

$2x(x-4)+5(x-4)$

$(2x+5)(x-4)$

Factorising the denominator. Multiplying the 2 (the coefficient of the x^2 term) by the -20 gives -40. -8 and 5 multiply to -40 and add to the -3 (the coefficient of x). Splitting the middle x term into this number of x. Then factorising the first two terms and the last two terms. Bringing the 2x and +5 together and multiplying them by the common bracket

Use table mode. f(x) = 40/x. Start: 1. End: 30. Step: 1

This helps to find the two numbers which multiply to the -40 and add to the -3 by listing out the factor pairs of 40. One of the pair needs to be negative to multiply to a negative

Cancelling out the (x - 4) from the numerator and denominator leaves this

(b) $\dfrac{x+3}{2x+5}$ .... **[5]**

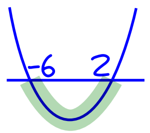
TURN OVER FOR QUESTION 20

20 Solve this inequality.

$x^2 + 4x - 12 \leqslant 0$

Give your answer using set notation.
You must show your working.

$\frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times -12}}{2 \times 1}$ ← $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← Solving the quadratic $x^2 + 4x - 12 = 0$ using the quadratic formula. a = 1, b = 4 and c = -12



← Sketching a positive x^2 graph and indicating the solutions of x = -6 and x = 2

The part of the graph highlighted in green is where it is less than or equal to 0. Writing the possible value of x as an inequality using set notation

$\{x : -6 \leq x \leq 2\}$ **[5]**

END OF QUESTION PAPER