AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

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GCSE MATHEMATICS

H

Higher Tier   Paper 2 Calculator

Thursday 8 November 2018   Morning   Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided

1 What does $(A \cap B)$ represent in $P(A \cap B)$?

Circle your answer.

The intersection of A and B

[1 mark]

A or B or both

A but not B

not A and not B

A and B

2 *P* is (4, 9) and *Q* is (–2, 1)

Circle the midpoint of *PQ*.

[1 mark]

(1, 5) (3, 4) (3, 5) (6, 8)

$\boxed{(4 + -2)/2 = 1}$

Working out the mean of the x-coordinates works the x-coordinate of the midpoint. There is only one option with an x-coordinate of 1

3 Which of these is a geometric progression?

Circle your answer.

[1 mark]

1 3 5 7 9

1 3 6 10 15

1 4 9 16 25

1 3 9 27 81

Each term is multiplied by 3 to get the next term

Do not write outside the box

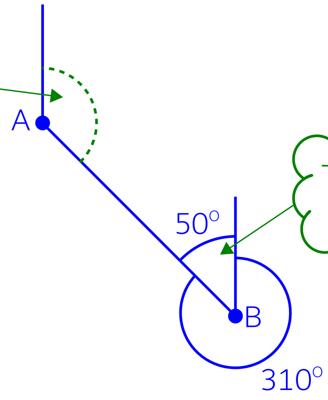
4 The bearing of *A* from *B* is 310°

Circle the bearing of *B* from *A*.

[1 mark]

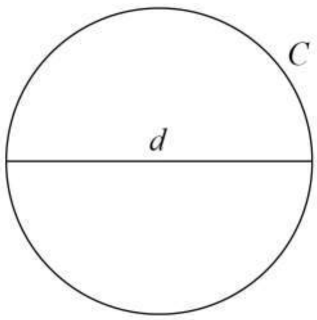
050° 110° 130° 220°

This is the angle we are trying to find. It is co-interior to the 50° so is 130 as co-interior angles add up to 180



This angle is 50 as there are 360 degrees around a point and 360 - 310 = 50

5 A circle has circumference C and diameter d.



$C = kd$

What **value** does the constant k represent?

[1 mark]

Answer π

Circumference = π x diameter

5

Do not write outside the box

6 Here is some information about 20 trains leaving a station.

Number of minutes late, t	Number of trains	Midpoint	
$0 \leqslant t < 5$	12	2.5	12 x 2.5 = 30
$5 \leqslant t < 10$	7	7.5	7 x 7.5 = 52.5
$10 \leqslant t < 15$	1	12.5	1 x 12.5 = 12.5
$t \geqslant 15$	0		

There are no trains in this category so it can be ignored

Each category has a range of 5. Dividing 5 by 2 then adding this on to each of the lowest number in each category works out the midpoints. 5/2 = 2.5. 0 + 2.5 = 2.5. 5 + 2.5 = 7.5. 10 + 2.5 = 12.5

6 (a) Work out an estimate of the mean number of minutes late.

[3 marks]

$$\frac{30+52.5+12.5}{12+7+1+0}$$

Mean = total/number. An estimate of the total is found by multiplying the midpoint by the frequency for each category then adding them all together. The number is the total number of trains

Answer 4.75 minutes

Do not write outside the box

6 (b) The station manager looks at the information in more detail.

Number of minutes late, t	**Number of trains**
$0 \leqslant t < 2$	12
$2 \leqslant t < 4$	0
$4 \leqslant t < 6$	7
$6 \leqslant t < 8$	0
$8 \leqslant t < 10$	0
$10 \leqslant t < 12$	1

He works out an estimate of the mean using this information.

How does his estimate compare with the answer to part (a)?

Tick **one** box.

[1 mark]

☐ Higher than part (a)

☐ Same as part (a)

☑ Lower than part (a)

As the midpoints of the categories for the 12, 7 and 1 trains are lower

☐ Not possible to tell

Turn over for the next question

4

Do not write outside the box

7 Work out the values of a and b in the identity

$$5(7x + 8) + 3(2x + b) \equiv ax + 13$$

[4 marks]

$35x + 40 + 6x + 3b$

Expanding the brackets. Equating the coefficients: there are 41x (35x + 6x) on the left side and ax on the right so a must be 41

$40 + 3b = 13$

The constants on the left side are 40 + 3b and on the right they are 13. These must be equal

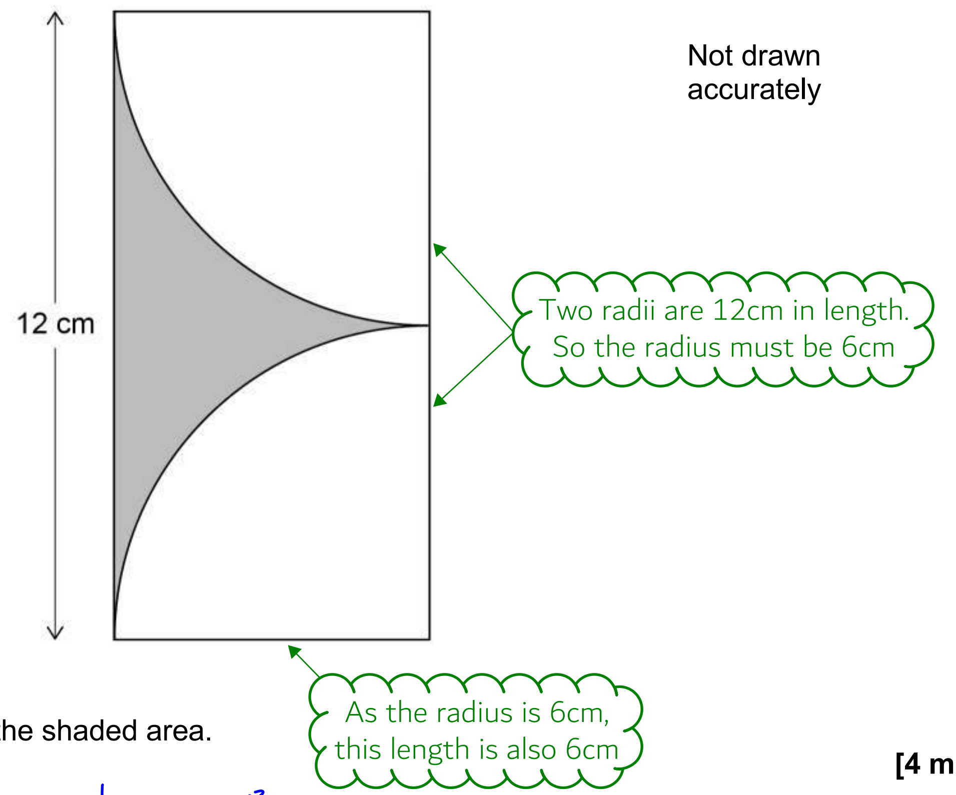
$b = \frac{13-40}{3}$

Rearranging to find b by subtracting 40 from both sides then dividing both sides by 3

$a =$ 41 $b =$ −9

Do not write outside the box

8 Two identical quarter circles are cut from a rectangle as shown.



Work out the shaded area.

[4 marks]

$12 \times 6 - 2 \times \frac{1}{4} \times \pi \times 6^2$

Subtracting the area of the two quarter circles from the area of the rectangle leaves the shaded area

Area of rectangle = length x width. The length is 12cm and the width is 6cm

Area of the two quarter circles. πr^2 works out the area of the full circle. Multiplying by 1/4 finds one of the quarter circles. Multiplying by 2 finds both of the quarter circles

Answer 15.5 cm^2

8

Do not write outside the box

9 The diagrams show the position of a tap when off and fully on.

The tap is fully on when the angle of turn is 180°

When fully on, water flows out of the tap at 14 litres per minute.

The rate at which water flows out is in direct proportion to the angle of turn.

The tap is turned 135°

The water flows into a tank with a capacity of 79.8 litres.

Will it take **less than** $7\frac{1}{2}$ minutes to fill the tank?

You **must** show your working.

[4 marks]

$s \, {}^{d} \, t$

This is basically a speed distance time problem as the rate it is filled is the speed, the capacity is the distance and we are trying to calculate the time. From the formula triangle, time = distance/speed

$$\frac{79.8}{\frac{135}{180} \times 14} = 7.6$$

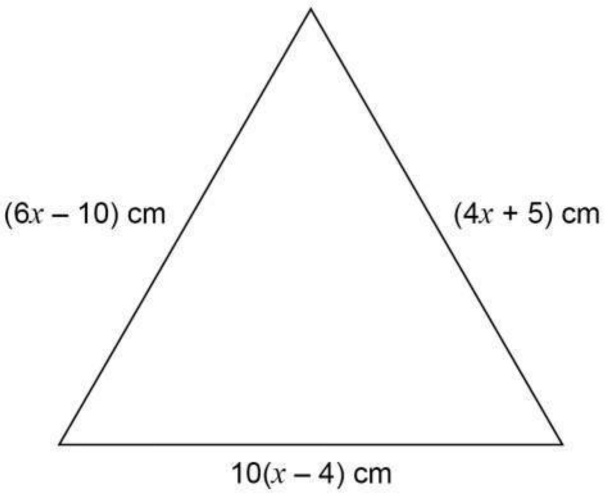
Dividing the capacity of 79.8 litres (distance) by the rate it is filled (speed). The rate it is filled is proportional to the angle of turn and the tap is turned 135 degrees out of the 180 degrees so as when it is fully on it lets out 14 litres per minute, 135/180 x 14 works out the rate

No

It would take 7.6 minutes, which is more than $7\ {}^{1}/_{2}$ minutes

Do not write outside the box

10 This triangle is equilateral.



Not drawn accurately

Is the perimeter of the triangle greater than one metre?

You **must** show your working.

[5 marks]

$6x - 10 = 4x + 5$

The triangle is equilateral so all the sides are equal. Setting two of the sides equal to each other creates an equation in terms of x which can be solved

$2x = 15$

Subtracting 4x from both sides to bring all the x terms to the same side then adding 10 to both sides

$x = 7.5$

Dividing both sides by 2 finds x

$(6(7.5) - 10) \times 3 = 105$

Substituting 7.5 for x in the side (6x - 10) to work out its length. Multiplying by 3 as all the side are equal so this works out the perimeter

Yes

One metre is 100 centimetres. 105 cm is greater than this

9

Do not write outside the box

11 An approximation for the value of π is given by

$$4\left(1-\frac{22}{57}+\frac{22}{85}-\frac{22}{105}+\frac{22}{117}-\frac{22}{242}\right)$$

Use your calculator to show that this approximation is within 0.1 of 3.14

[2 marks]

$3.14 - 3.041\ldots = 0.09\ldots$

Subtracting the approximation from 3.14 gives 0.09816038107

Typing the approximation into the calculator gives 3.041839619

Subtracting the approximation from 3.14 works out the difference. As this is less than 0.1, it is within 0.1 of 3.14

12 Work out $\dfrac{9.12 \times 10^{10}}{3.2 \times 10^{4}}$

Give your answer in standard form.

[2 marks]

2850000

Typing it into the calculator exactly as it is above gives this

To be in standard form, it needs to be in the form a x 10^n, where $1 \le a < 10$ and n is an integer. To get 2.85, which is between 1 and 10, it needs to be divided by 10 6 times so 2.85 needs to be multiplied by 10^6 to make up for this

Answer 2.85×10^6

Do not write outside the box

13 Ashraf is going to put boxes into a crate.

The crate is a cuboid measuring 2.5 m by 2 m by 1.2 m

Each box is a cube of length 50 cm

He does these calculations.

volume of crate	=	$2.5 \times 2 \times 1.2$
	=	$6\ m^3$
volume of one box	=	$0.5 \times 0.5 \times 0.5$
	=	$0.125\ m^3$
number of boxes	=	$6 \div 0.125$
	=	48

He claims,

"I can put 48 boxes in the crate."

Evaluate Ashraf's method **and** claim.

[2 marks]

They are wrong as 50cm doesn't fit into 1.2m a whole number of times.

So the volume will not be completely filled. There will be a gap on the 1.2m length. He will not be able to put as many as 48 boxes in the crate

14 The cross section of a prism has n sides.

Circle the expression for the number of edges of the prism.

[1 mark]

$2n$ $3n$ $n + 2$ $2n + 3$

Consider a cuboid (which is a type of prism): the cross section has 4 sides and the prism has 12 sides. Consider a triangular prism: the cross section has 3 sides and the prism has 9 sides. 3n is the only one which works for both shapes

7

Turn over ►

Do not write outside the box

15 The volume of a medal is 45 cm^3

The medal is made from copper and tin.

volume of copper : volume of tin = 22 : 3

The density of copper is 8.96 g/cm^3

The density of tin is 7.31 g/cm^3

Work out the mass of the medal.

[4 marks]

From the formula triangle, mass = density x volume

$D \; {}^{M} \; V$

$8.96 \times \frac{22}{25} \times 45 + 7.31 \times \frac{3}{25} \times 45$

Mass of the copper is added to the mass of the tin to get the mass of the medal

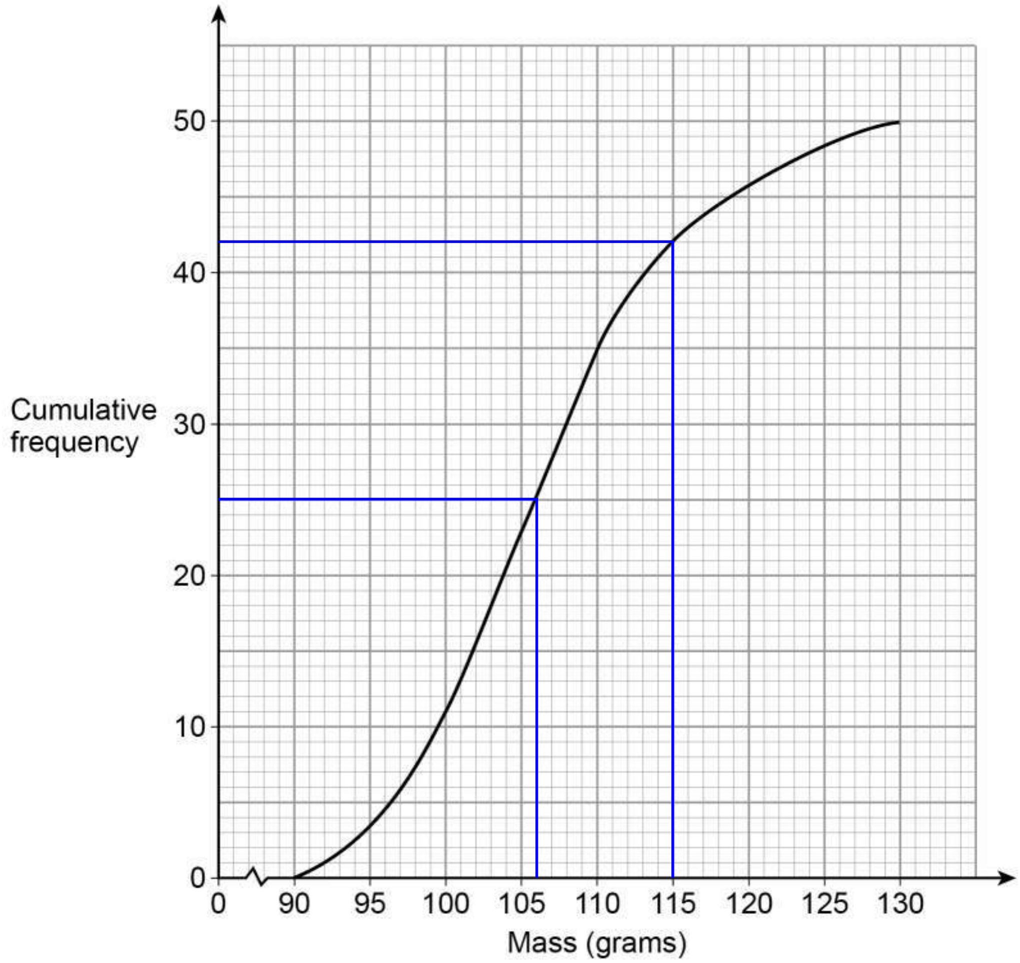
Mass of the copper. The density of 8.96g/cm^3 is multiplied by the volume. The volume of copper is represented by 22 parts out of the total 25 parts in the ratio so is 22/25 of the 45cm^3

Mass of the tin. The density of 7.31g/cm^3 is multiplied by the volume. The volume of tin is represented by 3 parts out of the total 25 parts in the ratio so is 3/25 of the 45cm^3

Answer 394.29 grams

Do not write outside the box

16 The cumulative frequency graph shows information about the masses of 50 apples.



16 (a) Use the graph to estimate the median mass of the apples.

[1 mark]

The median is roughly half way through the data. 50/2 = 25. So drawing a line across from 25 to the line then down gives an estimate for the median

Answer 106 grams

16 (b) Estimate the proportion of the apples that have a mass greater than 115 grams.

[2 marks]

Drawing a line up from 115 to the line then across estimates that there are 42 apples which are 115g or less. 50 - 42 = 8 so there would be 8 out of the 50 apples which are more than 115g

Answer $\frac{8}{50}$

7

Turn over ▶

Do not write outside the box

17 a is a prime number.

b is an even number.

$N = a^2 + ab$

Circle the correct statement about N.

[1 mark]

could be even or odd   always even

always prime   always odd

A prime number could be odd or even (as 2 is prime and even and all the other primes are odd). Odd x odd = odd so a^2 could be odd but even x even = even so a^2 could be even. Even x even = even and odd x even = even so ab will be even. Even + even = even and odd + even = odd so N could be even or odd

18 A bag contains 20 discs.

10 are red, 7 are blue and 3 are green.

18 (a) Marnie takes a disc at random before putting it back in the bag.

Nick then takes a disc at random before putting it back in the bag.

Olly then takes a disc at random.

Work out the probability that they all take a red disc.

[2 marks]

$\frac{10}{20} \times \frac{10}{20} \times \frac{10}{20}$

There are 10 red disks out of a total of 20 disks so the probability of getting red is 10/20. As the disk is put back in the bag, the probability is the same for Marnie, Nick and Olly. Red AND red AND red; AND means to multiply

Answer $\frac{1}{8}$

Do not write outside the box

18 (b) All 20 discs are in the bag.

Reggie takes three discs at random, one after the other.

After he takes a disc he does **not** put it back in the bag.

Reggie's first disc is blue.

Work out the probability that all three discs are different colours.

[3 marks]

$\frac{10}{19} \times \frac{3}{18} + \frac{3}{19} \times \frac{10}{18}$

It is given the first is blue. The next two must be different and can't be blue. They could be red AND green OR green AND red. AND means to multiply the probabilities of each event and OR means to add the probabilities. As there is 1 fewer disk each time (he does not put it back in the bag), the denominator decreases to 19 then to 18

Answer $\frac{10}{57}$

6

Do not write outside the box

19

Lunch

Choose one starter and one main course

There are four starters and ten main courses to choose from.

Two of the starters and three of the main courses are suitable for vegans.

What percentage of the possible lunches have **both** courses suitable for vegans?

[3 marks]

$\frac{2 \times 3}{4 \times 10} \times 100$

Using the product rule for counting, 2 x 3 gives the number of possibilities which are suitable for vegans and 4 x 10 gives the number of possibilities in total. Expressing the number suitable for vegans as a fraction of the total number then converting it into a percentage by multiplying by 100

Answer 15 %

20 n is a positive integer.

Prove algebraically that $2n^2\left(\frac{3}{n} + n\right) + 6n\left(n^2 - 1\right)$ is a cube number.

[3 marks]

$6n + 2n^3 + 6n^3 - 6n$ Expanding the brackets

$8n^3$ Collecting the like terms and simplifying

$(2n)^3$ Expressing as a number cubed shows it is a cube number

Do not write outside the box

21 y is inversely proportional to $\sqrt{x}$

$y = 4$ when $x = 9$

21 (a) Work out an equation connecting y and x.

[3 marks]

$y = \frac{k}{\sqrt{x}}$

Inversely proportional means 'proportional to 1 over'. $1/\sqrt{x}$ can be multiplied by anything but nothing can be added and it will still be proportional. So multiply it by k, which represents any number

$4\sqrt{9} = k$

Multiplying by $\sqrt{x}$ and substituting 4 for y and 9 for x to find k

k = 12 so substituting 12 for k in the original equation

Answer $y = \frac{12}{\sqrt{x}}$

21 (b) Work out the value of y when $x = 25$

[2 marks]

$\frac{12}{\sqrt{25}}$

Substituting 25 for x in the equation found in part (a)

Answer 2.4

Turn over for the next question

11

Do not write outside the box

22 Simplify fully $\dfrac{x^5 - 4x^3}{3x - 6}$

[3 marks]

$\dfrac{x^3(x^2-4)}{}$

Factorising the numerator by bringing out x^3 as a common factor

$\dfrac{x^3(x+2)(x-2)}{3(x-2)}$

Factorising the numerator further using difference of two squares. Factorising the denominator by bringing out 3 as a common factor

Simplifying the fraction by cancelling out (x - 2), which is a common factor to the numerator and denominator

Answer $\dfrac{x^3(x+2)}{3}$

23 *PQR* is a straight line.

PQ : *QR* = 3 : 1

$\overrightarrow{PQ} = \mathbf{a}$

Not drawn accurately

Circle the vector $\overrightarrow{RQ}$

[1 mark]

$\frac{1}{3}\mathbf{a}$ $\quad$ $\frac{1}{4}\mathbf{a}$ $\quad$ $-\frac{1}{3}\mathbf{a}$ $\quad$ $-\frac{1}{4}\mathbf{a}$

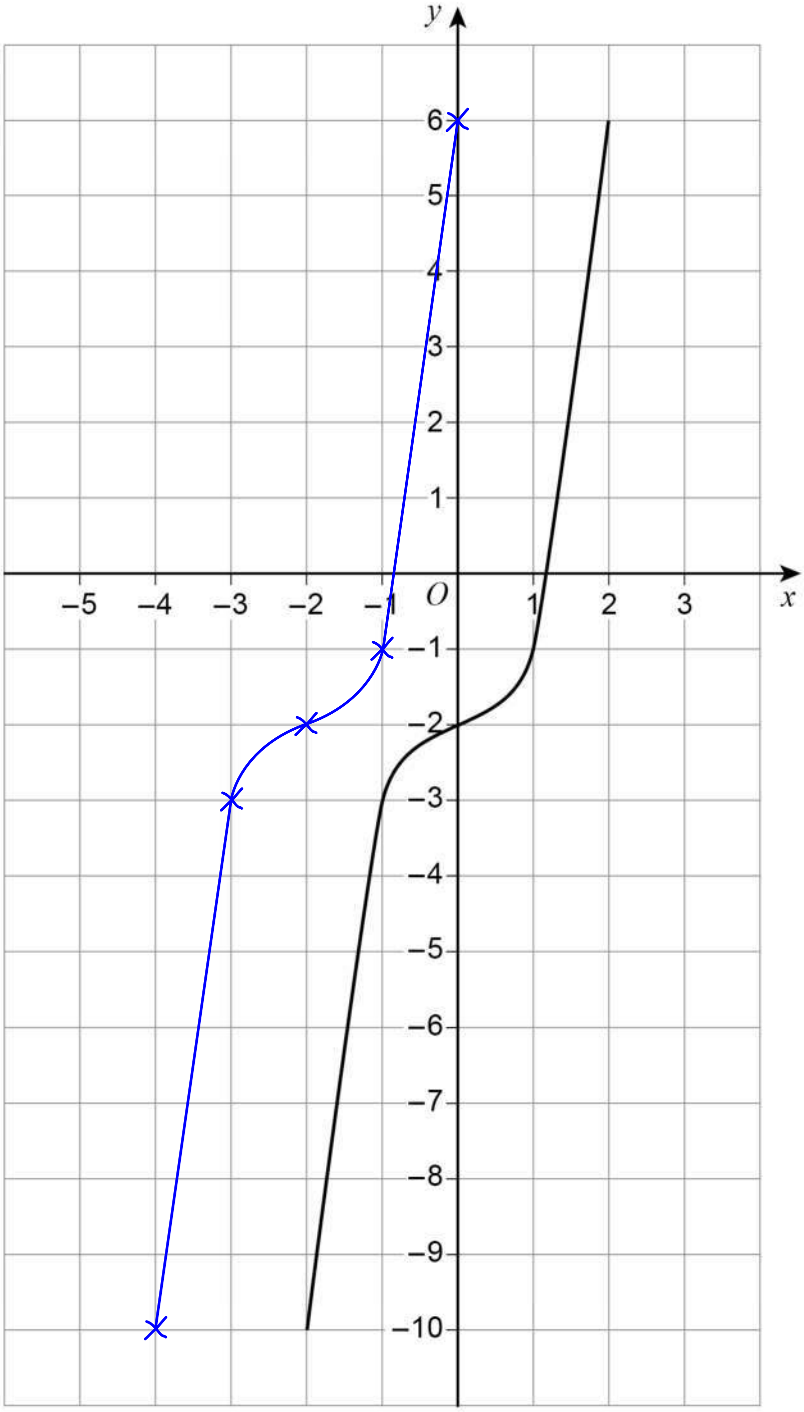
$\overrightarrow{PQ}$ is a and is represented by 3 parts of the ratio. $\overrightarrow{QR}$ is represented by 1 part so is 1/3 a. $\overrightarrow{RQ}$ is the same size but opposite direction to $\overrightarrow{QR}$ so is -1/3 a

Do not write outside the box

24 Here is a sketch of $y = f(x)$

The curve passes through the points

(–2, –10) (–1, –3) (0, –2) (1, –1) (2, 6)



On the grid, sketch the curve $y = f(x + 2)$

[2 marks]

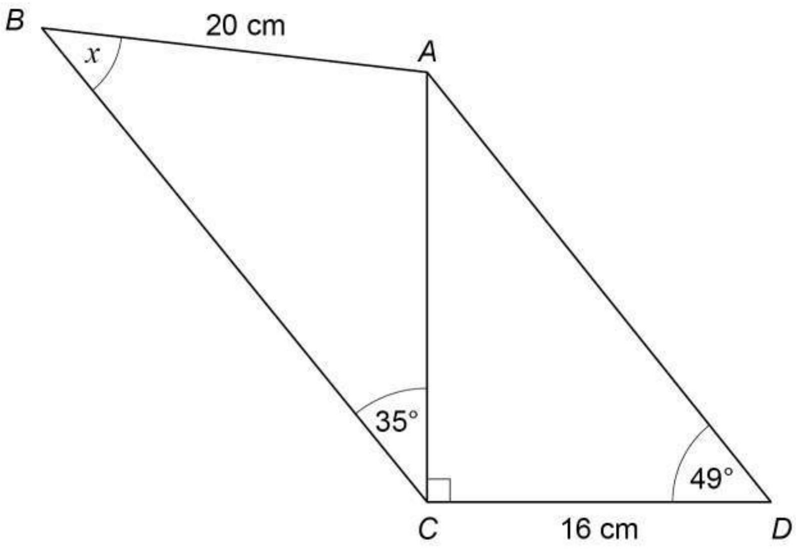
The graph translates 2 to the left as adding 2 to all the x values means it gets to the y values 2 sooner

6

Turn over ►

Do not write outside the box

25 *ABC* and *ACD* are triangles.



Not drawn accurately

Work out the size of angle x.

[5 marks]

S O H C A H T O A

AC can be found by using right-angled trigonometry in triangle ACD. Listing out SOH CAH TOA and ticking A as we have the adjacent and O as we are trying to find the opposite. There are two ticks on TOA so that formula triangle can be used. Covering O tells us that opposite = (tan of the angle) x adjacent

$\frac{\sin A}{a} = \frac{\sin B}{b}$

The sine rule can be used to find angle x as there are opposite pairs of sides and angles. Writing the sine rule with the angles as numerators to make it easier to rearrange to find an angle

$x = \sin^{-1}\left(\frac{\sin 35}{20} \times \tan 49 \times 16\right)$

Rearranging to make A the subject by multiplying both sides by a then doing the inverse sin of both sides. Substituting x for A, 35 for B, 20 for b and side AC (which is tan49 x 16) for a

Answer 31.9 degrees

Do not write outside the box

26 $\quad f(x) = \dfrac{x}{x+2} \qquad g(x) = x^2 - 2$

Work out fg(x)

Give your answer in the form $a + bx^n$ where a, b and n are integers.

[3 marks]

$\dfrac{x^2-2}{x^2}$

Putting g(x) into f(x) by substituting x^2 - 2 for x in f(x). The -2 and +2 cancel out leaving x^2 as the denominator

Dividing the terms on the numerator by x^2 separately. x^2/x^2 = 1 and $2/x^2 = 2x^{-2}$

Answer $1 - 2x^{-2}$

27 The point $\left(3, \dfrac{1}{64}\right)$ lies on the curve $y = k^x$ where k is a constant.

Show that the point $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ lies on the curve.

[3 marks]

$\dfrac{1}{64} = k^3$

The point (3 , 1/64) lies on the curve and therefore satisfies the equation. Substituting 1/64 for y (as this is the y-coordinate of the point) and 3 for x (as this is the x-coordinate of the point) in the equation

$k = \sqrt[3]{\dfrac{1}{64}} = \dfrac{1}{4}$

Rearranging to make k the subject by cube rooting both sides

$\left(\dfrac{1}{4}\right)^{\frac{1}{2}} = \dfrac{1}{2}$

Substituting in 1/2 for x (as this is the x-coordinate of the second point) and 1/4 for k finds that the y value will be 1/2. Therefore the point (1/2 , 1/2) lies on the curve

11

Do not write outside the box

28 Izzy runs an 80-metre race in 14 seconds.

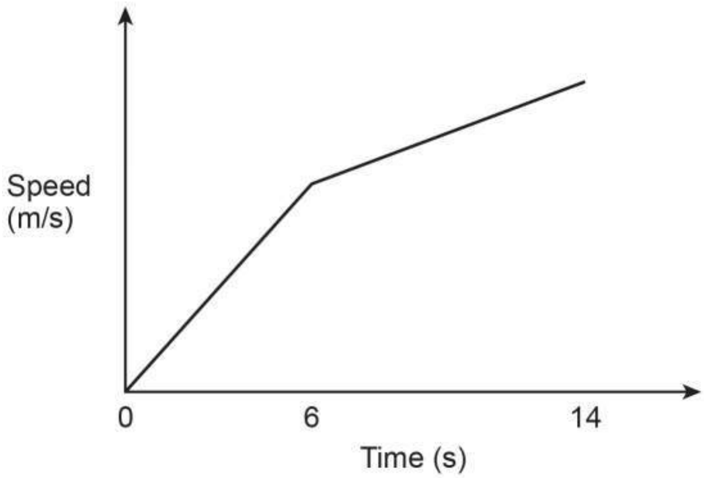
During the first 6 seconds her speed increases at a constant rate.

During the last 8 seconds her speed increases at a different constant rate.

Her speed at 14 seconds is 2 m/s more than her speed at 6 seconds.

Here is a sketch of her speed-time graph.

Not drawn accurately



28 (a) Work out her acceleration during the last 8 seconds.

State the units of your answer.

[2 marks]

Acceleration = (change in speed)/(change in time). The last 8 seconds is from 6 seconds to 14 seconds, in which her speed goes up by 2. The units of speed and time are divided ($m/s \div s = m/s^2$)

Answer $\frac{2}{8}$ m/s^2

Do not write outside the box

28 (b) When Izzy finishes the 80-metre race, her speed is v m/s

Work out the value of v.

[4 marks]

$\frac{1}{2} \times 6 \times (v-2) + \frac{1}{2}(v-2+v) \times 8$

The distance is equal to the area under the graph, which can be split into a triangle and trapezium. Area of triangle = 1/2 x base x height. Area of trapezium = 1/2 (a + b)h, where a and b are the parallel sides and h is the distance between them. v is the final speed at 14 seconds so the speed at 6 seconds is v - 2

$3v - 6 + 8v - 8$

$11v - 14 = 80$

Simplifying the expression of the distance in terms of v then setting it equal to 80 as it is a 80-metre race

$v = \frac{80+14}{11}$

Rearranging to make v the subject by adding 14 to both sides then dividing both sides by 11

Answer $8.\dot{5}\dot{4}$

END OF QUESTIONS

6