

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

GCSE MATHEMATICS

H

Higher Tier

Paper 1 Non-Calculator

Tuesday 21 May 2019

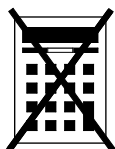
Morning

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- mathematical instruments



You must **not** use a calculator.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
TOTAL	

Advice

In all calculations, show clearly how you work out your answer.



Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

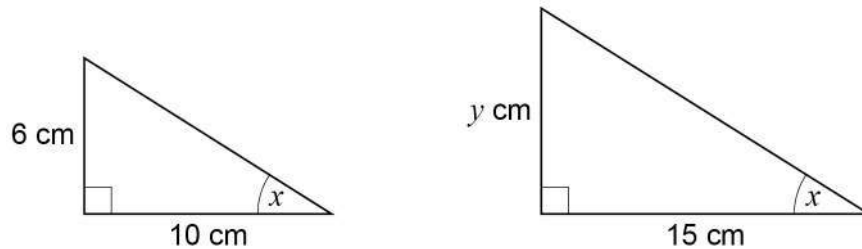
Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

Answer **all** questions in the spaces provided

- 1 Here are two right-angled triangles.

Not drawn
accurately



Circle the value of y .

[1 mark]

11

7.5

9

4

As they both have a right angle and one of the other angles is x , the missing angle must also be the same. Therefore the triangles are similar. $15/10 = 1.5$ so this is the scale factor. $6 \times 1.5 = 9$

- 2 Work out the value of $\left(1\frac{2}{3}\right)^2 = \frac{5}{3} \times \frac{5}{3} = \frac{25}{9}$

Circle your answer.

[1 mark]

$1\frac{4}{9}$

$3\frac{1}{3}$

$2\frac{4}{9}$

$2\frac{7}{9}$

Converted into an improper fraction then multiplied by itself. To multiply fractions, multiply the numerators and denominators. 9 goes into 25 twice with a remainder of 7

- 3 Work out the arc length, in metres, of a semicircle of radius 6 metres.

Circle your answer.

[1 mark]

3π

6π

12π

18π

The arc on a semicircle is half of the circumference of the full circle.
Circumference = $2\pi r$ where r is the radius
 $2 \times \pi \times 6 = 12\pi$. $12\pi/2 = 6\pi$



- 4 Circle the fraction that is equivalent to 4.625

[1 mark]

$$\frac{39}{8}$$

$$\frac{37}{8}$$

$$\frac{185}{4}$$

$$\frac{17}{4}$$

$$8 \overline{) 39.0} \quad 8 \overline{) 37.000}$$

Converting the fractions into decimals from left to right. $39/8$ is 4.8... so can't be 4.625. $37/8$ converts into 4.625

- 5 (a) Write 0.00097 in standard form.

[1 mark]

Answer 9.7 × 10⁻⁴

Multiplied by 10 4 times to get 9.7, a number between 1 and 10. So it must be multiplied by 10⁻⁴ (which basically means divide by 10 4 times) to make up for this

- 5 (b) Work out $\frac{3 \times 10^5}{4 \times 10^3}$

Give your answer as an ordinary number.

[2 marks]

$$0.75 \times 10^2$$

$3/4 = 0.75$
Using $a^x/a^y = a^{x-y}$
 $10^5/10^3 = 10^{5-3} = 10^2$

0.75 multiplied by
10 twice gives 75

Answer 75

Turn over ►



6 Anna plays a game with an ordinary, fair dice.

If she rolls 1 she wins.

If she rolls 2 or 3 she loses.

If she rolls 4, 5 or 6 she rolls again.

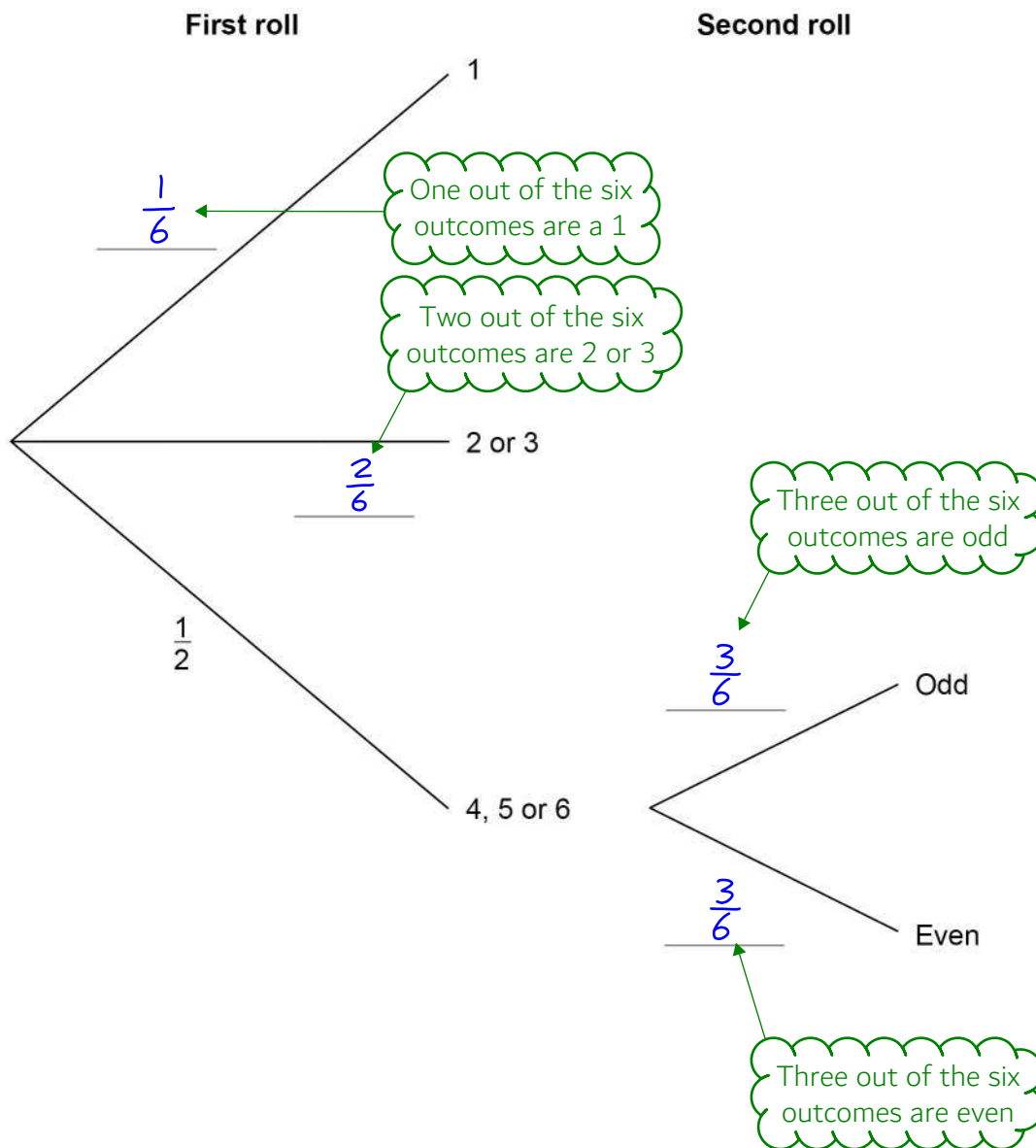
When she has to roll again,

if she rolls an odd number she wins

if she rolls an even number she loses.

6 (a) Complete the tree diagram with the four missing probabilities.

[2 marks]



6 (b) Is Anna more likely to win or to lose?

You **must** work out the probability that she wins.

[4 marks]

$$\frac{1}{6} + \frac{1}{2} \times \frac{3}{6}$$

AND means to multiply, OR means to add. To win, roll a 1 OR roll 4, 5, 6 AND odd. Substituting in the probabilities from the tree diagram gives this

$$\frac{2}{12} + \frac{3}{12} = \frac{5}{12}$$

$1/2 \times 3/6 = 3/12$ as the numerators and denominators are multiplied. $1/6$ is converted into $2/12$ by multiplying the numerator and denominator by 2 to make the denominator the same as the $3/12$ so they can be added to get $5/12$

Lose

The only outcomes for the game are win or lose. $5/12$ is less than half as half of 12 is 6 and 5 is less than this. Therefore the probability of losing must be more than half so will be more likely

Turn over for the next question

Turn over ►



- 7 Three friends arrive at a party.
Their arrival increases the number of people at the party by 20%
In total, how many people are now at the party?

[2 marks]

3×6

20% of the people is 3. As it has increased by 20%, the number of people is now at 120%. Multiplying 20% by 6 gives 120% so 3 is also multiplied by 6 to find out how many people there are

Answer 18

- 8 Work out the value of $(3^{12} \div 3^5) \div (3^2 \times 3)$

[3 marks]

$3^7 \div 3^3 = 3^4$

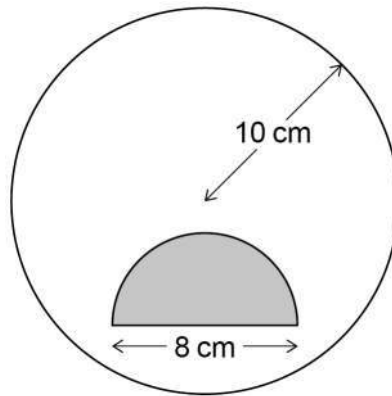
$$\begin{aligned} a^x / a^y &= a^{x-y} \\ a^x \times a^y &= a^{x+y} \\ 3^{12} / 3^5 &= 3^{12-5} = 3^7 \\ 3^2 \times 3^1 &= 3^{2+1} = 3^3 \\ 3^7 / 3^3 &= 3^{7-3} = 3^4 \end{aligned}$$

$$3^4 = 3 \times 3 \times 3 \times 3 = 9 \times 9 = 81$$

Answer 81

9

A shaded semicircle is inside a circle as shown.

Not drawn
accuratelyThe **radius** of the circle is 10 cmThe **diameter** of the semicircle is 8 cm

How many times bigger is the unshaded area than the shaded area?

[4 marks]

$$\frac{1}{2} \times \pi \times 4^2 = 8\pi$$

Area of circle = πr^2 , where r is the radius.
8cm is the diameter of the semicircle so halving this gets the radius of 4cm. As it is a semicircle, finding half of the area of the full circle finds its area

$$\pi \times 10^2 = 100\pi$$

This works out the area of the circle

$$100\pi - 8\pi = 92\pi$$

This works out the unshaded area

$$\frac{92\pi}{8\pi} = \frac{92}{8}$$

$$\begin{array}{r} 11.5 \\ 8 \overline{) 92.0} \end{array}$$

Dividing the unshaded area by the shaded area works out how many times larger it is. π cancels out from the numerator and denominator

Answer _____

11.5

Turn over for the next question

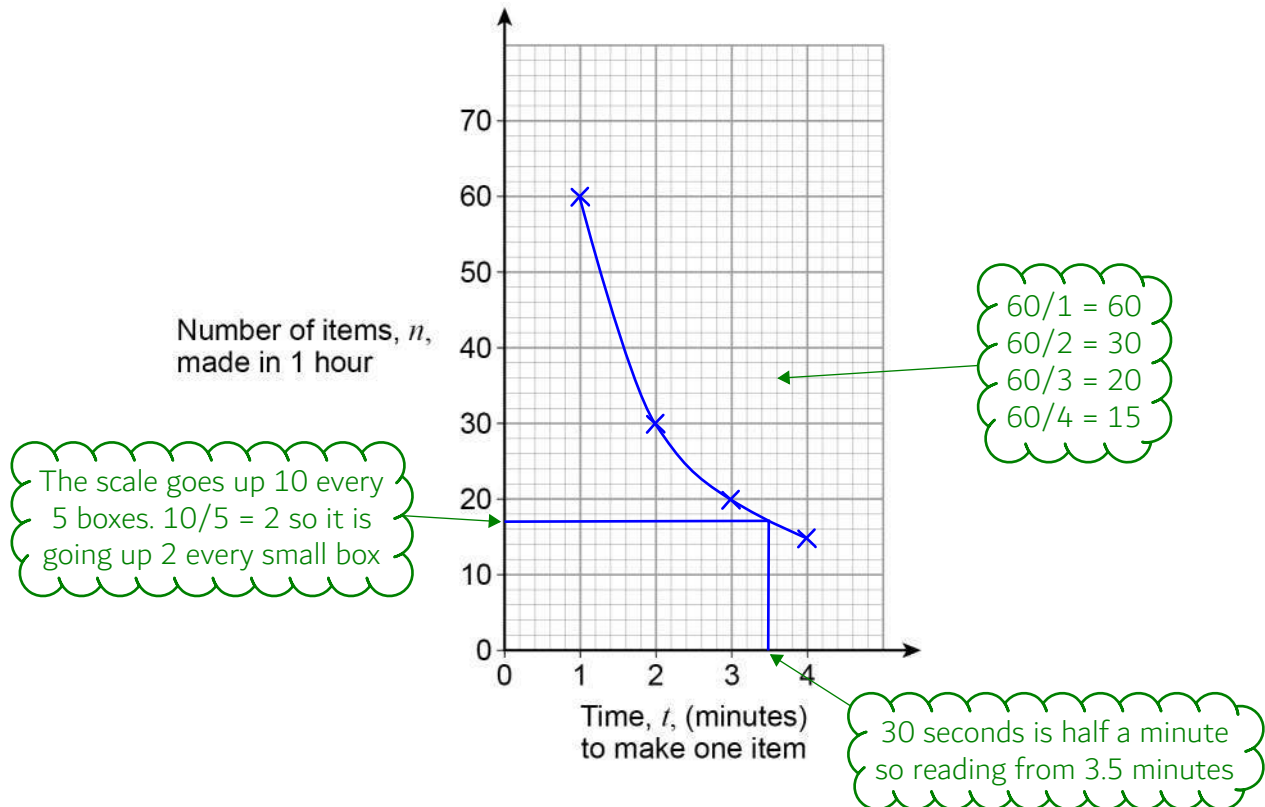
Turn over ►



- 10** The number of items, n , made in 1 hour by a machine is given by $n = \frac{60}{t}$
- t is the time in minutes the machine takes to make one item.
- The value of t changes for different types of item.

- 10 (a)** On the grid below, draw the graph of $n = \frac{60}{t}$ for values of t from 1 to 4

[2 marks]



- 10 (b)** The machine takes 3 minutes 30 seconds to make one item.
- Use your graph to estimate the value of n .

[2 marks]

Answer _____ 17 _____



- 11 Ed and Fay shared £330 in the ratio 7 : 4
Ed gives Fay some of his money.
Fay now has the same amount as Ed.

How much does Ed give Fay?

$$\begin{array}{r} 30 \\ 11 \overline{)330} \end{array}$$

$$30 \times 7 = 210$$

$$30 \times 4 = 120$$

$$\begin{array}{r} 210 \\ -120 \\ \hline 90 \end{array}$$

$$-120$$

$$90$$

$$\begin{array}{r} 45 \\ 2 \overline{)90} \end{array}$$

7 + 4 = 11 so there are 11 parts in total. Dividing the £330 by 11 works out what 1 part is worth

This works out the value of 7 parts and 4 parts. So Ed had £210 and Fay had £120

Subtracting Fay's money from Ed's money works out the difference

Dividing the difference by 2 halves it and therefore works out how much Ed gave Fay

[3 marks]

Just to check:

$$£210 - £45 = £165$$

$$£120 + £45 = £165$$

Answer £ 45

- 12 The next term of a sequence is made by adding the previous two terms.
Which of these sequences follows this rule?
Circle your answer.

[1 mark]

-9 2 -7 -5 -12

-3 5 -2 3 1

0 -3 -3 0 -3

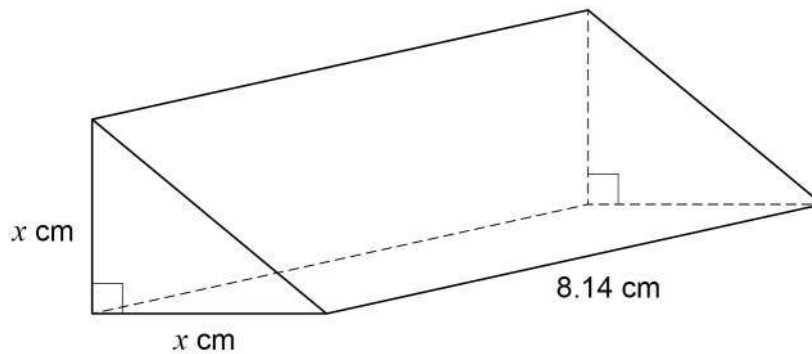
-1 -1 -2 -3 1

$$\begin{array}{l} -9 + 2 = -7 \\ 2 + -7 = -5 \\ -7 + -5 = -12 \end{array}$$



13

The triangular cross section of a prism is an isosceles right-angled triangle.



The volume of the prism is 102 cm^3

Use approximations to estimate the value of x .

You **must** show your working.

[3 marks]

$$\frac{1}{2} \times x \times x \times 8$$

Volume of prism = cross sectional area \times length
The cross section is a triangle.
Area of triangle = $\frac{1}{2} \times$ base \times height
The length is approximately 8cm.
This is an expression of the volume of the prism
in terms of x

$$4x^2 = 100$$

Simplifying the expression gives the left side of
this equation. It is equal to approximately 100 cm^3

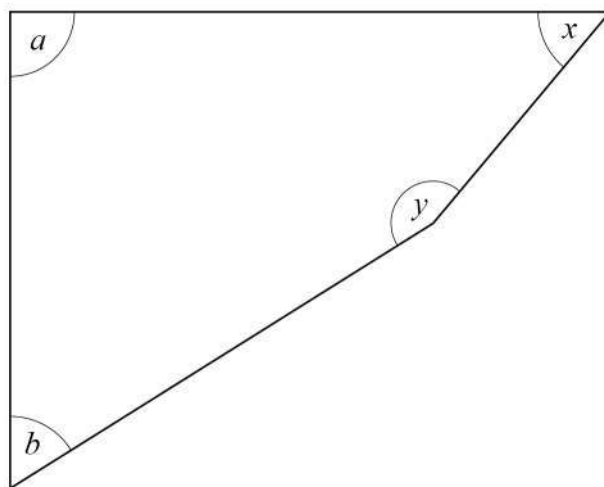
Dividing both sides by 4 then square rooting finds x

Answer _____

5



14 Here is a quadrilateral.



Not drawn accurately

$a = 90^\circ$ and $a : b = 5 : 3$

$x : y = 1 : 3$

Show that $b = x$

[3 marks]

$5 \overline{) 90}$	18	360	$4 \overline{) 216}$	$b = 54, x = 54$
	$\times 3$	$- 90$		
	$\underline{54}$	210		
		$- 54$		
		$\underline{216}$		

5 parts represents a, which is 90. Dividing by 5 works out 1 part of the ratio. Then multiplying by 3 to find the 3 parts which represent b

$3 + 1 = 4$ so there are 4 parts in total which represent the total of x and y. Dividing 216 by 4 works out the value of 1 part, which represents x

There are 360 degrees in total in a quadrilateral. Subtracting the angles a and b leaves the total of angles x and y



15 Here is some information about the test marks of 120 students.

Mark, m	$0 < m \leq 10$	$10 < m \leq 20$	$20 < m \leq 30$	$30 < m \leq 40$	$40 < m \leq 50$
Frequency	20	28	40	20	12

15 (a) Complete the cumulative frequency table.

[1 mark]

Mark, m	$m \leq 10$	$m \leq 20$	$m \leq 30$	$m \leq 40$	$m \leq 50$
Cumulative frequency	20	48	88	108	120

Cumulative frequency means to add up the frequencies as they go. $20 + 28 = 48$. $48 + 40 = 88$. $88 + 20 = 108$. $108 + 12 = 120$

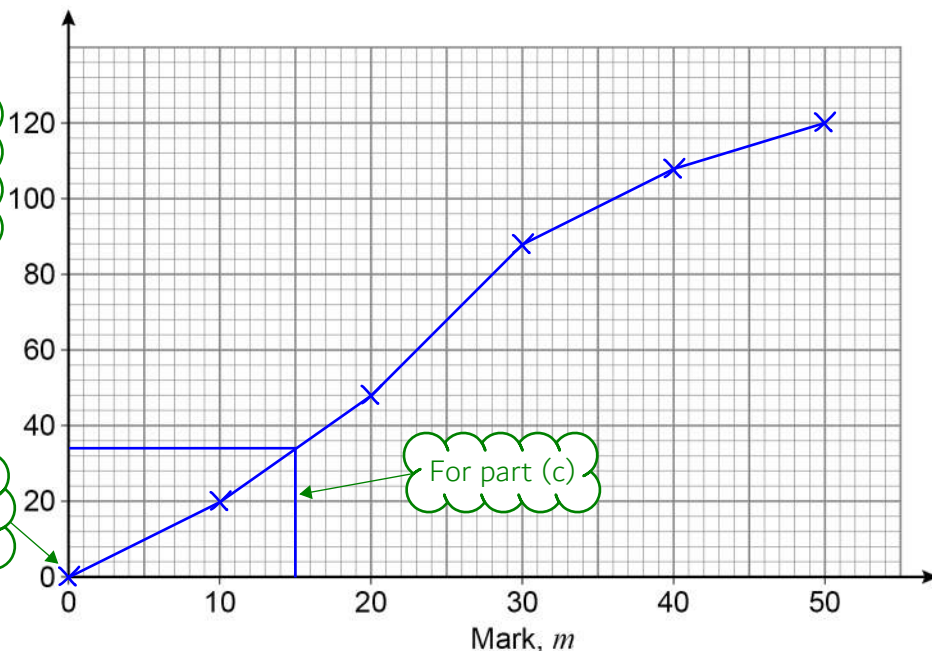
15 (b) Draw a cumulative frequency graph.

[2 marks]

The scale goes up 20 every 5 boxes. $20/5 = 4$ so the scale is going up in 4 every small box

Cumulative frequency

There were no students with a mark less than 0



The cumulative frequencies are plotted at the end points of each category as it is reached by the end but not before. Each of the points can be joined up using a series of straight lines



15 (c) Students who scored 15 marks or fewer take another test.

Use your graph to estimate how many students take another test.

[2 marks]

Read up from 15 to the line
then across to the frequency

Answer 34

16 Simplify fully

$$\frac{4x - 8x^2}{12x - 6}$$

[3 marks]

$$\frac{-4x(2x-1)}{6(2x-1)}$$

Factorising the numerator and denominator. The $-8x^2$ is the highest order of x so has priority: therefore $-4x$ is brought out as a factor instead of $4x$

$(2x - 1)$ cancels out from the numerator and denominator. Then the numerator and denominator can be divided by 2

Answer $\frac{-2x}{3}$

Turn over for the next question



17 Toby is forming and solving equations.

17 (a)

The product of half of a number and three more than the number
is the same as
the square of the number

Toby uses y to represent the number.

Write an equation that Toby could form.

[2 marks]

Product means multiplied together 'The same as' means equals

Answer $\frac{1}{2}y(y+3) = y^2$

Half of a number 3 more than the number The square of the number

17 (b) Toby forms another equation.

$$x = \frac{9}{8x}$$

He wants to work out the values of x .

Here is his working.

$$x = \frac{9}{8x}$$

$$8x^2 = 9$$

$$8x = 3 \text{ or } 8x = -3$$

$$x = \frac{3}{8} \text{ or } x = -\frac{3}{8}$$

What error has he made in his working?

[1 mark]

Should have divided by 8 before square rooting



18 Here is an identity.

$$x^2 - y^2 \equiv (x + y)(x - y)$$

18 (a) Use the identity to work out the value of $193^2 - 7^2$

You **must** show your working.

[2 marks]

$$(193 + 7)(193 - 7) \leftarrow \begin{array}{c} \text{x is 193 and y is 7} \end{array}$$

$$\begin{array}{r} 186 \\ \times 200 \\ \hline \end{array} \leftarrow \begin{array}{c} 193 + 7 = 200 \\ 193 - 7 = 186 \end{array}$$

Answer 37200

18 (b) Factorise $100a^2 - 81b^2$

$$\begin{array}{c} x^2 = 100a^2 \\ x = 10a \\ y^2 = 81b^2 \\ y = 9b \end{array}$$

[1 mark]

Answer $(10a + 9b)(10a - 9b)$

19 Circle the fraction that is equivalent to $0.\dot{1}$

[1 mark]

$$\frac{1}{9}$$

$$\frac{1}{99}$$

$$\frac{1}{10}$$

$$\frac{11}{100}$$

It can't be this one as is nearly $1/100$, which equals to 0.01

It can't be this one as $1/10 = 0.1$

It can't be this one as $11/100 = 0.11$

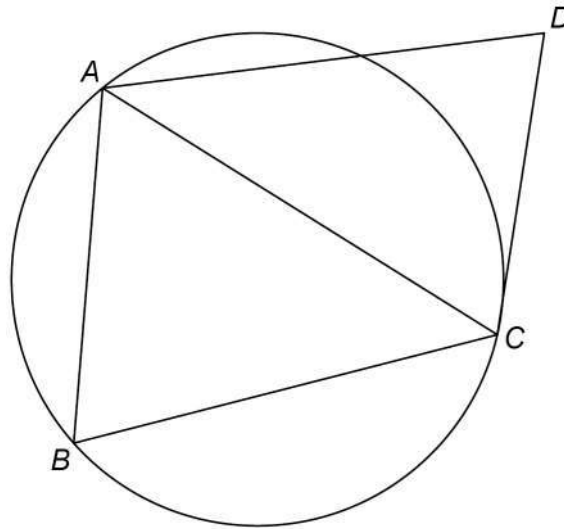


20

A , B and C are points on a circle.

CD is a tangent.

Not drawn
accurately



20 (a) Assume that triangle ABC is isosceles with $AC = BC$

Prove that AB is parallel to DC .

[4 marks]

Angles $ABC = BAC$ as the base angles of isosceles triangles are equal

Angles $ABC = ACD$ as the alternate segment theorem

Angles $BAC = ACD$ so there are alternate angles

Therefore AB is parallel to DC



20 (b) In fact, triangle ABC is equilateral.

Tick the **two** boxes for the statements that **must** be correct.

[1 mark]

AB is parallel to DC

This was proven in (a) as the angles in the triangle will still be equal and the alternate segment theorem will still apply

AC bisects angle BCD

Both halves of BCD (BCA and ACD) are both equal as all the angles in an equilateral triangle are equal and the alternate segment theorem

AC bisects angle BAD

CD could have any length so angle CAD could be anything. It doesn't have to be the same as BAC so it doesn't have to be true that AC bisects BAD

21 Solve the simultaneous equations

$$2x + 3y = 5p$$

Equation 1

$$y = 2x + p$$

Equation 2

where p is a constant.

Give your answers in terms of p in their simplest form.

[4 marks]

$$2x + 3(2x + p) = 5p$$

Substitute the right side of Equation 2 for y in Equation 1

$$8x = 2p$$

Expand the bracket to get $6x + 3p$. Collect the $2x$ and $6x$ to get $8x$ and subtract the $3p$ from both sides

$$x = \frac{2p}{8} = \frac{p}{4}$$

Divide both sides by 8 then simplify

$$y = 2\left(\frac{p}{4}\right) + p$$

Substitute $p/4$ for x in Equation 2

$$2 \times p/4 = p/2$$

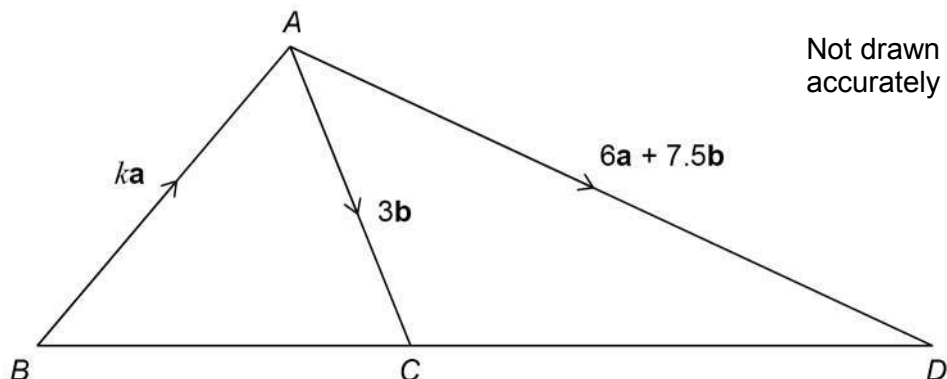
Adding half of p to p gets this

$$x = \frac{p}{4} \quad y = 1\frac{1}{2}p$$

Turn over ▶



22 ABC and ACD are triangles.
 k is a constant.



22 (a) Show that $\vec{CD} = 6a + 4.5b$

[1 mark]

$$\begin{aligned} & -3b + 6a + 7.5b \\ & \hline & 6a + 4.5b \end{aligned}$$

$\vec{CD} = \vec{CA} + \vec{AD}$
 $\vec{CA} = -3b$ as it is in the opposite direction to \vec{AC}

22 (b) BCD is a straight line.

Work out the value of k .

You **must** show your working.

[3 marks]

$$\vec{BC} = ka + 3b$$

$$\vec{BC} = \vec{BA} + \vec{AC}$$

$$\begin{aligned} & 3 \overline{) 4.5} \\ & \underline{6} \\ & 60 \\ & \underline{60} \\ & 0 \end{aligned}$$

As BCD is a straight line, \vec{BC} must be in the same direction as \vec{CD} . So \vec{BC} is a scaled down version of \vec{CD} . This works out the scale factor by comparing the coefficients of b

The scale factor from BC to CD is 1.5 so dividing the coefficient of a by 1.5 works out k

Answer 4



23 Simplify $8^4 \div 32^{\frac{2}{5}}$

Give your answer in the form 2^m where m is an integer.

[3 marks]

$$(2^3)^4 \div (2^5)^{\frac{2}{5}}$$

Expressing both 8 and 32 as powers of 2

$$2^{12} \div 2^2$$

$$(a^x)^y = a^{xy}$$

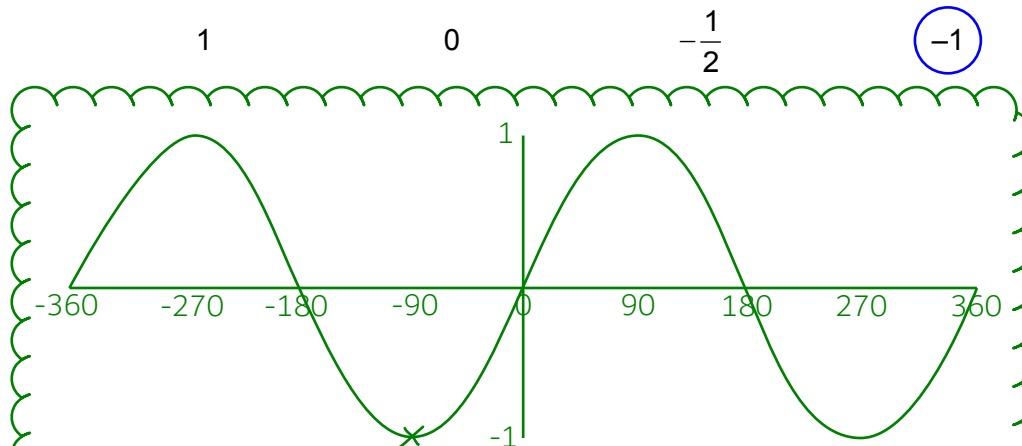
$$a^x / a^y = a^{x-y}$$

Answer 2^{10}

24 $f(x) = \sin(x - 90^\circ)$

Circle the value of $f(0^\circ)$

[1 mark]



The sine curve keeps repeating every 360 degrees. Substituting 0 into $f(x)$ gives $\sin(-90)$. Looking at the graph above, this equals to -1

Turn over for the next question



25 (b) Work out the length AB.

Give your answer in the form $a\sqrt{5}$ where a is an integer.

You **must** show your working.

[4 marks]

$$y = -\frac{1}{2}x + c$$

Substituting the gradient from part (a) into the general equation of a straight line

$$c = 8 + \frac{1}{2}(4) = 10$$

Substituting the x and y values from the point P (4, 8) and rearranging to find c

$$0 = -\frac{1}{2}x + 10$$

Substituting 10 for c in the original equation. Working out where point A meets the x-axis by setting $y = 0$

$$x = 20$$

$$\sqrt{20^2 + 10^2} = \sqrt{500}$$

Pythagoras' Theorem can be used to work out the length AB as OAB is a right-angled triangle. OA = 20, OB = 10

$$\sqrt{100}\sqrt{5}$$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

This is used in reverse to split root 500 into these two surds

$$\text{Root } 100 = 10$$

c is the y-intercept so OB runs from 0 to 10 on the y axis so has length of 10

Answer $10\sqrt{5}$ units

Turn over for the next question



26

The turning point of the graph $y = (x + a)^2 + b$ has x -coordinate -2
 $(3, 1)$ is another point on the graph.

Work out the y -coordinate of the turning point.

[3 marks]

$$-2 + a = 0$$

$$a = 2$$

$$b = 1 - (3 + 2)^2$$

The turning point is where the square bracket has the minimum value, which is 0 (the lowest a squared number can be is 0).
 Substituting in the x -coordinate of the turning point

Rearranging the equation to find b and substituting in the x and y values from the point $(3, 1)$

When the square bracket is 0 (at the minimum point, which is the turning point),
 $y = b$ and $b = 1 - 5^2 = 1 - 25 = -24$

Answer _____

-24



27

Angle x is acute.

$$\cos x = \sin 60^\circ \times \tan 30^\circ$$

Work out the size of angle x .You **must** show your working.**[3 marks]**

	0	30	45	60	90
Sin:	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos:	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

$$\tan 30 = \frac{\sqrt{1}}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$$

The angles we need to remember the trig values for are 0, 30, 45, 60 and 90. To find the sin values, list 0, 1, 2, 3, 4 then square root them all and put them over 2. The cos values are the same but the other way around

$$\tan 30 = \sin 30 / \cos 30$$

$$\sin 60 \times \tan 30$$

$$\cos 60 = 1/2$$

Answer 60 degrees

END OF QUESTIONS

