AQA

.CG Maths.
Worked Solutions

Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

I declare this is my own work.

GCSE MATHEMATICS

H

Higher Tier Paper 2 Calculator

Wednesday 7 June 2023 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments
- the Formulae Sheet (enclosed).

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26	
TOTAL	

Please note that these worked solutions have neither been provided nor approved by AQA and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

.CG Maths.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Write 30 : 12 in the form $n : 1$

[1 mark]

The 12 is divided by 12 to get 1 so the 30 must also be divided by 12.
This can be left as a fraction as the result will not be a whole number

Answer $\frac{30}{12}$: 1

+4 +5 +6

2 Four consecutive triangular numbers are 6 10 15 21

Write down the next triangular number.

[1 mark]

1 more is added between each term in the sequence. 21 + 7 = 28

Answer 28

Do not write outside the box

3 Write down the reciprocal of $\frac{4}{7}$

[1 mark]

Answer $\frac{7}{4}$

Reciprocal means to do 1 divided by, which flips the fraction

4 The price of a toy increases by 12.5% to £19.53

Work out the **original** price of the toy.

[2 marks]

$100 + 12.5$ ← The original price is 100%. Adding the increase of 12.5% works out that it had been increased to 112.5%

$19.53 \div 112.5$ ← Dividing the increased price by the percentage the original had increased to works out that £0.1736 is 1% of the original

0.1736×100 ← Multiplying the value of 1% by 100 works out the 100%, which is the original price

Answer £ 17.36

Turn over for the next question

5

Do not write outside the box

5 Jess saves 2p, 5p and 10p coins.

She has

- 45 10p coins
- 8 times as many 2p coins as **10p coins**
- £17.70 in total.

Work out total **value** of 2p coins : total **value** of 5p coins

Give your answer in its simplest form.

[4 marks]

$45 \times 0.10 = 4.50$

10p is £0.10. Multiplying this by 45 works out that the value of the 45 10p coins is £4.50

8×45

This works out that there are 360 2p coins

$360 \times 0.02 = 7.20$

2p is £0.02. Multiplying this by 360 works out that the value of the 360 2p coins is £7.20

$17.70 - 4.50 - 7.20$

Subtracting the value of the 10p coins and the value of the 2p coins from the total value of all the coins works out that the value of the 5p coins is £6

$\frac{7.20}{6} = \frac{6}{5}$

Ratios simplify in a similar way to fractions. Putting the value of the 2p coins over the value of the 5p coins in the calculator simplifies it to 6/5. So the ratio must be 6 : 5 in its simplest form

Answer 6 : 5

Do not write outside the box

6 (a) Part of a regular polygon is shown.

Not drawn accurately

Assume that the polygon is an octagon.

Work out the size of an **exterior** angle.

[2 marks]

$360 \div 8$

Angle x is an exterior angle. The exterior angles of any polygon add up to 360°. As there are 8 sides in an octagon there are also 8 exterior angles. As it is regular all the exterior angles are the same so dividing 360° by the 8 exterior angles works out each one

Answer 45 °

6 (b) In fact, the polygon has **more** sides than an octagon.

What does this mean about the size of an exterior angle?

Tick **one** box.

[1 mark]

- [ ] It is more than the answer to part (a)
- [ ] It is the same as the answer to part (a)
- [x] It is less than the answer to part (a)
- [ ] It could be any of the above

There are more than 8 exterior angles so the 360° will be divided by more than 8. Dividing by more will make the exterior angles less

7

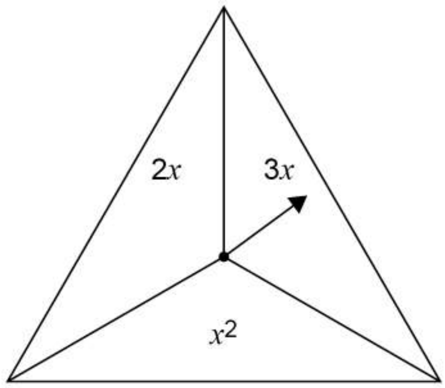
Turn over ▶

.CG Maths.

Do not write outside the box

7 In a game,

- an ordinary fair six-sided dice is rolled
- the fair spinner shown is spun.



The score is the dice number **substituted** into the spinner expression.

7 (a) Complete the table to show all of the possible scores.

[2 marks]

	1	**2**	**3**	**4**	**5**	**6**
$\mathbf{2}x$	2	4	6	8	10	12
$\mathbf{3}x$	3	6	9	12	15	18
x^2	1	4	9	16	25	36

For the first row, all the numbers are multiplied by 2. For the second row, all of the numbers are multiplied by 3. For the third row, all of the numbers are squared

Do not write outside the box

7 (b) A player wins the game if their score is 10 or more.

Work out the probability that they win the game.

[1 mark]

8 out of the 18 possible scores are 10 or more

Answer $\frac{8}{18}$

7 (c) The game is played 711 times.

Estimate the number of games that are won.

[2 marks]

$\frac{8}{18} \times 711$

Multiplying the probability of winning by the number of games played estimates how many of them are won

Answer 316

8 $(a - 3)x^2 + 2b \equiv 5x^2 + 12$

Work out the values of a and b.

[2 marks]

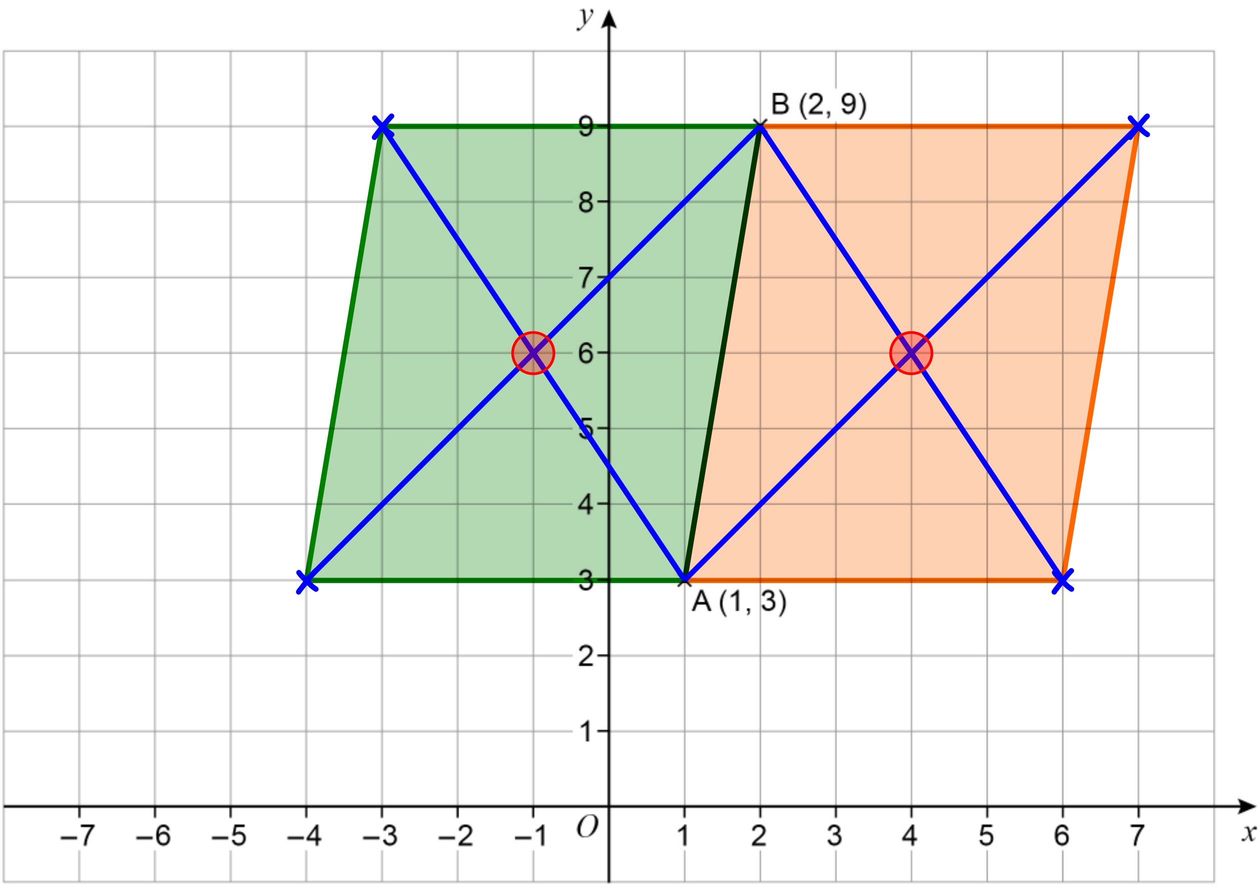
Both halves of the identity are identical. So the $5x^2$ on the right must be the same as the $(a - 3)x^2$ on the left. Therefore $a - 3 = 5$ then adding 3 to both sides finds that $a = 8$. The $2b$ on the left must be the same as the 12 on the right. Therefore $2b = 12$ then dividing both sides by 2 finds that $b = 6$

$a =$ 8 $b =$ 6

7

Do not write outside the box

9 A (1, 3) and B (2, 9) are points on a centimetre grid.



ABCD is a parallelogram.

AD and BC are **horizontal** and each has length 5 cm

The diagonals of ABCD cross at E.

Work out the **two** possible pairs of coordinates of E.

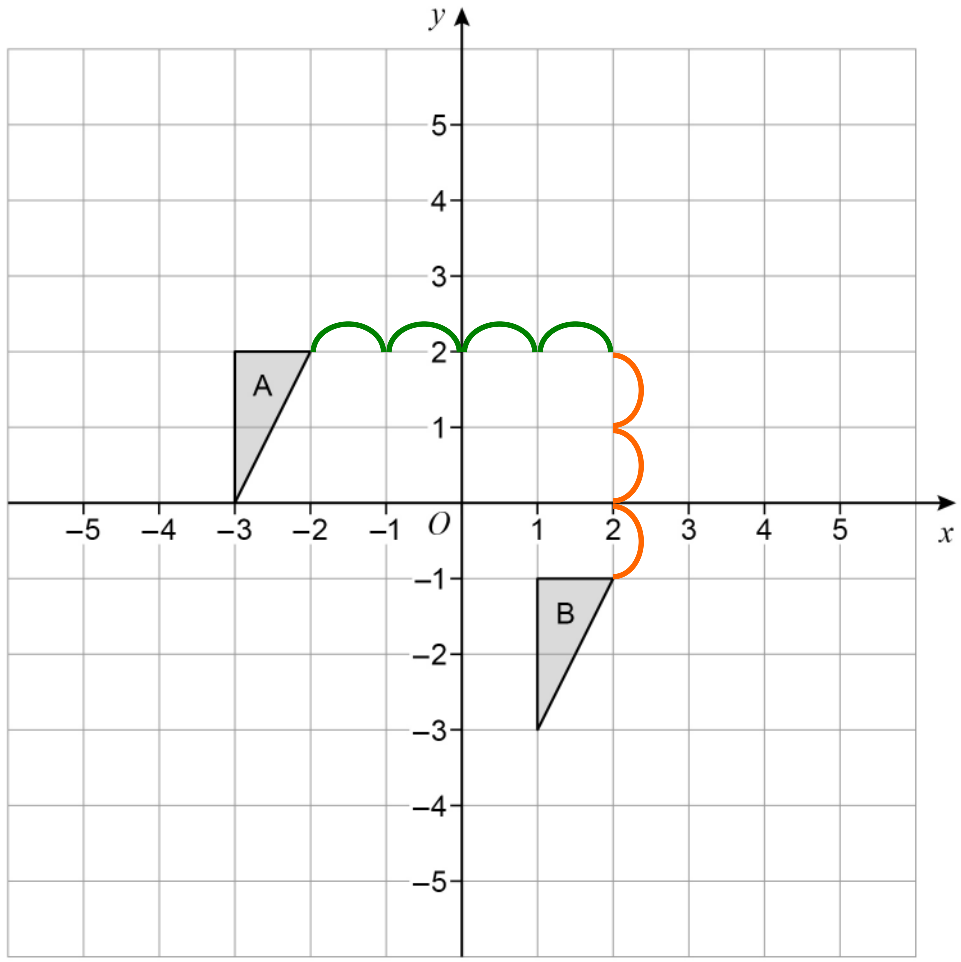
[4 marks]

There are two possible parallelograms (one shown in green and the other shown in orange, which do not have to be drawn). The diagonals of both are shown in blue and where they cross is circled in red

Answer (–1 , 6) and (4 , 6)

10 Write down the translation vector that maps shape A onto shape B.

[2 marks]



Answer $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Vectors are in the form $\binom{x}{y}$. It has translated 4 in the x-direction (shown in green) and -3 in the y-direction (shown in orange)

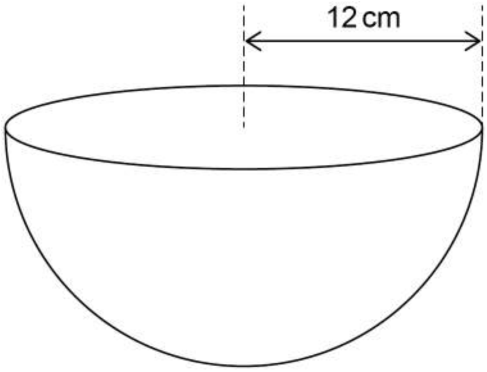
6

Do not write outside the box

11

Volume of a sphere $= \frac{4}{3}\pi r^3$

A bowl is a hemisphere with radius 12 cm



Water is poured into the bowl

at a rate of 325 cm^3 per second

for 8 seconds.

Does the water fill **more than** 70% of the bowl?

You **must** show your working.

[4 marks]

$325 \times 8 = 2600$

Multiplying the rate in which the water is poured into the bowl by the amount of time it is poured for works out that 2600 cm^3 is poured into the bowl

$\frac{1}{2} \times \frac{4}{3}\pi \times 12^3 \times \frac{70}{100} = 2533.3$

Substituting in the radius of 12 cm into the formula for the volume of a sphere, doing half of this to find the volume of the hemisphere then doing 70% of this finds that 70% of the bowl is 2533.3 cm^3. Percentage is out of 100 so putting the 70% over 100 converts it into a fraction, which when multiplied by finds 70%

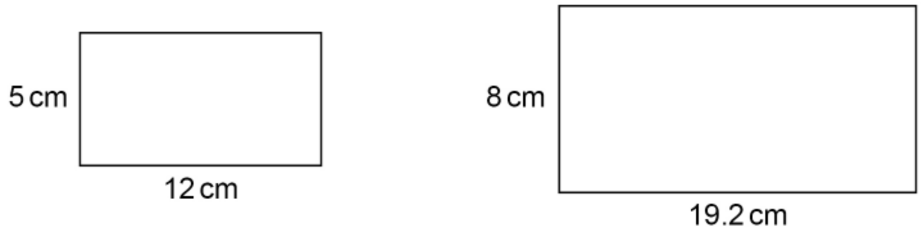
Yes

The 2600 cm^3 poured into the bowl is more than the 2533.3 cm^3 which is 70% of the bowl

12 Show that these two rectangles are similar.

[2 marks]

Not drawn accurately



$8 \div 5 = \frac{8}{5}$

$19.2 \div 12 = \frac{8}{5}$

Dividing the sides on the larger rectangle by the same sides on the smaller rectangle shows that the same scale factor has been used for all of the sides and therefore they must be similar

13 A factory packs x boxes of teabags per hour.

Each box contains 80 teabags.

Show that the factory packs $\frac{4x}{3}$ teabags per minute.

[2 marks]

$\frac{80x}{60} = \frac{4x}{3}$

Multiplying x by the 80 teabags in each box expresses how many teabags are packed per hour. There are 60 minutes in an hour so dividing this expression by 60 expresses how many teabags are packed per minute. This simplifies to 4x/3

Turn over for the next question

8

Do not write outside the box

14 A company has 123 employees.

Information about their hourly rates of pay is shown in the table.

Hourly rate, £p	**Number of employees**
$10 \leqslant p < 14$	66
$14 \leqslant p < 20$	32
$20 \leqslant p < 40$	15
$40 \leqslant p < 100$	10
	Total = 123

The owner of the company uses the data to make two statements.

Statement A

"Over 30% of employees have an hourly rate that is more than £17"

Statement B

"The average hourly rate of pay is more than £20"

14 (a) Show working that supports **Statement A**.

[3 marks]

$\frac{3}{6} \times 32$

Estimating that 3/6 of the second interval (14 ⩽ p < 20) is more than £17 as the class width is 6 and 17 is 3 from the end of the interval. Doing this fraction of the 32 to estimate that 16 employees have an hourly rate 17 < p < 20

$16 + 15 + 10$

Adding the estimated 16 employees who have an hourly rate 17 < p < 20 to the 15 who have hourly rate 20 ⩽ p < 40 and the 10 who have hourly rate 40 ⩽ p < 100 as all of these intervals are more than £17. This estimates that 41 employees have an hourly rate of more than £17

$\frac{41}{123} \times 100 = 33.\dot{3}\%$

Putting the estimated 41 employees who have an hourly rate more than £17 over the total 123 employees expresses the fraction estimated to have an hourly rate more than £17. Multiplying this by 100 converts it into a percentage, which is over 30%

Do not write outside the box

14 (b) Why might **Statement A not** be true?

[1 mark]

$15+10$

$\frac{25}{123} \times 100 = 20.3\ldots\%$

All of the employees with hourly rate $14 \leqslant p < 20$ could have hourly rates less than £17 so repeating the calculation done in (a) but with just $20 \leqslant p < 40$ and $40 \leqslant p < 100$ (which must both be more than £70) to show that the lowest possible percentage is not more than 30%

All of the employees with hourly rate $14 \leqslant p < 20$ could have hourly rates less than £17

14 (c) Work out an estimate of the mean to support **Statement B**.

[3 marks]

$$\frac{\frac{10+14}{2} \times 66 + \frac{14+20}{2} \times 32 + \frac{20+40}{2} \times 15 + \frac{40+100}{2} \times 10}{123} = 20.2$$

Doing the mean of the upper bound and the lower bound for each interval works out the midpoint of each interval. These means are found by adding the upper bound to the lower bound then dividing by 2. Multiplying the midpoints by the frequency of each interval works out an estimate for the total hourly rate of all of the employees in each interval. Adding all these estimates gives an estimate of the total hourly rate of all the employees. Dividing this by the 123 employees works out an estimate of the mean

14 (d) Why is the mean **not** the best average to represent the data?

[1 mark]

Most of the employees had an hourly rate less than £20

So the mean is not a very representative average. It is skewed higher because of the high midpoint of the last interval which goes up to 100

8

Turn over ►

.CG Maths.

Do not write outside the box

15 Expand $(x^2 - 9xy)(2x + 5y)$

[2 marks]

$x^2 \times 2x = 2x^3$
$x^2 \times 5y = 5x^2y$
$-9xy \times 2x = -18x^2y$
$-9xy \times 5y = -45xy^2$

Answer $2x^3 + 5x^2y - 18x^2y - 45xy^2$

There is no need to simplify by collecting the like terms

16 Line A

has equation $y = ax - 1$

passes through the point (7, 13)

Line B has equation $5y - 3x = 4$

Show that line A has a greater gradient than line B.

[3 marks]

The general equation of a straight line is y = mx + c, where m is the gradient and c is the y-intercept

$a = \frac{y+1}{x}$ Rearranged to make a the subject by adding 1 to both sides then dividing by x as a represents the gradient of line A

$= \frac{13+1}{7}$ Substituted in the x and y-coordinates of the point (7 , 13) as this is on the line so must satisfy the equation

$= 2$ a = 2 so the gradient of line A is 2

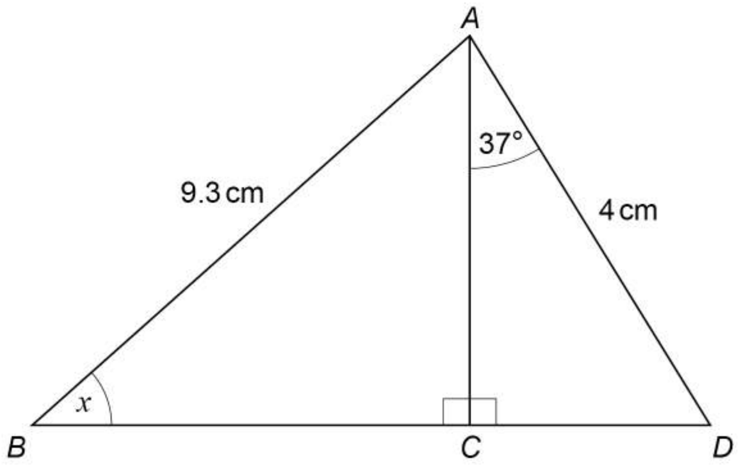
$5y = 3x + 4$ Adding 3x to both sides of the equation of line B to get the y term on its own

$y = 0.6x + 0.8$ Dividing all terms on both sides by 5 makes y the subject and therefore puts it into the form y = mx + c

Line A has a gradient of 2
Line B has a gradient of 0.6

Do not write outside the box

17



Not drawn accurately

Work out the size of angle x.

[4 marks]

S Ó H́ C Á H́ T O Á

Using right-angled trigonometry in triangle ACD to work out side AC.
Ticking H as side AD is the hypotenuse. Ticking A as side AC is the adjacent

$\cos 37 \times 4 = 3.1\ldots$ (A)

There are two ticks on the CAH formula triangle so this one can be used.
From the formula triangle, adjacent = cos of the angle x hypotenuse.
Storing the exact value as A on the calculator

S Ó H́ C A H́ T O A

Using right-angled trigonometry in triangle ABC to work out angle x.
Ticking O as side AC is the opposite. Ticking H as side AB is the hypotenuse

$\sin x = \frac{3.1\ldots}{9.3}$

There are two ticks on the SOH formula triangle so this one can be used.
From the formula triangle, sin of the angle = opposite/hypotenuse.
Using the exact value of side AC which was stored as A on the calculator

$x = \sin^{-1}\left(\frac{3.1\ldots}{9.3}\right)$

Rearranging to find x by doing $\sin^{-1}$ of both sides

$x =$ 20.1 °

9

Turn over ▶

Do not write outside the box

18 Rearrange $y = \dfrac{x+8}{x}$ to make x the subject.

[3 marks]

$xy = x + 8$ ← Multiplying both sides by x to eliminate it as the denominator

$xy - x = 8$ ← Subtracting x from both sides to get all the terms involving x on the same side

$x(y-1) = 8$ ← Factorising by bringing x out as a factor

Dividing both sides by (y - 1) to make x the subject

Answer $x = \dfrac{8}{y-1}$

Do not write outside the box

19 Here are the first four terms of a quadratic sequence.

3 20 47 84

Work out an expression for the nth term of the sequence.

[4 marks]

Quadratic sequences are in the form $an^2 + bn + c$

17 27

Listing out the first differences. The difference between 3 and 20 is 17 and the difference between 20 and 47 is 27

10

The second difference (the difference of the first differences) is 10 as this is the difference between 17 and 27

$5n^2 : 5, 20$

a is half of the second difference. Listing out the sequence of $5n^2$. n is 1 on the first term. $5(1)^2 = 5$. n is 2 on the second term. $5(2)^2 = 20$

$-2, 0 : 2n - 4$

Listing what needs to be added to each term of the $5n^2$ sequence to get the original sequence. 5 + -2 = 3 and 20 + 0 = 20. The sequence -2, 0 increases by 2 so must involve 2n. The 0th term (the one before the first term) would be -4 so the nth term of this sequence must be 2n - 4

Adding the sequence of 2n - 4 to the sequence of $5n^2$ gives the original sequence

Answer $5n^2 + 2n - 4$

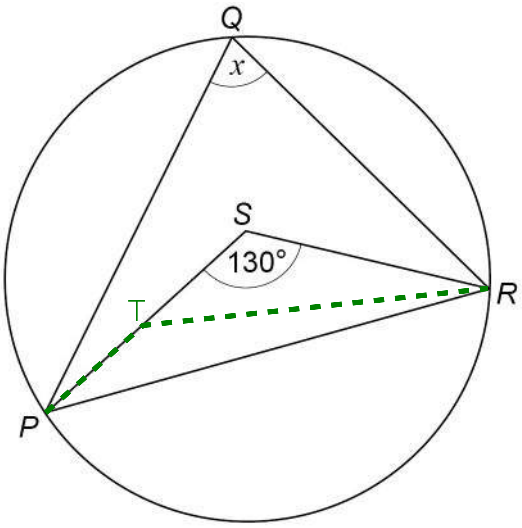
7

Do not write outside the box

20 (a) *P*, *Q* and *R* are points on a circle.

S is a point inside triangle *PQR*.

Not drawn accurately



Assume that *S* is the centre of the circle.

Work out the size of angle x.

[1 mark]

130 ÷ 2 ← The angle at the circumference is half the angle at the centre

x = 65 °

20 (b) In fact, the centre of the circle is on *PS* but **not** at *S*.

What does this mean about the size of angle x ?

Tick **one** box.

[1 mark]

☐ It is the same as the answer to part (a)

☑ It is greater than the answer to part (a)

☐ It is smaller than the answer to part (a)

☐ It is impossible to tell

Moving the centre of the circle toward P makes the angle greater than 130° and will therefore make angle x greater. Angle PTR looks greater than angle PSR

Do not write outside the box

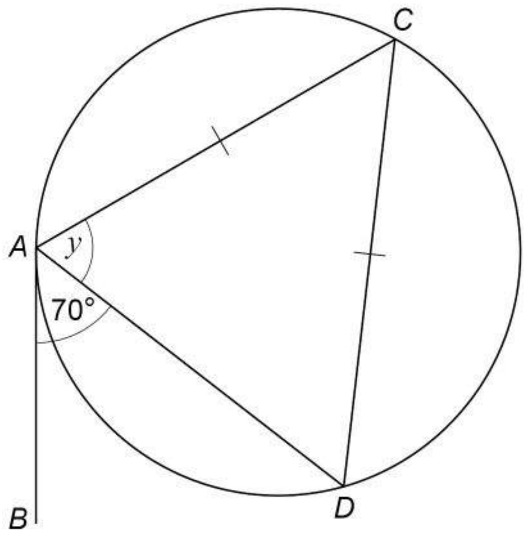
20 (c) For a different circle,

AB is a tangent at A

C and D are on the circumference of the circle

$AC = CD$

Not drawn accurately



Here is Simon's method to work out the size of angle y.

Angle $ADC = 70°$ (alternate segment theorem)

Therefore $y = 70°$ (angles in an isosceles triangle)

Is he correct?

Give a reason for your answer.

[1 mark]

No, he used the alternate segment theorem incorrectly

Using the alternate segment theorem shows that angle ACD = 70°, not angle ADC

3

Turn over ►

.CG Maths.

Do not write outside the box

21 Magana decides to put £500 into an account that pays compound interest.

She wants to have **at least** £560 in the account after 3 years.

Work out to 1 decimal place the **minimum** annual interest rate she needs.

[3 marks]

$$500\times\left(\frac{100+x}{100}\right)^3=560$$

Using the compound interest formula. The original amount is 100%. Adding x% (where x is the minimum annual interest rate) to this increases it by x%. Putting this over 100 converts the percentage to a fraction, which when multiplied by increases the £500 by x%. Raising the fraction to the power of 3 as it needs to be increased 3 times. This is set equal to £560 as this is the minimum amount she wants to have

$$\frac{100+x}{100}=\sqrt[3]{\frac{560}{500}}$$

Rearranging to find x. Dividing both sides by 500 then cube rooting

$$x=\sqrt[3]{\frac{560}{500}}\times100-100$$

Multiplying both sides by 100 then subtracting 100

Answer 3.9 %

The answer of 3.84... must be rounded up to 3.9% as 3.8% would not be enough

Do not write outside the box

22 An approximate value of a root of an equation, x, can be found using the iterative formula

$$x_{n+1} = \sqrt[3]{5(x_n)^2 - 2x_n - 3}$$

The starting value is $x_1 = 4$

22 (a) Work out the values of x_2 and x_3

[2 marks]

Enter 4 then press = (or EXE). Enter $\sqrt[3]{5Ans^2 - 2Ans - 3}$. Press = (or EXE) to get x_2. Press it again to get x_3

This sets 4 as x_1 then substitutes it in the right side of the iteration formula to get x_2. Then x_2 is substituted into the right side of the iteration formula to get x_3

$x_2 =$ 4.10

$x_3 =$ 4.18

22 (b) By continuing the iteration, show that the value of x is more than 4.25

[1 mark]

$x_4 = 4.234$

$x_5 = 4.276$

Carrying on from x_3, enter $\sqrt[3]{5Ans^2 - 2Ans - 3}$. Keep pressing = (or EXE)

This gives the values of x_4 and x_5 and so on. We can write down the next 2 after x_3 to show that it is rising and that it goes over 4.25

6

Turn over ▶

23 Here are three sets of cards.

Set A	1	1	3	5	5	5	6	8
Set B	1	2	4	6	8	8	9	
Set C	3	4	5	6				

In a game, a player has two options.

Option 1
Pick two cards from Set A

Option 2
Pick one card from Set B
and
pick one card from Set C

The cards are picked at random.

The player wins if the total of their two cards is exactly 10

Do not write outside the box

Which option gives a better chance of winning?

Option 1 ✓ Option 2

Show working to support your answer.

[4 marks]

① 5&5

Listing out the cards which can be picked for option 1 to give a total of 10

$\frac{3}{8} \times \frac{2}{7} \times 100 = 10.7\%$

The probability of the first 5 is 3/8 as 3 out of the 8 cards are a 5. The probability of the second 5 is 2/7 as there is one fewer 5 after the first pick and there is one fewer card in total. 'And' means to multiply. Multiplying by 100 converts the probability into a percentage, which makes it easier to compare

② 4&6 or 6&4

Listing out the cards which can be picked for option 2 to give a total of 10

$\left(\frac{1}{7} \times \frac{1}{4} + \frac{1}{7} \times \frac{1}{4}\right) \times 100 = 7.1\%$

The probability of picking a 4 for the first pick is 1/7 as 1 out of the 7 cards are a 4 in set B. The probability of picking a 6 on the second pick is 1/4 as 1 out of the 4 cards are a 6 in set C. The probabilities for the 6 then 4 are the same. 'And' means to multiply, 'or' means to add. Multiplying by 100 converts the probability into a percentage, which makes it easier to compare

10.7% is greater than 7.1%, so option 1 has a better chance of winning by getting a total of 10

Turn over for the next question

4

Do not write outside the box

24 $a = 65$ to the nearest integer

$b = 30$ to 1 significant figure

Work out the **upper bound** for $2a^2 - b^2$

You **must** show your working.

[3 marks]

$$2\left(65+\frac{1}{2}\right)^2 - \left(30-\frac{10}{2}\right)^2$$

Substituting in the upper bound of a and the lower bound of b (as a greater amount will be given if subtracting less). The upper bound of a is expressed by adding half of the resolution of a (which is 1 as it is to the nearest integer and integers go up in 1s). The lower bound of b is expressed by subtracting half of the resolution of b (which is 10 as the 1st significant figure is in the 10s place)

Answer 7955.5

Do not write outside the box

25 Show that $\frac{x-5}{x-2} + \frac{x+5}{x+2}$

simplifies to $\frac{ax^2-b}{x^2-4}$ where a and b are integers.

[3 marks]

$$\frac{(x-5)(x+2)}{(x+2)(x-2)} + \frac{(x+5)(x-2)}{(x+2)(x-2)}$$

To add fractions the denominators need to be the same. Multiplying both the numerator and denominator of the first fraction by (x + 2) and both the numerator and denominator of the second fraction by (x - 2)

$$\frac{x^2+2x-5x-10+x^2-2x+5x-10}{x^2-2x+2x-4}$$

Expanding all the brackets

$$\frac{2x^2-20}{x^2-4}$$

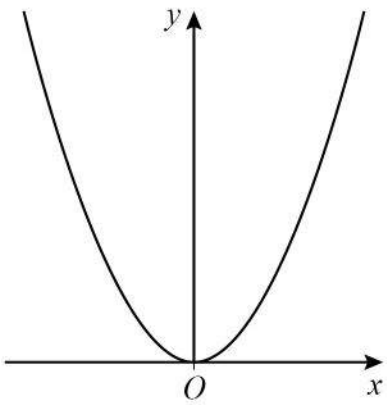
Simplifying by collecting like terms

Turn over for the next question

6

Do not write outside the box

26 Here is a sketch of $y = x^2$



26 (a) The minimum point of $y = x^2$ is at (0, 0)

Write down the coordinates of the minimum point of $y = x^2 + 2$

[1 mark]

2 has been added to the right side so y is increased by 2

Answer (0 , 2)

26 (b) The graph $y = x^2$ is reflected in the x axis.

Write down the equation of the graph after this transformation.

[1 mark]

Answer $y=-x^2$

Making the whole of the right side negative reflects it in the x axis

26 (c) $y = x^2$ is now transformed to give $y = (x + 3)^2$

Describe fully this single transformation.

[2 marks]

Translation 3 to the left

3 is added to x so this moves 3 to the left

END OF QUESTIONS

4